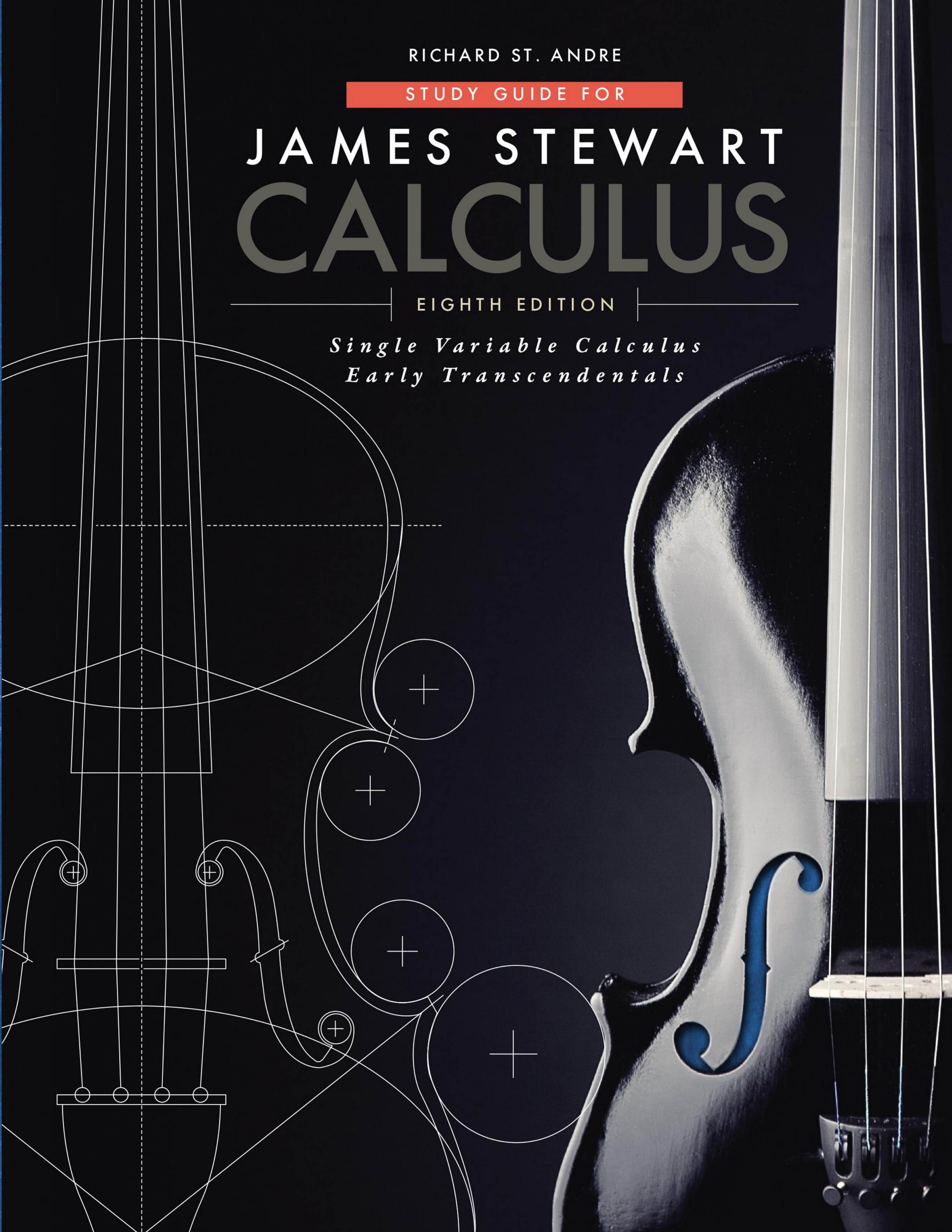
RICHARD ST. ANDRE

STUDY GUIDE FOR

JAMES STEWART CALCULUS

EIGHTH EDITION

Single Variable Calculus
Early Transcendentals



Study Guide

for

Stewart's

Single Variable Calculus

Early Transcendentals

Eighth Edition

Richard St. Andre
Central Michigan University

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Preface

This *Study Guide* is designed to supplement the first eleven chapters of *Calculus: Early Transcendentals*, 8th edition, by James Stewart. It may also be used with *Single Variable Calculus: Early Transcendentals,* 8th edition. If you later go on to multivariable calculus, you will want to obtain the multivariable volume of this *Study Guide.*

Your text is well written in a very complete and patient style. This *Study Guide* is not intended to replace it. You should read the relevant sections of the text and work problems, lots of problems. What this *Study Guide* does do is capture the main concepts and formulas of each section of your text and provide complete examples and short, concise questions that will help you understand the essential concepts and fundamental calculations. Every question has an explained answer. Some of our solutions begin with parenthetical comments offset by < . . . > and in italics to explain the approach to take to solve the problem. The two-column format allows you to cover the answer portion of a question while working on it and then uncover the given answer to check your solution. Working in this fashion leads to greater success than simply perusing the solutions. Students have found this *Study Guide* especially helpful when reviewing for examinations.

Technology, such as graphing calculators and computer algebra systems (CAS), can help the understanding of calculus concepts by drawing accurate graphs, solving or approximating solutions to equations, doing numerically intensive calculations, and performing symbolic manipulations. Although technology is not an emphasis, there are a few exercises that ask you to use it to help master calculus concepts.

As a quick check of your understanding of a chapter we have included a Practice Test of questions located at the end of the chapter. These are nearly 400 multiple choice-type questions—the kind you might see on an exam in a calculus class. You are "on your own" in the sense that an answer, but no detailed solution, is provided for each question.

I hope that you find this *Study Guide* helpful in understanding the concepts and completing the exercises in *Calculus: Early Transcendentals*, 8th edition.

Richard St. Andre

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Chapter 1 — Functions and Models

Section 1.1 Four Ways to Represent a Function

The concept of a function is central to all of calculus. This section defines what is a function and several concepts that go along with that definition.

Concepts and Calculations

A. Definition of a function; Function value; Domain; Range; Independent variable; Dependent variable

B. Four ways to describe a function; Tables of values; Graphs

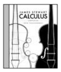
C. Implied domain

D. Piecewise defined function

E. Symmetry (even and odd functions)

F. Increasing; Decreasing

Summary and Focus Questions



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A. A function f is a rule that associates pairs of numbers: to each number x in a set (the **domain**) there is associated another real number, denoted $f(x)$, the **value** of f at x.

The set of all images (that is, the set of all $f(x)$ values) is the **range** of f. The variable x is the **independent variable** and $y = f(x)$ is the **dependent variable.** When we say "H is a function of t," that means H is the name of the function and we can write $H(t)$ for the values of the function.

EXAMPLE. The function S, "to each nonnegative number, associate its square root," has domain $[0, \infty)$ and rule of association given by:

$$S(x) = \sqrt{x}$$

The value of S at $x = 25$ is 5 because $S(25) = \sqrt{25} = 5$. S has no value at -7 because $\sqrt{-7}$ is not a real number. Another way to say this is "-7 is not in the domain of S." ■

EXAMPLE. The function T, given by the table at the right, has domain $\{-1, 1, 3, 5\}$ and range of $\{2, 4, 6\}$. One possible rule to describe T is $T(x) = |x| + 1$ for x in $\{-1, 1, 3, 5\}$.

x	y
-1	2
1	2
3	4
5	6

To determine whether a given graph is the graph of a function use this *Vertical Line Test*: If every vertical line passes through the graph in at most one point, the graph represents a function. In other words, no vertical line intersects the graph more than once. ■

1) Sometimes, Always, Never:
Both (2, 5) and (2, 7) can be pairs of a function f.

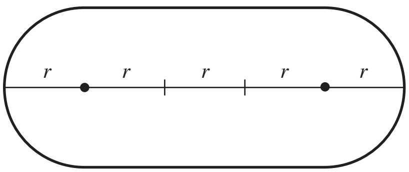
Never. $f(2)$ cannot be both 5 and 7.

2) True or False:
$x = y^2$ defines y as a function of x.

<*Look to see whether there is an x-value with multiple corresponding y-values.*>

False, because $(4, 2)$ and $(4, -2)$ satisfy the equation. Note that x is a function of y.

3) A track is in the shape of two semicircular ends and straight sides as in the figure. Find the distance around the track as a function of the radius.



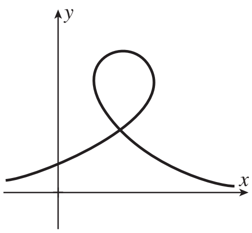
<*Determine the length of each part of the track in terms of r and add them up.*>

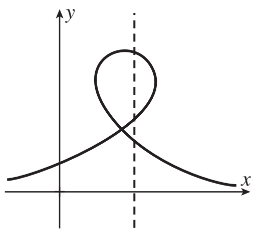
The length of each straight side is $3r$ and each semicircular end is πr. The distance d is $d(r) = 2(3r + \pi r) = (6 + 2\pi)r$.

4) Which of these is the graph of a function?

<*Apply the Vertical Line Test.*>

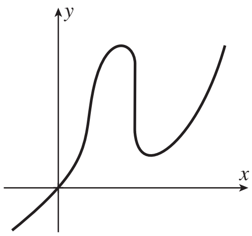
a)

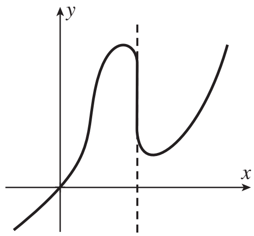




Not a function.

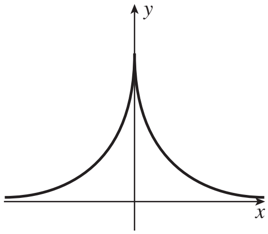
b)





Not a function.

c)



Is a function.

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B. The four ways to describe the rule of association for a function are

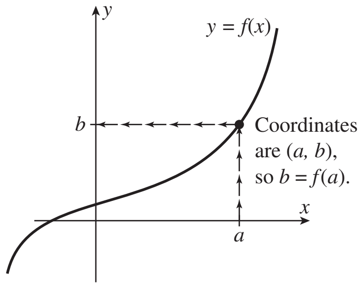
i. in words, a verbal description.

ii. a table of values listing domain values and corresponding range values.

iii. the **graph** of $y = f(x)$; that is, all points (x, y) in the Cartesian plane that make $y = f(x)$ true. The y-coordinate of a point on the graph of function f is the value of f associated with the x-coordinate.

iv. an algebraic formula or equation.

It is important to remember that if (a, b) is a point on the graph of $y = f(x)$, then $b = f(a)$. Conversely, if $f(c) = d$, then (c, d) is on the graph of f.



EXAMPLE. Here are four ways to represent the rule of association for the square root function.

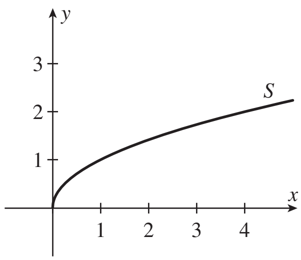
i. "to each nonnegative real number, associate its square root."

ii.

x	0	1	2	3	...
y	0	1	$\sqrt{2}$	$\sqrt{3}$	...

For this function with infinite domain, we can only list a few x-values with corresponding y-values and hope that the meaning is clear.

iii.



A graph will usually represent a portion of the function.

iv. $S(x) = \sqrt{x}$, where $x \geq 0$. ■

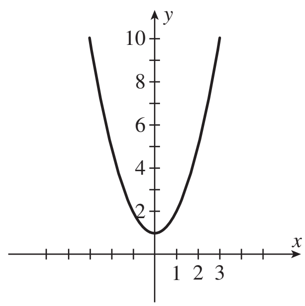
5) Write three other ways to describe each function:

a) "to each real number, associate one more than its square."

i. Partial list of values.

x	y
−3	10
−2	5
−1	2
0	1
1	2
2	5
3	10

ii. *<Plot points and draw a curve through them; it helps to recognize the graph is a parabola.>*



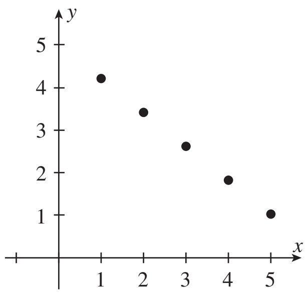
iii. $f(x) = x^2 + 1$, where x is a real number.

b)

x	1	2	3	4	5
y	5	4	3	2	1

i. "to each of $x = 1, 2, 3, 4, 5$ associate 6 minus the value of x."

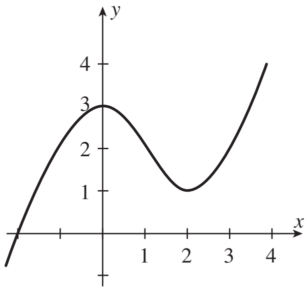
ii.



Note: Unless we know the domain is all real numbers, we should not "connect the dots" to form a line.

iii. For $x = 1, 2, 3, 4, 5$, let $f(x) = 6 - x$.

6) Given the graph of f below, find these values of f.



<Locate each x-value on the x-axis and use the curve to determine the corresponding y-value.>

a) $f(2)$

1 (because the pair (2, 1) is on the graph).

b) $f(0)$

3.

c) $f(3)$

2.

d) $f(f(0))$ (Hint: First determine $f(0)$.)

Because $f(0)$ is 3, $f(f(0)) = f(3) = 2$.

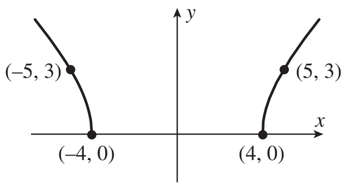
e) $f(7)$

7 is not pictured on the x-axis; we can only surmise from the trend of the graph that $f(7)$ will be a large positive number.

7) Sketch the graph of $f(x) = \sqrt{x^2 - 16}$.

<Plot points and recognize the equation as a hyperbola.>

Here are several computed values: (4, 0), (5, 3), (6, $\sqrt{20}$), (−4, 0), (−5, 3).
Squaring both sides of $y = \sqrt{x^2 - 16}$ gives $y^2 = x^2 - 16$, or $x^2 - y^2 = 16$. The graph is the top half of a hyperbola.



Note: About all we can do now to sketch graphs is plot points and recognize the general shape of certain graphs from the form of the equation. Calculus will help us draw better graphs.

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C. The domain of a function f, if unspecified, is understood to be the largest possible set of the real numbers x for which $f(x)$ exists.

EXAMPLE. $f(x) = \dfrac{1}{\sqrt{x-5}}$ has implied domain $(5, \infty)$ because $\sqrt{x-5}$ is defined only for $x \geq 5$. Because $\sqrt{x-5}$ cannot be zero, $x > 5$. The range of $f(x)$ is $(0, \infty)$. ■

8) Find the domain of $f(x) = \sqrt{x^2 - 16}$.

<*Look for where the square root can be calculated.*>

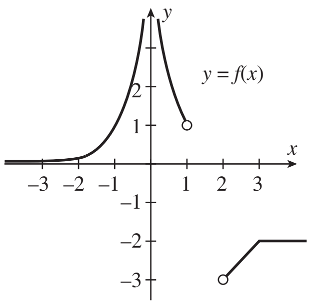
For $f(x)$ to exist, $x^2 - 16 \geq 0$. Thus $(x-4)(x+4) \geq 0$. The solution set is $x \leq -4$ or $x \geq 4$. The domain is $(-\infty, -4] \cup [4, \infty)$.

9) Find the domain of $f(x) = \dfrac{1}{|x| + 1}$.

<*Look for where the denominator is not zero.*>

Because $|x| \geq 0$ for all x, $|x| + 1$ is never 0. The domain is all real numbers.

10) What is the domain and range of the function whose graph is given below?



<*Describe all the x-values used by points on the graph and all the y-values used.*>

The domain appears to be $(-\infty, 0) \cup (0, 1) \cup (2, \infty)$. This is all the x-values for which there is a point (x, y) on the graph. The range is $(-3, -2] \cup (0, \infty)$—the set of all y-coordinates used in the graph.

11) Write a rule for a function whose unspecified domain is all real numbers except 3 and 6.

<*Make sure that $f(3)$ and $f(6)$ cannot be calculated.*>

One such function is

$f(x) = \dfrac{1}{(x-3)(x-6)}$. There are many other functions, such as

$g(x) = \dfrac{x^3}{(x-3)^2(6-x)}$.

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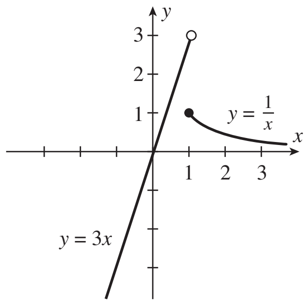
D. A **piecewise defined** function has its domain divided into disjoint (nonoverlapping) parts and uses a different formula on each part. For example:

$$f(x) = \begin{cases} 3x & \text{if } x < 1 \\ \dfrac{1}{x} & \text{if } x \geq 1 \end{cases}$$

has a domain of all real numbers split up into $(-\infty, 1)$ and $[1, \infty)$. Use the appropriate rule to draw the graph over each part.



12) The cell phone company charges a base rate of \$42.60 per month with an additional charge of \$0.09 per minute after 400 minutes. Express the monthly bill as a function of the number of minutes.

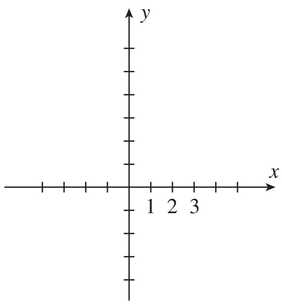
<Separately find the rates with less than and with more than 400 *minutes.>*

Let $x =$ the number of minutes. Then

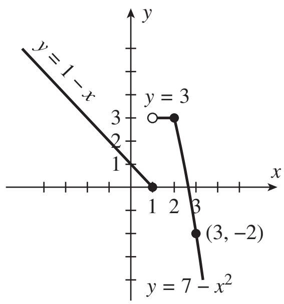
$$M(x) = \begin{cases} 42.60 & \text{if } x \in [0, 400] \\ 42.60 + 0.09(x - 400) & \text{if } x \in (400, \infty) \end{cases}$$

13) Sketch a graph of

$$f(x) = \begin{cases} 1 - x & \text{if } x \leq 1 \\ 3 & \text{if } 1 < x \leq 2 \\ 7 - x^2 & \text{if } x > 2 \end{cases}$$



<Sketch pieces of each of the three functions $y = 1 - x$, $y = 3$, *and* $y = 7 - x^2$.*>*



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E. Sometimes one part of a graph of a function will be a reflected image of another part:

$y = f(x)$ is **even** means $f(x) = f(-x)$ for all x in the domain.

An even function is symmetric about the y-axis—if the point (a, b) is on the graph of f, then so is the point $(-a, b)$.

$$y = f(x) \text{ is } \textbf{odd} \text{ means } f(x) = -f(-x) \text{ for all } x \text{ in the domain.}$$

An odd function is symmetric about the origin—if the point (a, b) is on the graph of f, then so is the point $(-a, -b)$.

14) If $(7, 3)$ is on the graph of an odd function, what other point must also be on the graph?

$(-7, -3)$. This is because $3 = f(7) = -f(-7)$, so $f(-7) = -3$.

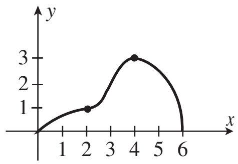
15) Is $f(x) = 10 - 2x$ even? odd?

<Determine whether the definitions of odd and even hold.>

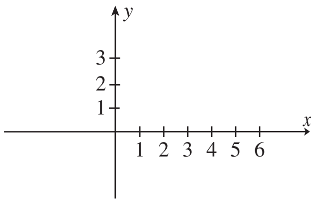
i. $f(x) = 10 - 2x$
$f(-x) = 10 - 2(-x) = 10 + 2x$
$f(x)$ is *not even* because $f(x) \neq f(-x)$.

ii. $-f(-x) = -(10 + 2x) = -10 - 2x$
$f(x)$ is *not odd* because $f(x) \neq -f(-x)$.

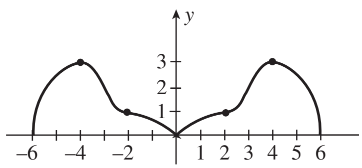
16) Complete the graph of $y = f(x)$ given below,



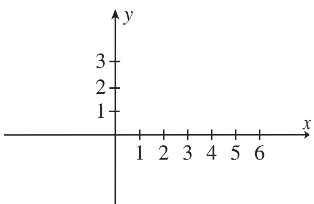
a) if f is even.



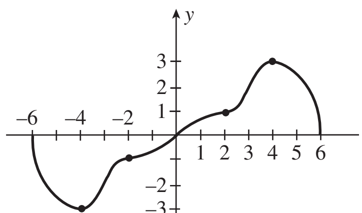
<Reflect the graph about the y-axis.>



b) if f is odd.



<Reflect the graph about the origin $(0, 0)$.*>*



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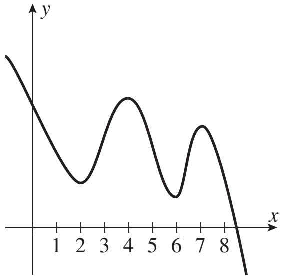
F. A function f is **increasing** on an interval I if for all $x_1, x_2 \in I$, $x_1 < x_2$ implies $f(x_1) < f(x_2)$. In other words, as x gets larger, so do the corresponding $f(x)$ values.

f is **decreasing** means that $x_1 < x_2$ implies $f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$. In other words, as x gets larger, $f(x)$ gets smaller.

17) Given the graph of f below, on what intervals is f increasing? decreasing?



<Look for intervals on the x-axis where the corresponding curve is going upward and intervals where it is going downward.>

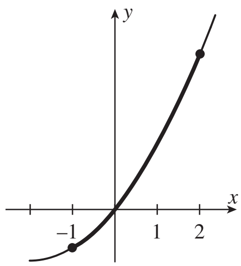
f is increasing on $[2, 4]$ and on $[6, 7]$.

f is decreasing on each of these intervals: $(-\infty, 2]$, $[4, 6]$, $[7, \infty)$.

18) Is $f(x) = x^2 + 4x$ increasing on $[-1, 2]$?

<Graph the function and observe whether it is increasing or decreasing.>

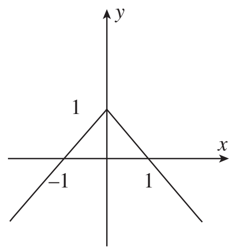
Yes. The graph of f is given below. As x increases from -1 to 2, $f(x)$ increases from -3 to 12.



19) Find where $f(x) = 1 - |x|$ is increasing and where it is decreasing.

<Sketch the graph.>

The graph of $f(x) = 1 - |x|$ is

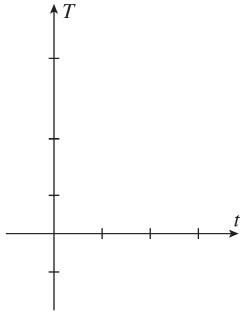


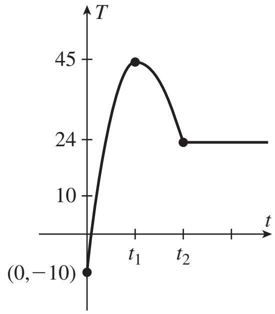
$f(x)$ is increasing on $(-\infty, 0]$ and decreasing on $[0, \infty)$.

20) Can a function be both increasing and decreasing on an interval?

No, not as we have defined the terms:
If $x_1 < x_2$, we cannot have both $f(x_1) < f(x_2)$ and $f(x_1) > f(x_2)$.

21) A pizza frozen at $-10°C$ is cooked in a microwave oven until it reaches 45°C, then allowed to cool to a room temperature of 24°C. Draw a graph of the temperature (T) as a function of time (t).





At $t = 0$, the pizza is frozen at $-10°C$.
At time t_1, the temperature reaches 45°C.
At t_2, the temperature reaches 24°C and is constant thereafter.

Section 1.2 Mathematical Models

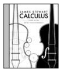
This section describes several types of functions that are useful in describing (modeling) real-world phenomenon.

Concepts and Calculations

A. Characteristics of these modeling functions: linear, quadratic, cubic, polynomial, power, root, rational, algebraic, trigonometric, exponential, logarithmic, and transcendental

B. Fit a math model to a real-world phenomenon; Interpolation; Extrapolation

Summary and Focus Questions



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A. A mathematical model is an abstraction of a real-world phenomenon into symbols and equations. Our models relate two variables with a function. For example, $C = \frac{5}{9}(F - 32)$ models the relationship between temperature measured in degrees Fahrenheit (F) and degrees Celsius (C). Thus, when the temperature is 50°F, the temperature is $C(50) = \frac{5}{9}(50 - 32) = 10$°C.

EXAMPLE. If a hardware store makes a profit of \$1.38 for each latch hasp it sells, then $P(x) = 1.38x$ models the profit on selling x hasps. ■

EXAMPLE. The radius (r) of a spherical balloon is related to the volume (V) of air in the balloon by $V = \frac{4}{3}\pi r^3$. The volume of a balloon with radius 3 inches is $V(3) = \frac{4}{3}\pi(3)^3 = 36\pi$ in.^{3} ■

Many types of functions can serve as models; some include:

Type	Description	Examples
Linear	$f(x) = mx + b$	$f(x) = 5x$ $f(x) = -\pi x + 3$
Quadratic	$f(x) = ax^2 + bx + c,\ a \neq 0$	$f(x) = 2x^2 - x + 7$
Cubic	$f(x) = ax^3 + bx^2 + cx + d,\ a \neq 0$	$f(x) = -3x^3 + 2x^2 - 5x + 1$
Polynomial	$f(x) = a_n x^n + \dots + a_1 x + a_0$ (n is the degree)	$f(x) = 6x^2 + 3x + 1$ $f(x) = -5x^{12} + 7x$
Power	$f(x) = x^n$ $n = 1, 2, 3, \dots$	$f(x) = x^7$ $f(x) = x\ (= x^1)$
Root	$f(x) = x^a$ where $a = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$	$f(x) = \sqrt{x}\ (= x^{1/2})$ $f(x) = x^{1/10}$

Rational	$f(x) = \frac{P(x)}{Q(x)}$, where P, Q are polynomials	$f(x) = \frac{7x+6}{x^2+10}$ $f(x) = \frac{3}{2-x}$
Algebraic	$f(x)$ is obtained from polynomials, using algebra ($+, -, \cdot, /$, roots)	$f(x) = \sqrt{16 - x^2}$ $f(x) = \sqrt[3]{\frac{x^3}{x+1}}$
Trigonometric	sine, cosine, tangent, cotangent, secant, cosecant	$f(x) = \tan x$ $f(x) = \cos 3x$
Exponential	$f(x) = a^x$ (for $a > 0$)	$f(x) = 2^x$ $f(x) = (1/3)^{x+1}$
Logarithmic	$f(x) = \log_a(x)$ (for $a > 0$)	$f(x) = \ln x$ $f(x) = \log_4(x+1)$

A special case of a rational function is the **reciprocal** function $f(x) = \frac{1}{x}$.

1) True or False:
$f(x) = x^3 + 6x + x^{-1}$ is a polynomial.

False (because of the x^{-1} term).

2) True or False:
$f(x) = x + \frac{1}{x}$ is a rational function.

<*Rewrite in the form* $\frac{P(x)}{Q(x)}$.>

True. $\left(x + \frac{1}{x} = \frac{x^2+1}{x}\right)$.

3) True or False:

a) Every rational function is an algebraic function.

True.

b) Every linear function is a polynomial.

True.

4) $f(x) = 6x^{10} + 12x^4 + x^{11}$ has degree ______.

<*Look for the largest exponent.*>

11.

5) True or False:
$f(x) = 2^{\pi}$ is an exponential function.

False. 2^{π} is a constant (approximately 8.825).

6) Which functions are not algebraic?

a) $f(x) = \sin 2x$

b) $g(x) = \sqrt{1-x}$

c) $h(x) = 2^x + x^2$

a) and c). (f is trigonometric and h has an exponential term.)

7) True or False:
$p(x) = 3$ is linear.

<*Determine if the function fits the form* $f(x) = mx + b$.>

True. ($m = 0$, $b = 3$)

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B. Success fitting models to given problems comes with practice and gaining insight into how variables and data values are related. Sometimes the graph of a phenomenon initially given in tabular data will look rather like a certain type of model. For example, if the data appears to be periodic, a trigonometric function will likely be involved. Something that seems to grow very rapidly may involve an exponential or a polynomial of a high degree.

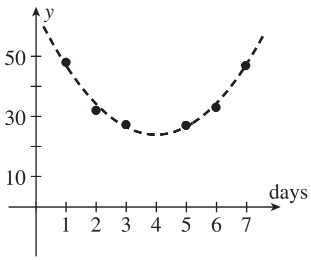
EXAMPLE. A tenant pays a non-refundable \$750 deposit upon leasing an apartment and \$1050 per month. Rent is paid in advance. A linear model for the total amount paid at the beginning of the xth month is:

$$C(x) = 750 + 1050x.$$

■

EXAMPLE. The amounts of sulfur dioxide particulates in the air around Rose City are given in the table at the right. A scatter plot of the data suggests that a model for the amount of particulates after x days is a quadratic (dashed curve is added).

Day	Parts per million
1	48
2	32
3	28
5	27
6	33
7	47



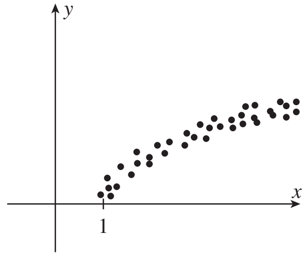
Using a calculator or software, a quadratic that best fits the data (quadratic regression) is $y = 2.55x^2 - 20.48x + 65.03$.

For the missing day, $x = 4$, we estimate that the amount of sulfur dioxide is $y(4) = 2.55(4)^2 - 20.48(4) + 65.03 \approx 23.91$ parts per million. Estimations for values of x much beyond day 7 would involve a great deal of uncertainty. ■

8) Which type of function model is associated with each scatter diagram?

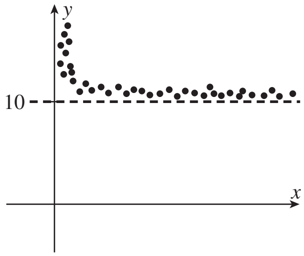
<Know the general shapes of the graphs of the various functions in this section.>

a)



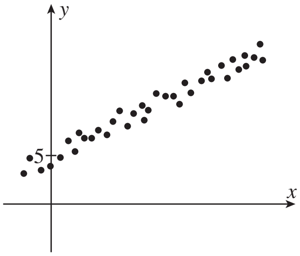
Logarithmic, such as $y = \log_2 x$, for $x \geq 1$.

b)



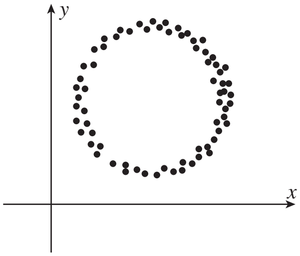
Rational, such as $y = 10 + \frac{1}{x} = \frac{10x + 1}{x}$, for $x > 0$.

c)



Linear, such as $y = \frac{1}{2}x + 5$.

d)

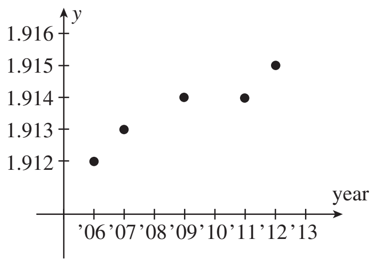


While there appears to be a relationship between x and y, there is no *functional* relationship between x and y.

9) To determine whether coins lose weight as they age (from factors such as wear from handling), the average weight of a penny was calculated for samples of 100 coins for five of the years between 2006 and 2012.

Year	Average weight in sample
2006	1.912
2007	1.913
2009	1.914
2011	1.914
2012	1.915

a) Draw a scatter diagram and determine whether the model seems linear.



The model appears to be rather linear.

b) Assuming a linear model, find the model that passes through the 2006 and 2012 data points.

<Find the slope and then the equation of the line.>

The slope of the line through (2006, 1.912) and (2012, 1.915) is

$$\frac{1.915 - 1.912}{2012 - 2006} = \frac{.003}{6} = .0005.$$

The equation is

$$y - 1.915 = .0005(x - 2012)$$

or

$$y = .0005x + .909.$$

c) Estimate the average weight of a 2008 penny and a 2016 penny.

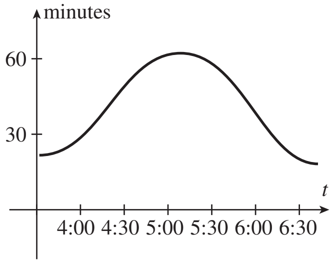
For $x = 2008$,
$y = .0005(2008) + .909 = 1.913$.
For $x = 2016$,
$y = .0005(2016) + .909 = 1.917$.

d) Does using the model to estimate the weight of a 2025 penny with the function in part **b)** seem reasonable?

No. Pennies in the future will probably have the same weight as new ones do today.

10) In light traffic it takes Karen about 20 minutes to get home from work. Sketch a graph of a model of the duration of her trip home from work (in minutes) as a function of the starting time of the trip—4:00, 4:30, 5:00, 5:30, 6:00, 6:30.

<It takes longer to get home during rush hour so the graph must show those trip starting times correspond to larger trip durations.>



We have assumed that it takes her about 20 minutes in light traffic at 4:00 and 6:30 and nearly an hour in heavy rush hour traffic to drive from work to home (5:00). The model is not linear.

Section 1.3 New Functions from Old Functions

This section describes several ways to combine functions to produce a function with a more complex expression, and conversely, writing a "complicated" function in terms of ones with simpler expressions. Knowing how to do this is essential for calculus.

Concepts and Calculations

A. Translations and stretchings of functions; Reflection

B. Combining functions (sum, difference, product, quotient, and composition)

Summary and Focus Questions

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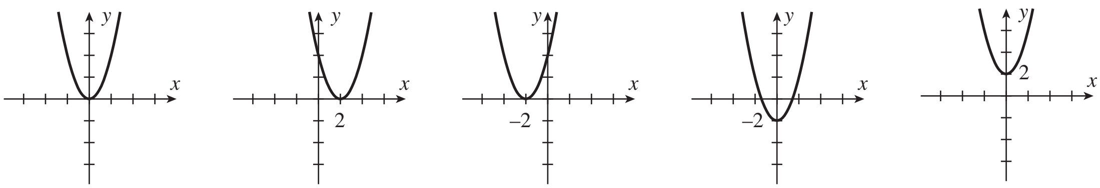
A. For $c > 0$, adding or subtracting the constant c to either the independent or dependent variable of $y = f(x)$ shifts the graph either left, right, up, or down.

Function	Graph of the resulting function
$y = f(x - c)$	Shift graph of $y = f(x)$ *right* by c units
$y = f(x + c)$	Shift graph of $y = f(x)$ *left* by c units
$y = f(x) + c$	Shift graph of $y = f(x)$ *upward* by c units
$y = f(x) - c$	Shift graph of $y = f(x)$ *downward* by c units

EXAMPLE. The function $f(x) = x^2$ is translated by 2 units:



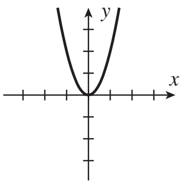
$y = f(x)$ $= x^2$ *original*	$y = f(x - 2)$ $= (x - 2)^2$ *shifted right*	$y = f(x + 2)$ $= (x + 2)^2$ *shifted left*	$y = f(x) - 2$ $= x^2 - 2$ *shifted down*	$y = f(x) + 2$ $= x^2 + 2$ *shifted up*

■

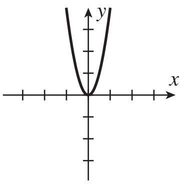
For $c > 1$, multiplying either the independent or dependent variable of $y = f(x)$ by the constant c will stretch or compress a graph either horizontally or vertically.

Function	Graph of the resulting function
$y = cf(x)$	Stretch *vertically* graph of $y = f(x)$ by a factor of c.
$y = \frac{1}{c}f(x)$	Compress *vertically* graph of $y = f(x)$ by a factor of c units.
$y = f(cx)$	Compress *horizontally* graph of $y = f(x)$ by a factor of c units.
$y = f\left(\frac{x}{c}\right)$	Stretch *horizontally* graph of $y = f(x)$ by a factor of c units.

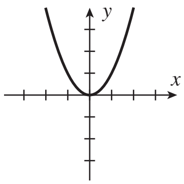
EXAMPLE. The function $f(x) = x^2$ is stretched/compressed by 2 units:



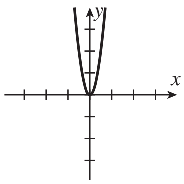
$y = f(x)$
$= x^2$
original



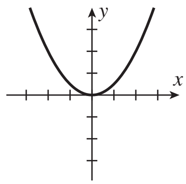
$y = 2f(x)$
$= 2x^2$
stretched vertically



$y = \frac{1}{2}f(x)$
$= \frac{1}{2}x^2$
compressed vertically



$y = f(2x)$
$= (2x)^2$
compressed horizontally



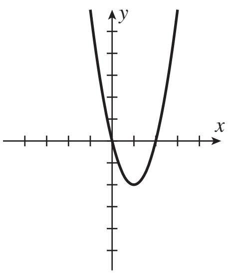
$y = f(\frac{1}{2}x)$
$= (\frac{1}{2}x)^2$
stretched horizontally

■

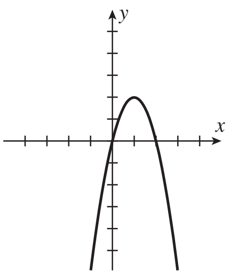
Changing the sign of either the independent or dependent variable of $y = f(x)$ will reflect the graph about an axis.

Function	Graph of the resulting function
$y = -f(x)$	Reflect graph of $y = f(x)$ about the x-axis.
$y = f(-x)$	Reflect graph of $y = f(x)$ about the y-axis.

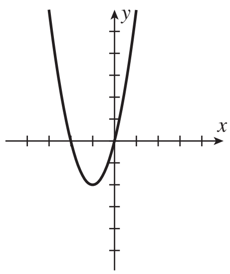
EXAMPLE. The function $f(x) = x^2 - 4x$ is reflected about each axis:



$y = f(x) = x^2 - 4x$
original



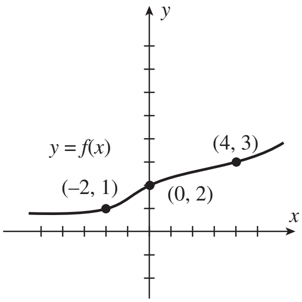
$y = -f(x) = -(x^2 - 4x)$
reflected about x-axis



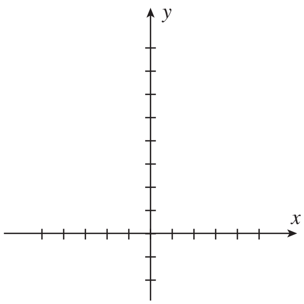
$y = f(-x) = (-x)^2 - 4(-x)$
reflected about y-axis

■

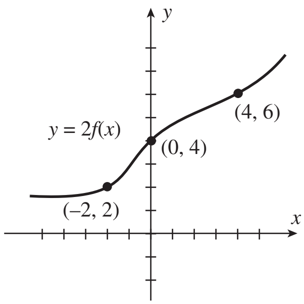
1) Given the graph below, sketch each:



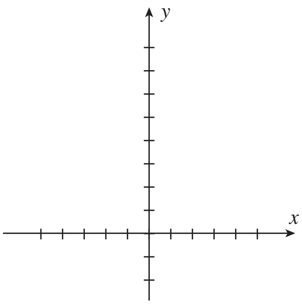
a) $y = 2f(x)$



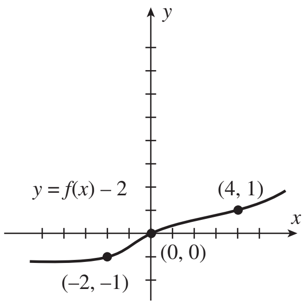
<Stretch in y-direction by a factor of 2.>



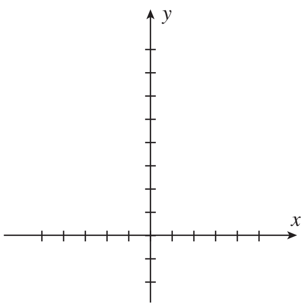
b) $y = f(x) - 2$



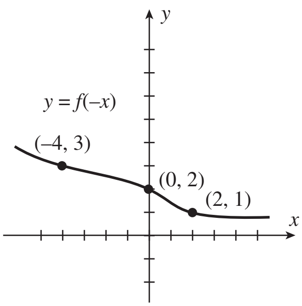
<Lower graph by 2 units.>



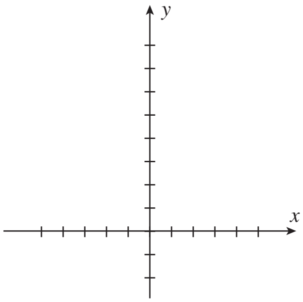
c) $y = f(-x)$



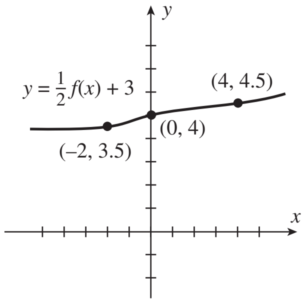
<Reflect graph about y-axis.>



d) $y = \frac{1}{2}f(x) + 3$



<Compress graph by half and then raise by 3 units.>



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B. Given two functions $f(x)$ and $g(x)$:

The **sum** $f + g$ associates to each x the number $f(x) + g(x)$.

The **difference** $f - g$ associates to each x the number $f(x) - g(x)$.

The **product** fg associates to each x the number $f(x) \cdot g(x)$.

The **quotient** $\frac{f}{g}$ associates to each x the number $\frac{f(x)}{g(x)}$, provided that $g(x) \neq 0$.

EXAMPLE. The sum, difference, product, and quotient of $f(x) = x^2$ and $g(x) = 2x - 8$ are:

$$(f + g)(x) = x^2 + 2x - 8$$
$$(f - g)(x) = x^2 - 2x + 8$$
$$fg(x) = x^2(2x - 8)$$
$$\frac{f}{g}(x) = \frac{x^2}{2x - 8}, \text{ for } x \neq 4.$$

■

It is important to recognize the components of a function so that it can be broken down into simpler functions.

EXAMPLE. The function $h(x) = \dfrac{x^2(x+1)}{2+x}$ may be written as $h = \dfrac{fg}{k}$, where

$$f(x) = x^2,$$
$$g(x) = x + 1,$$
$$\text{and } k(x) = 2 + x.$$

■

For $f(x)$ and $g(x)$, the **composition** of f and g, written $f \circ g$, is the function that associates to each x the same number that f associates to $g(x)$, that is:

$$(f \circ g)(x) = f(g(x))$$

To compute $(f \circ g)(x)$, first compute $g(x)$, then compute f of that result.

EXAMPLE. If $f(x) = x^2 + 1$ and $g(x) = 3x$, then:

For $x = 5$, $(f \circ g)(5) = f(g(5)) = f(3(5)) = f(15) = 15^2 + 1 = 226$.

For $x = -2$, $(f \circ g)(-2) = f(g(-2)) = f(-6) = (-6)^2 + 1 = 37$.
In general, $(f \circ g)(x) = f(g(x)) = f(3x) = (3x)^2 + 1$. ■

To determine the components f and g of a composite function, $h(x) = (f \circ g)(x)$, requires an examination of which function is applied first.

EXAMPLE. For $h(x) = \sqrt{x + 5}$, we may use $g(x) = x + 5$ (it is applied first, before the square root) and $f(x) = \sqrt{x}$. Then

$$h(x) = \sqrt{x + 5} = \sqrt{g(x)} = f(g(x)).$$ ■

2) Find $f + g$ and $\frac{f}{g}$ for $f(x) = 8 + x$, $g(x) = \sqrt{x}$. What is the domain of each?

$(f + g)(x) = 8 + x + \sqrt{x}$ with domain $= [0, \infty)$.

$\frac{f}{g}(x) = \frac{8 + x}{\sqrt{x}}$ with domain $= (0, \infty)$.

3) Write $f(x) = \dfrac{2\sqrt{x+1}}{x^2(x+3)}$ as a combination of simpler functions.

<*Look for simple components like $\sqrt{x + 1}$ and $x + 3$.*>

There are many answers to this question.
One is $f = \frac{hk}{mn}$ where $h(x) = 2$, $k(x) = \sqrt{x + 1}$, $m(x) = x^2$, $n(x) = x + 3$.

4) Find $f \circ g$ where:

a) $f(x) = x - 2x^2$ and $g(x) = x + 1$.

<*Use the definition of composition.*>

$$(f \circ g)(x) = f(g(x)) = f(x + 1) = (x + 1) - 2(x + 1)^2.$$

b) $f(x) = \sin x$ and $g(x) = \sin x$.

$(f \circ g)(x) = f(g(x)) = f(\sin x) = \sin(\sin x)$.
This is not the same as $\sin^2(x)$.

5) Rewrite $h(x) = (x^2 + 3x)^{2/3}$ as a composition, $f \circ g$.

<*Look for basic components; the expression $x^2 + 3x$ is one.*>

Let $f(x) = x^{2/3}$ and $g(x) = x^2 + 3x$.

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 3x) = (x^2 + 3x)^{2/3}.$$

Thus $h = f \circ g$.
There are other answers. This one is the best.

6) Find $f \circ f$ where $f(x) = x^2 + 2x$.

<*Use the definition of composition.*>

$$(f \circ f)(x) = f(f(x)) = f(x^2 + 2x) = (x^2 + 2x)^2 + 2(x^2 + 2x).$$

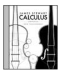
Section 1.4 Exponential Functions

An exponential function contains a variable in an exponent of the formula for the function. This section defines exponential functions and gives some of their properties. Exponential functions may be used to describe many types of phenomenon, including growth and decay of a population over time.

Concepts and Calculations

A. Definition, properties, and graph of an exponential function
B. Model growth and decay with an exponential function
C. Definition of e

Summary and Focus Questions



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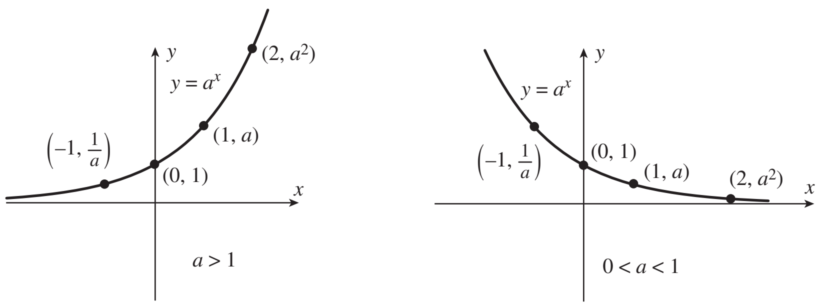
A. Let a be a positive constant. The **exponential function with base a** is $f(x) = a^x$ and is defined as follows:

Type of exponent x	Definition of a^x	Example
$x = n$, a positive integer	$a^n = a \cdot a \cdot a \ldots$ (n times)	$4^3 = 4 \cdot 4 \cdot 4$
$x = 0$	$a^0 = 1$	$5^0 = 1$
$x = -n$ (n, a positive integer)	$a^{-n} = \frac{1}{a^n}$	$3^{-2} = \frac{1}{3^2}$
$x = \frac{p}{q}$, a rational number	$a^{p/q} = \sqrt[q]{a^p}$	$5^{2/3} = \sqrt[3]{5^2}$
x, irrational	a^x is approximately a^r, where r is a rational number near a.	$3^{\sqrt{2}} \approx 3^{\frac{1414}{1000}} = 3^{1.414}$

The shape of the graph of $y = a^x$ depends on whether $a > 1$ or $0 < a < 1$:



Properties of exponentials include:

$a^{x+y} = a^x a^y$ $\qquad a^{x-y} = \dfrac{a^x}{a^y}$ $\qquad a^{xy} = (a^x)^y$ $\qquad (ab)^x = a^x b^x$

1) By definition, $2^{4/3} =$ ____ and $2^{-5} =$ ____.

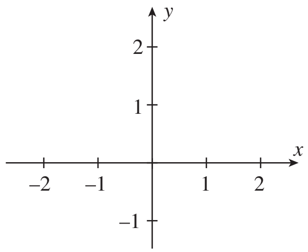
<*Use the definition of* a^x.>

$\sqrt[3]{2^4}$ and $\dfrac{1}{2^5}$.

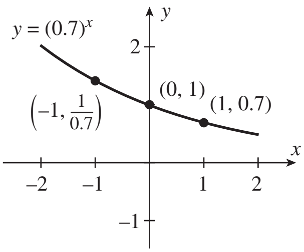
2) The rational number 1.72 is near $\sqrt{3}$. An approximation of $3^{\sqrt{3}}$ is ______.

$3^{\sqrt{3}} \approx 3^{1.72} \approx 6.617$.

3) Graph $y = (0.7)^x$.



<*Note that* $0.7 < 1$. *Be familiar with the shapes of exponential functions.*>



4) True or False:

a) $(x + y)^a = x^a + y^a$.

b) $x^{a-b+c} = \dfrac{x^a x^c}{x^b}$.

c) $(a^b)^c = a^{b^c}$.

<*Know the properties of exponentials.*>

False.

True.

False.

5) For large positive values of x, which is larger, $100x^4$ or $\dfrac{4^x}{100}$?

<*Know that an exponential function* a^x *(for* $a > 1$*) will always eventually have values larger than any given polynomial function.*>

For $x \geq 15$, $\dfrac{4^x}{100} > 100x^4$.

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B. Exponential functions can be used to model growth and decay of the number of individuals in a population as a function of time.

EXAMPLE. A tribble colony is known to double its size every 60 days. For an initial population of 100 tribbles, what is the population after 1 year (360 days)?

If $P(t)$ is the population after t days,

$$P(0) = 100$$
$$P(60) = 200 = 100 \cdot 2$$
$$P(120) = 400 = 100 \cdot 2^2$$

and in general $P(t) = 100 \cdot 2^{t/60}$.

Thus, $P(360) = 100 \cdot 2^{360/60} = 100 \cdot 2^6 = 6{,}400$. ■

6) The half life of a certain radioactive isotope is 100 years. What will be the mass of a 12 mg sample after 500 years?

<Find the formula for P(t); then use t = 500.>

Let $P(t)$ be the mass after t years. We are given $P(0) = 12$. $P(100) = 6 = 12 \cdot 2^{-1}$. $P(200) = 3 = 12 \cdot 2^{-2}$ and, in general, $P(t) = 12 \cdot 2^{-t/100}$.
$P(500) = 12 \cdot 2^{-500/100} = \frac{12}{2^5} = \frac{3}{8}$ mg.

7) The price of Macrosoft stock at the close of each day for a week is given in this table.

Mon	Tue	Wed	Thu	Fri
2.30	3.96	6.57	11.33	19.26

If x represents the number of days since Monday, which exponential model best describes the stock prices?

A) $y = 2.3^x$

B) $y = 2.3(1.7)^x$

C) $y = (2.3)^{-x}$

D) $y = (2.3)(1.7)^{-x}$

<Be familiar with the shapes of exponential functions.>

B) Here is a table of values for $y = 2.3(1.7)^x$:

x	0	1	2	3	4
y	2.30	3.91	6.65	11.30	19.21

Note: Functions C and D cannot be correct because they are decreasing. Function A grows too quickly.

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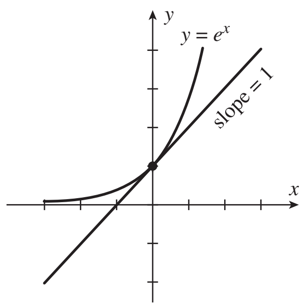
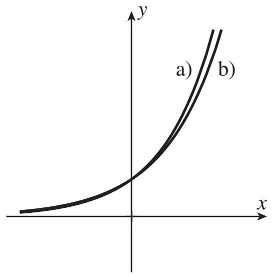
C. The number $\boldsymbol{e}$ is that unique real number for which the line tangent to $y = e^x$ at $(0, 1)$ has slope 1. Later we will see that the value of e, correct to five decimal places, is $e \approx 2.71828$.



8) For the graphs below, which is the graph of $y = e^x$ and which is $y = 3^x$?



<Be familiar with the shapes of exponential functions. The greater the base, the steeper the graph.>

Because $e < 3$, a) is the graph of $y = 3^x$ and b) is graph of $y = e^x$.

9) Write the following as e to a power: $\frac{(e^x \cdot e^3)^2}{e}$.

<Use laws of exponents.>

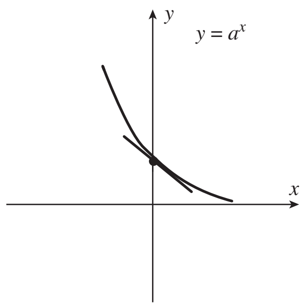
$$\frac{(e^x \cdot e^3)^2}{e} = \frac{(e^{x+3})^2}{e} = \frac{e^{2x+6}}{e} = e^{2x+6-1}$$
$$= e^{2x+5}.$$

10) If the slope of the tangent line to $y = a^x$ at $(0, 1)$ is 1, then $a =$ _____.

<Use the definition of e.>

e.

11) Slope of tangent line to the graph of $y = a^x$ at $(0, 1)$ is -1. What is a?



$a = e^{-1}$. This graph is a reflection of the graph of $f(x) = e^x$ about the y-axis. Hence $y = f(-x) = e^{-x} = (e^{-1})^x$.

Section 1.5 Inverse Functions and Logarithms

If y is a function of x described in any one of the several ways (such as by rule, by table of values, or by graph), then there are times when the same description may be used with the roles of x and y reversed, so that x is a function of y.

For example, if C is the circumference of a circle and r is the radius then $C = 2\pi r$ may be used to define C as a function of r. The same equation may be rewritten to define r as a function of C: $r = \frac{C}{2\pi}$. The two functions are inverses of each other.

This section defines inverses and sets conditions when the inverse is a function. For $a > 0$ and $a \neq 1$, $y = \log_a x$ is defined as the inverse of the exponential function $y = a^x$.

Concepts and Calculations

A. One-to-one functions; Horizontal Line Test
B. Inverse of a function
C. Definition, properties, and graph of $y = \log_a x$; Natural logarithm
D. Inverse trigonometric functions

Summary and Focus Questions

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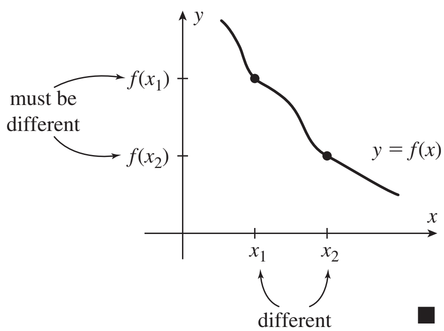
A. A function f is **one-to-one** means that for all x_1, x_2 in the domain of f,

$$\text{if } x_1 \neq x_2, \text{ then } f(x_1) \neq f(x_2).$$

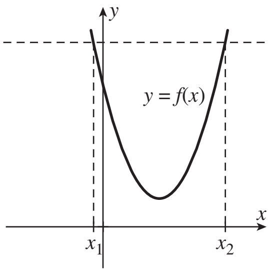


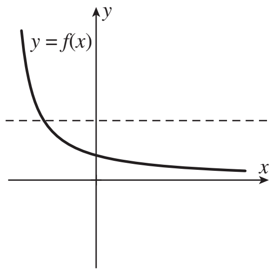
EXAMPLES. a) $f(x) = 2x + 5$ is one-to-one because if $x_1 \neq x_2$ then $2x_1 + 5 \neq 2x_2 + 5$.
b) The function $g(x) = x^2$ is not one-to-one. For instance, $4 \neq 4$, but both $g(4) = 16$ and $g(-4) = 16$.

Increasing and decreasing functions are one-to-one.

If you have the graph of a function, the **Horizontal Line Test** can help decide if the function is one-to-one:

A function f is one-to-one if no horizontal line intersects the graph of f more than once.





Horizontal Line Test fails:
$x_1 \neq x_2$, but $f(x_1) = f(x_2)$.
$y = f(x)$ is not one-to-one.

Horizontal Line Test succeeds:
$y = f(x)$ is one-to-one because no horizontal line intersects the graph more than once. ■

1) Is $f(x) = \sin x$ one-to-one?

<Determine whether two different x-values correspond to the same y-value.>

No. For example, $0 \neq \pi$, yet $\sin 0 = 0 = \sin \pi$.

2) Is the function $y = g(x)$ given by this table one-to-one?

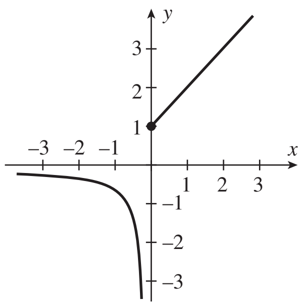
x	**g(x)**
1	10
2	9
3	8
4	7
5	8

No, $3 \neq 5$ but both $g(3)$ and $g(5)$ are 8.

3) True, False: If $y = f(x)$ is one-to-one, then $y = f(x)$ is either increasing or decreasing.

False. This function f is one-to-one but is neither increasing nor decreasing:

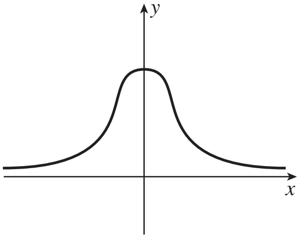
$$f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$$



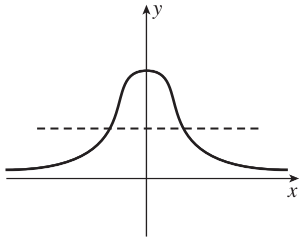
4) Is the given figure the graph of a one-to-one function?

<Pass horizontal lines through the graph. See if they touch the graph in more than one place.>

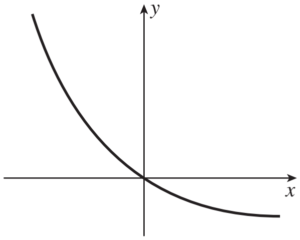
a)



The figure is the graph of a function but the function is not one-to-one.

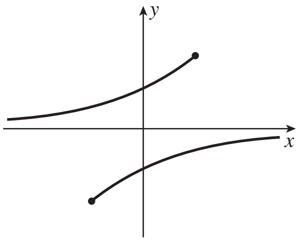


b)



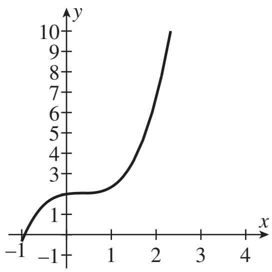
The figure is the graph of a one-to-one function.

c)

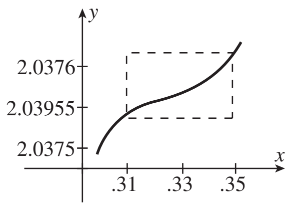


The figure is not the graph of a function and hence not the graph of a one-to-one function.

5) Graph $f(x) = x^3 - x^2 + 0.335x + 2$ using a calculator in the window $[-1, 3]$ by $[-1, 10]$. Is the function one-to-one?



In this window it is difficult to tell. Zooming in near $x = 0.33$ we see that f is indeed always increasing and hence one-to-one.



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B. If f is one-to-one with domain A and range B then the **inverse function** f^{-1} has domain B and range A, and

$$f^{-1}(b) = a \text{ if and only if } f(a) = b.$$

Therefore:

The pair (a, b) satisfies $y = f(x)$ if and only if the pair (b, a) satisfies $y = f^{-1}(x)$.
The pair (a, b) is in the table that defines f if and only if the pair (b, a) is in the table that defines f^{-1}.
The point (a, b) is on the graph of f if and only if the point (b, a) is on the graph of f^{-1}.

These inverse properties hold
$f^{-1}(f(x)) = x$ for all x in the domain of f.
$f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.

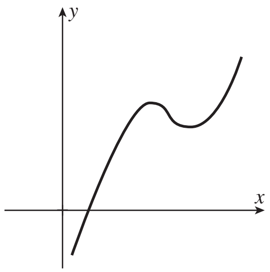
The steps for finding the rule for $y = f^{-1}(x)$ are

i. write $y = f(x)$
ii. solve the equation $y = f(x)$ for x
iii. interchange x and y.

EXAMPLE. Find the inverse of the function $f(x) = x^3 - 4$.

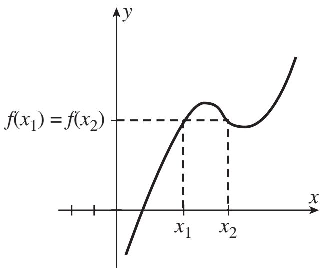
i. $y = x^3 - 4$.
ii. Solve $y = x^3 - 4$ for x:
$y = x^3 - 4$
$y + 4 = x^3$
$x = \sqrt[3]{y+4}$.
iii. Interchange x and y:
$y = \sqrt[3]{x+4}$, so $f^{-1}(x) = \sqrt[3]{x+4}$. ■

6) Is the graph below the graph of a function with an inverse function?



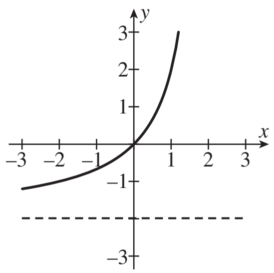
<*Use the Horizontal Line Test.*>

No. The function is not one-to-one.

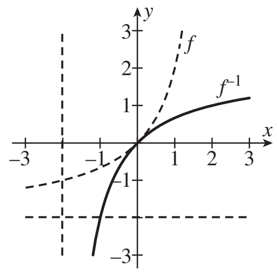


$x_1 \neq x_2$, but $f(x_1) = f(x_2)$. Therefore, the function does not have an inverse.

7) Given the following graph of f, sketch the graph of f^{-1} on the same axis.



<Reflect the graph about the line $y = x$.>



8) If f^{-1} is a function and $f(5) = 8$, then $f^{-1}(8) =$ ______.

<Use the definition of f^{-1}.>

5.

9) Find the inverse function of $f(x) = 6x + 30$.

i. $y = 6x + 30$.

ii. Solve for x:

$$y = 6x + 30$$
$$y - 30 = 6x$$
$$x = \tfrac{1}{6}y - 5$$

iii. Interchange y and x:

$y = \frac{1}{6}x - 5$, so $f^{-1}(x) = \frac{1}{6}x - 5$.

10) True, False:
If f is one-to-one with domain A and range B, then $f(f^{-1}(x)) = x$ for all $x \in A$.

False. $f(f^{-1}(x)) = x$ for all $x \in \underline{B}$.

11) Is $f^{-1}(x) = \dfrac{1}{f(x)}$?

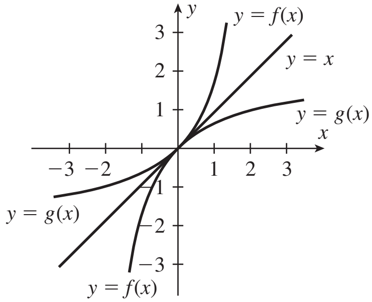
No.

12) Let $f(x) = x^3 + x$ and

$$g(x) = \sqrt[3]{\frac{x + \sqrt{x^2 + \frac{4}{27}}}{2}} + \sqrt[3]{\frac{x - \sqrt{x^2 + \frac{4}{27}}}{2}}.$$

Using a calculator or computer, draw the graphs of $y = f(x)$, $y = g(x)$, and $y = x$ on the same axes. What can you conclude about f and g?

From the graphs below, $g = f^{-1}$.



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C. The **logarithmic function** $y = \log_a x$ is defined as the inverse of the exponential $y = a^x$. For $a > 0$ and $a \neq 1$, $\log_a x = y$ means $a^y = x$.

This definition requires $x > 0$ since $a^y > 0$ for all y. You should think of $\log_a x$ as the exponent that you put on a to get x. Thus, the function $y = \log_a x$ has these properties:

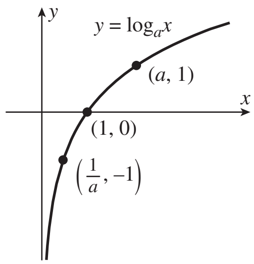
$\log_a(a^x) = x$, for all x.

$a^{\log_a x} = x$, for $x > 0$.

$\log_a(xy) = \log_a x + \log_a y$.

$\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$.

$\log_a x^c = c \log_a x$, for all c.



For $a > 1$, the graph of $y = \log_a x$ is given at the right.

In the special case where $a = e$, we use the notation $\mathbf{ln}\ \boldsymbol{x}$ to mean $\log_e x$. Thus, $\ln e = \log_e e = 1$ and $\log_a x = \dfrac{\ln x}{\ln a}$.

13) $\log_4 8 =$ ______.

<Use the definition of $\log_4 x$.>

Let $y = \log_4 8$.
Then $4^y = 8$
$2^{2y} = 2^3$
$2y = 3$
$y = \frac{3}{2}$.

14) True or False:

<Know the properties of logarithms.>

a) $\log_a x^3 = 3 \log_a x$.

True.

b) $\log_2(10) = \log_2(-5) + \log_2(-2)$.

False. $\log_2(-5)$ and $\log_2(-2)$ are not defined because -5 and -2 are negative (not in the domain of $\log_2$).

c) $\ln(a - b) = \ln a - \ln b$.

False.

d) $\ln e = 1$.

True. In $\log_e$ notation, this is $\log_e e = 1$.

e) $\ln\left(\frac{a}{b}\right) = -\ln\left(\frac{b}{a}\right)$.

True. $\ln\left(\frac{a}{b}\right) = \ln\left(\frac{b}{a}\right)^{-1} = -\ln\left(\frac{b}{a}\right)$.

f) $\log_2 0 = 1$.

False. $\log_2 0$ is not defined.

g) $\log_a(-x) = -\log_a x$

False. If $x < 0$, $\log_a x$ is not defined. If $x \geq 0$, $\log_a(-x)$ is not defined.

15) Write $2 \ln x + 3 \ln y$ as a single logarithm.

$2 \ln x + 3 \ln y = \ln x^2 + \ln y^3 = \ln(x^2y^3)$.

16) For $x > 0$, simplify using properties of logarithms: $\log_2\left(\frac{4x^3}{\sqrt{2}}\right)$.

$$\log_2\left(\frac{4x^3}{\sqrt{2}}\right) = \log_2 4x^3 - \log_2 \sqrt{2}$$
$$= \log_2 4 + \log_2 x^3 - \log_2 2^{1/2}$$
$$= \log_2 4 + 3 \log_2 x - \tfrac{1}{2} \log_2 2$$
$$= 2 + 3 \log_2 x - \tfrac{1}{2}(1)$$
$$= \tfrac{3}{2} + 3 \log_2 x.$$

17) Solve for x:

a) $e^{x+1} = 10$.

<Use the definition of ln to rewrite the equation and then solve for x.>
By definition $\log_e x = \ln x$, this means $\ln 10 = x + 1$, so $x = \ln 10 - 1$.

b) $\ln(x + 5) = 2$.

The equation is $\log_e (x + 5) = 2$, so $e^2 = x + 5$. Therefore, $x = e^2 - 5$.

c) $2^{3x} = 7$.

Take $\log_2$ of both sides and simplify:
$\log_2 2^{3x} = \log_2 7$
$3x(\log_2 2) = \log_2 7$
$3x = \log_2 7$
$x = \frac{1}{3} \log_2 7$.

18) $\log_2 10 = \frac{\ln\_\_\_\_}{\ln\_\_\_\_}$.

$$\log_2 10 = \frac{\ln 10}{\ln 2}$$

19) The magnitude of an earthquake on the Richter scale is $\log_{10}\left(\frac{I}{S}\right)$, where I is the intensity of the quake and S is the intensity of a "standard" quake. The Mexico City and San Francisco quakes were 6.4 and 7.1 on the Richter scale, respectively. How many times more powerful was the San Francisco quake than the Mexico City quake?

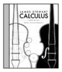
<Use properties of logarithms and algebra to simplify and solve.>
If the Mexico City quake intensity is I_M and the San Francisco quake intensity is I_F, we are given $\log_{10}\left(\frac{I_F}{S}\right) = 7.1$ and $\log_{10}\left(\frac{I_M}{S}\right) = 6.4$. We need to find a value k for which $I_F = k \cdot I_M$.

$$7.1 = \log_{10}\left(\frac{I_F}{S}\right) = \log_{10}\left(\frac{kI_M}{S}\right)$$
$$= \log_{10} k + \log_{10}\left(\frac{I_M}{S}\right)$$
$$= \log_{10} k + 6.4.$$

Thus $7.1 = \log_{10} k + 6.4$.

$\log_{10} k = 0.7$.

$k = 10^{0.7} \approx 5$ times as powerful.



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D. The table of values below shows that none of the sine, cosine, and tangent functions are one-to-one. However, when the domains are restricted to certain intervals (for the highlighted values), then the restricted functions are one-to-one and thus have inverses.

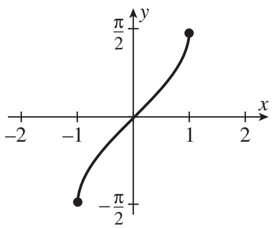
x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin x$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan x$	—	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	—	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Note that the numerators for sine and cosine in this table are always $\sqrt{0}$, $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, or $\sqrt{4}$.

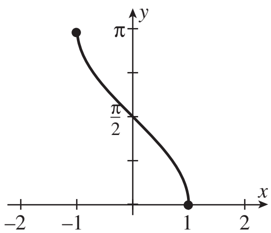
The domains and ranges of the six inverse trigonometric functions are:

Function	Domain	Range
$\sin^{-1}$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}$	all reals	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}$	all reals	$(0, \pi)$
$\sec^{-1}$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$
$\csc^{-1}$	$(-\infty, -1] \cup [1, \infty)$	$\left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$

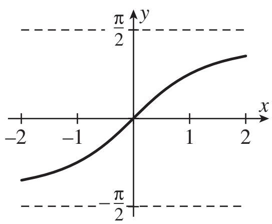
The first three functions are used frequently; their graphs are:



$y = \sin^{-1} x$



$y = \cos^{-1} x$



$y = \tan^{-1} x$

To calculate, for example, $\sin^{-1}\left(\frac{1}{2}\right)$, let $y = \sin^{-1}\left(\frac{1}{2}\right)$. Then $\sin y = \frac{1}{2}$. By checking the domain of $\sin^{-1}$, we see that $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, so $y = \frac{\pi}{6}$. Therefore, $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

EXAMPLES. Find a) $\tan^{-1}\sqrt{3}$ b) $\cos^{-1} 1.5$ c) $\csc^{-1} 2$

a) Let $\tan^{-1}\sqrt{3} = y$. Then $-\frac{\pi}{2} < y < \frac{\pi}{2}$ and $y = \sqrt{3}$. Therefore, $y = \frac{\pi}{3}$.

b) The value 1.5 is not in the domain of $\cos^{-1}$. Thus $\cos^{-1} 1.5$ does not exist.

c) Let $\csc^{-1} - 2 = y$. Then $\csc y = -2$, so $y = -\frac{1}{2}$. We are tempted to conclude that $y = -\frac{\pi}{6}$, but we must have $0 < y \le \frac{\pi}{2}$ or $\pi < y \le \frac{3\pi}{2}$.

Because $\sin \frac{7\pi}{6} = -\frac{1}{2}$, $\csc^{-1} - 2 = \frac{7\pi}{6}$. ■

20) True or False:

<*Use the definition of the inverse trigonometric function in each.*>

a) $\sin^{-1}\left(\frac{3}{2}\right)$ is not defined.

True. $\frac{3}{2}$ is not in the domain of $\sin^{-1}$.

b) For what x is $\tan^{-1} x = \frac{\pi}{2}$.

No such x exists because $\frac{\pi}{2}$ is not in the range of $\tan^{-1}$.

21) Determine each:

a) $\cos^{-1}\left(\frac{1}{2}\right)$

Let $y = \cos^{-1}\left(\frac{1}{2}\right)$. Then $0 \le y \le \pi$ and $\frac{1}{2} = \cos(y)$. From your knowledge of trigonometry, $y = \frac{\pi}{3}$.

b) $\tan^{-1}(-\sqrt{3})$

Let $y = \tan^{-1}(-\sqrt{3})$. Then $-\frac{\pi}{2} < y < \frac{\pi}{2}$ and $-\sqrt{3} = \tan y$. Thus $y = -\frac{\pi}{3}$.

c) $\sin^{-1}\left(\sin \frac{3\pi}{4}\right)$

Because $\sin \frac{3\pi}{4} = \frac{1}{\sqrt{2}}$, this problem asks you to find $y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$. Then $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $\sin y = \frac{1}{\sqrt{2}}$. Thus $y = \frac{\pi}{4}$.

d) $\sin^{-1}\left(\sin \frac{\pi}{3}\right)$

For $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, $\sin^{-1}(\sin x) = x$. Thus $\sin^{-1}\left(\sin \frac{\pi}{3}\right) = \frac{\pi}{3}$.

e) $\sec^{-1}\left(\sin \frac{\pi}{7}\right)$

This does not exist because $-1 \le \sin \frac{\pi}{7} \le 1$; $\sin \frac{\pi}{7}$ is not in the domain of $\sec^{-1}$.

22) Sometimes, Always, or Never:

a) $\cos(\cos^{-1} x) = x$

Sometimes. True for $-1 \leq x \leq 1$.

b) $\tan(\tan^{-1} x) = x$

Always.

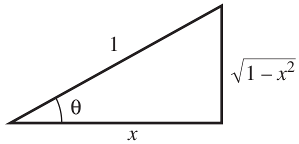
c) $\tan^{-1}(\tan x) = x$

Sometimes. True for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

23) For $0 < x < 1$, simplify $\tan(\cos^{-1} x)$.

<Drawing and labeling a right triangle will help.>

Let $\theta = \cos^{-1} x$. Then $0 < \theta < \frac{\pi}{2}$ because $0 < x < 1$.



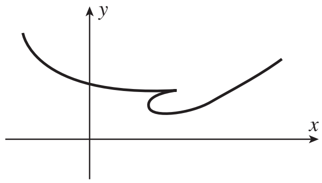
If $\cos\theta = x$ where $0 < \theta < \frac{\pi}{2}$ then $\theta = \cos^{-1} x$ and $\tan\theta = \frac{\sqrt{1-x^2}}{x}$.

Practice Test for Chapter 1

Section 1.1

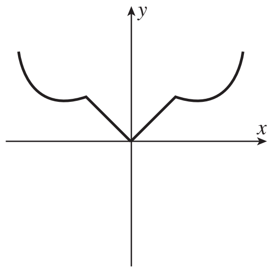
____ **1.** True or False:
$x^2 + 6x + 2y = 1$ defines y as a function of x.

____ **2.** True or False:
The graph at the right is the graph of a function.

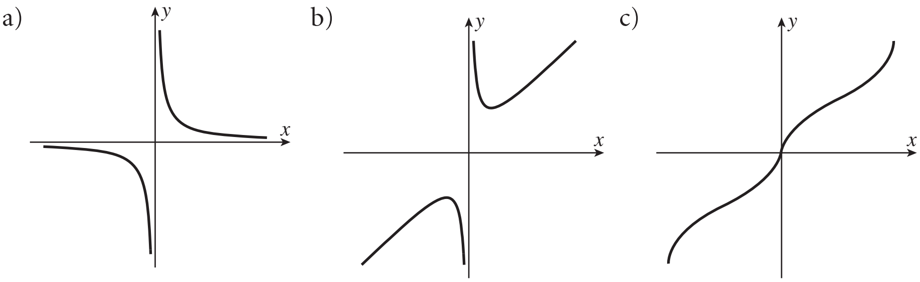


____ **3.** The implied domain of $f(x) = \frac{1}{\sqrt{1-x}}$ is:
a) $(1, \infty)$ b) $(-\infty, 1)$ c) $x \neq 1$ d) $(-1, 1)$

____ **4.** True or False:
The graph at the right is the graph of an even function.



____ **5.** Which graph represents $y = x + \frac{1}{x}$?



____ **6.** True or False:
$f(x) = x^2$ is decreasing for $-10 \leq x \leq -1$.

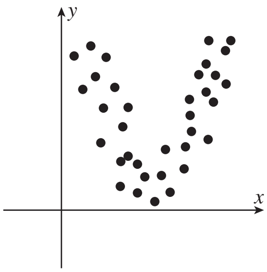
Section 1.2

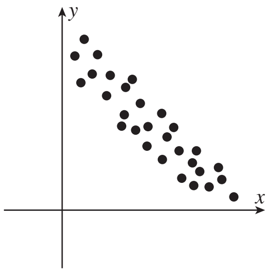
____ **1.** True or False:

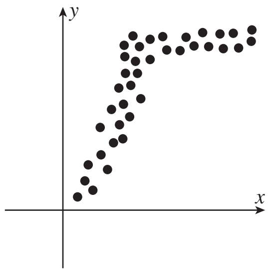
$$f(x) = \frac{x^2 + \sqrt{x}}{2x + 1} \text{ is a rational function.}$$

____ **2.** True or False:
The degree of $f(x) = 4x^3 + 7x^6 + 1$ is 7.

____ **3.** Which graph is best described by a linear model?

a) 

b) 

c) 

____ **4.** The best linear fit for the data at the right is $y = -3.4x + 39$.
An estimate for $x = 5$ is

a) 20 b) 22 c) 24 d) 25

x	y
1	36
2	32
3	28
4	26

Section 1.3

____ **1.** For $f(x) = 3x + 1$ and $g(x) = x$, $\dfrac{f}{g}(x) =$ _____?

a) $\dfrac{3 + x}{x}$ b) $\dfrac{x}{3x + 1}$ c) $\dfrac{3x + 1}{x}$ d) $\dfrac{3x^2 + 1}{x}$

____ **2.** For $f(x) = \sqrt{x^2 + x}$, we may write $f(x) = (h \circ g)(x)$, where:

a) $h(x) = \sqrt{x}$ and $g(x) = x^2 + x$ b) $h(x) = x^2 + x$ and $g(x) = \sqrt{x}$
c) $h(x) = x^2$ and $g(x) = \sqrt{x}$ d) $h(x) = x^2 + x$ and $g(x) = x^2$

____ **3.** Let $f(x) = 2 + \sqrt{x}$ and $g(x) = x + 3$. Then $(g \circ f)(x) =$ _____.

a) $2 + \sqrt{x} + 3$ b) $2 + \sqrt{x + 3}$ c) $3 + \sqrt{x + 2}$ d) $(x + 3)(2 + \sqrt{x})$

____ **4.** For $f(x) = x^2$ and $g(x) = 2x + 1$, $(f \circ g)(x) =$

a) $2x^2 + 1$ b) $(2x)^2 + 1$ c) $(2x + 1)^2$ d) $x^2(2x + 1)$

____ **5.** For $f(x) = \cos(x^2 + 1)$ we may write $f(x) = (h \circ g)(x)$, where:

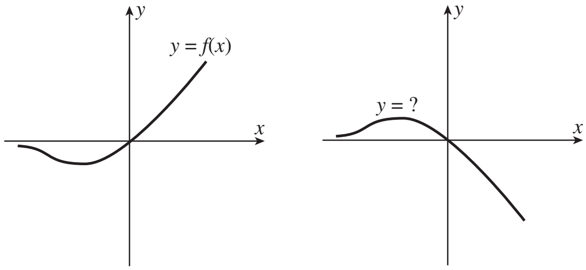
a) $h(x) = \cos x^2$ and $g(x) = x + 1$ b) $h(x) = \cos x$ and $g(x) = x^2 + 1$
c) $h(x) = x^2 + 1$ and $g(x) = \cos x$ d) $h(x) = x^2$ and $g(x) = \cos(x + 1)$

____ **6.** The graph of $y = f(3x)$ is obtained from the graph of $y = f(x)$ by:

a) stretching vertically by a factor of 3.
b) compressing vertically by a factor of 3.
c) stretching horizontally by a factor of 3.
d) compressing horizontally by a factor of 3.

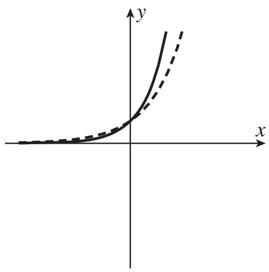
____ **7.** Given the graph of $y = f(x)$, the graph to its right is the graph of:

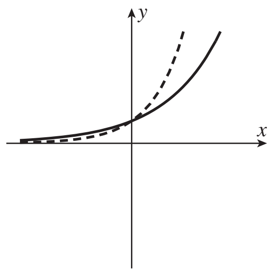
a) $y = -f(x)$
b) $y = f(-x)$
c) $y = -f(-x)$
d) $y = f(|x|)$

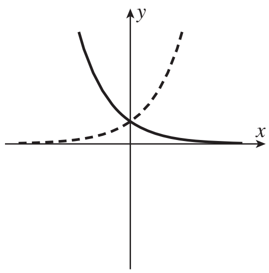


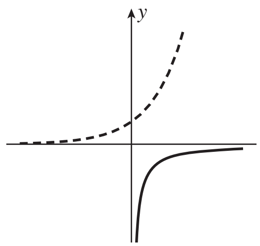
Section 1.4

____ **1.** Which is the best graph of $y = 1.5^x$? (The graph of $y = 2^x$ is shown as a dashed curve for comparison.)

a)


b)


c)


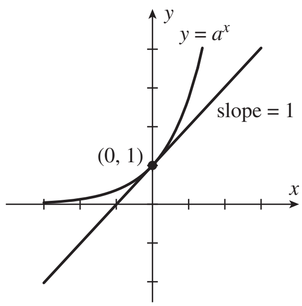
d)


____ **2.** A mosquito population of 100 grows to 500 after two weeks. If the population follows an exponential growth model, how many mosquitoes are there after 5 weeks?

a) 1069 b) 1100 c) 5590 d) 31250

____ **3.** Which number is the best choice for the value of a in the following graph?

a) e
b) 1
c) 2
d) 3



____ **4.** $e^x(e^x)^2 =$

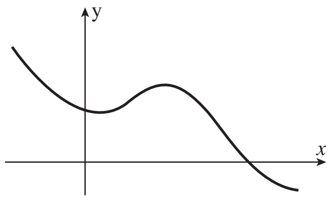
a) e^{x^3} b) e^{3x} c) $3e^x$ d) e^{4x}

Section 1.5

____ **1.** A function f is one-to-one means:

a) if $x_1 = x_2$, then $f(x_1) = f(x_2)$
b) if $x_1 \neq x_2$, then $f(x_1) = f(x_2)$
c) if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$
d) if $f(x_1) \neq f(x_2)$, then $x_1 \neq x_2$

____ **2.** True or False:
The function graphed at the right is one-to-one.



____ **3.** If $f(x) = \sqrt[3]{x + 3}$, then $f^{-1}(x) =$

a) $\dfrac{1}{\sqrt[3]{x + 3}}$ b) $x^3 + 3$ c) $(x + 3)^3$ d) $x^3 - 3$

____ **4.** Sometime, Always, or Never:
If f is one-to-one and the point (a, b) is on the graph of $y = f(x)$ then (b, a) is on the graph of $y = f^{-1}(x)$.

____ **5.** $\log_{27} 9 =$

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{3}{2}$ d) $\frac{1}{2}$

____ **6.** True or False:
$\ln(a + b) = \ln a + \ln b$.

____ **7.** Simplified, $\log_3 9x^3$ is:

a) $2 + 3 \log_3 x$ b) $6 \log_3 x$ c) $9 \log_3 x$ d) $9 + \log_3 x$

____ **8.** Solve for x: $e^{2x-1} = 10$.

a) $\frac{1}{2}(1 + 10^e)$ b) $3 + \ln 10$ c) $2 + \ln 10$ d) $\frac{1 + \ln 10}{2}$

____ **9.** Solve for x: $\ln(e + x) = 1$.

a) 0 b) 1 c) $-e$ d) $e^e - e$

Answers

Section 1.1
1. True
2. False
3. b
4. True
5. b
6. True

Section 1.2
1. False
2. False
3. b
4. b

Section 1.3
1. c
2. a
3. a
4. c
5. b
6. d
7. a

Section 1.4
1. b
2. c
3. a
4. b

Section 1.5
1. c
2. False
3. d
4. Always
5. b
6. False
7. a
8. d
9. a

Chapter 2 — Limits and Derivatives

Section 2.1 The Tangent and Velocity Problems

This section illustrates two types of problems solved using calculus:

- finding the slope of a line tangent to a curve, and
- finding the velocity of a moving object.

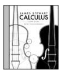
These are two examples of instantaneous rate of change that will be calculated as a "limit" of average rates of change.

Concepts and Calculations

A. Slope of secant line to the graph of a function; Slope of tangent line

B. Interpretation of slope as instantaneous velocity

Summary and Focus Questions



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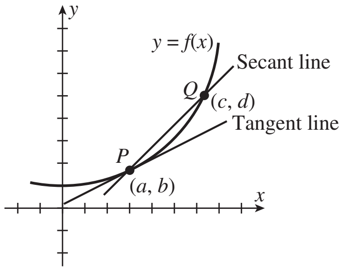

A. A **secant line** at $P(a, b)$ for the graph of $y = f(x)$ is the line through P and another point Q also on the graph. If Q has coordinates (c, d), then the **slope of the secant line** is

$$\frac{d-b}{c-a}.$$

EXAMPLE. For $f(x) = x^3$, the secant line through

$P(2, 8)$ and $Q(3, 27)$ has slope $\dfrac{27-8}{3-2} = 19$.

$P(2, 8)$ and $Q(2.5, 15.625)$ has slope $\dfrac{15.625-8}{2.5-2} = 15.25$.

$P(2, 8)$ and $Q(2.1, 9.261)$ has slope $\dfrac{9.261-8}{2.1-2} = 12.61$.

$P(2, 8)$ and $Q(2.01, 8.1206)$ has slope $\dfrac{8.1206-8}{2.01-2} = 12.06$. ■

A **tangent line** to a graph of a function touches the graph at a point much like a tangent line to a circle touches the circle at a point.

To find the slope of the tangent line at P, we repeatedly select points Q closer and closer to P; the slopes of the secant lines become better and better estimates of the slope of the tangent line at P. Finally, our guess for the slope of the tangent line is the number that the slopes of the secant lines are approaching as points Q get closer and closer to P.

EXAMPLE. For $f(x) = x^3$ and $P(2, 8)$, the slopes of the secant lines in the above example are 19, 15.25, 12.61, 12.06 as Q gets closer to P. We estimate the slope of the tangent line to f at P to be 12. ■

Once we have determined the slope m of the tangent line to the curve, then using the point P as a point on the line, the **equation of the tangent line** is

$$y - b = m(x - a).$$

1) Let P be the point (3, 10) on the graph of $y = f(x)$. Each point Q in the table below is also on the graph of f. Find the slopes of each secant line PQ.

Q	Slope of PQ
(6, 18)	
(5, 15)	
(4, 12.3)	
(3.5, 11.1)	
(3.1, 10.21)	

<Calculate $\frac{d-b}{c-a}$ for each point Q.>

Q	Slope of PQ
(6, 18)	$\frac{18-10}{6-3} = 2.67$
(5, 15)	$\frac{15-10}{5-3} = 2.5$
(4, 12.3)	$\frac{12.3-10}{4-3} = 2.3$
(3.5, 11.1)	$\frac{11.1-10}{3.5-3} = 2.2$
(3.1, 10.21)	$\frac{10.21-10}{3.1-3} = 2.1$

2) What is your guess for the slope of the tangent line to $y = f(x)$ at $x = 3$ in question 1?

The slopes of the secant lines appear to be approaching 2.

3) Let $P(1, 5)$ be a point on the graph of $f(x) = 6x - x^2$. Let $Q(x, f(x))$ be on the graph. Find the slope of the secant line PQ for each given x value for Q.

x	$f(x)$	Slope of PQ
3		
2		
1.5		
1.01		

x	$f(x)$	Slope of PQ
3	9	$\frac{9-5}{3-1} = 2$
2	8	$\frac{8-5}{2-1} = 3$
1.5	6.75	$\frac{6.75-5}{1.5-1} = 3.5$
1.01	5.0399	$\frac{5.0399-5}{1.01-1} = 3.99$

4) Use your answer to question 3 to guess the slope of the tangent to $f(x)$ at P.

The values appear to be approaching 4.

5) a) What is the equation of the tangent line to $f(x)$ at P in question 1?

<Use the coordinates of P and the slope (2) *from question 2 in the point slope form of the equation.>*

$y - 10 = 2(x - 3)$

b) What is the equation of the tangent line to $f(x)$ at P in question 3?

$y - 5 = 4(x - 1)$.

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B. Suppose an object is moving along an axis (a number line). If $f(x)$ represents the distance the object is located from its initial starting point at time x, then

the slope of the secant line between points P and Q is the **average velocity** of the object as it travels from P to Q.

the slope of the tangent line at the point P is the **instantaneous velocity** of the object at P.

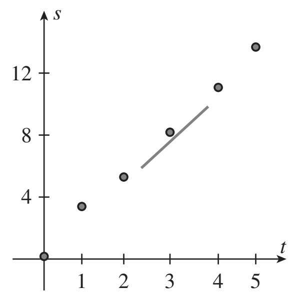
Thus, given a distance function:

To find an average velocity over an interval of time, calculate the slope of a secant line.

To find an instantaneous velocity at a particular time, calculate the slope of a tangent line.

EXAMPLE. The table below shows the position of a car starting from rest at a traffic light. Find the average velocity in the time interval [2, 4]. Estimate the instantaneous velocity at $t = 3$ from the graph of s

t (seconds)	0	1	2	3	4	5
s (meters)	0	3	5	8	10	13



The average velocity over [2, 4] is $\dfrac{10 - 5}{4 - 2} = \dfrac{5}{2} = 2.5$ m/s.

From the graph, the slope of the tangent line appears to be about 2. Thus the instantaneous velocity is about 2 m/s. ■

6) Let $f(x) = x^2 + 3x$ be the distance in feet a race car has traveled from its starting point after x seconds.

a) How far has the race car traveled after 2 seconds? After 4 seconds?

<Calculate function values.>

At $x = 2, f(2) = 2^2 + 3(2) = 10$ ft.
At $x = 4, f(4) = 4^2 + 3(4) = 28$ ft.

b) What is the average velocity over the time interval $x = 2$ to $x = 4$?

<Calculate the slope of the secant line.>

From part a), $f(2) = 10$ and $f(4) = 28$.

$$\frac{f(4) - f(2)}{4 - 2} = \frac{28 - 10}{4 - 2} = \frac{18}{2} = 9 \text{ ft/sec}.$$

c) What is the instantaneous velocity when $x = 2$?

<Calculate the slopes of several secant lines.>

Choose x-values approaching 2:

x	$f(x)$	Slope of Secant
3	18	$\frac{18-10}{3-2} = 8$
2.5	13.75	$\frac{13.75-10}{2.5-2} = 7.5$
2.1	10.71	$\frac{10.71-10}{2.1-2} = 7.1$

We guess the slope of the tangent line, and thus the instantaneous velocity, is 7 ft/sec.

7) Let $f(x) = mx + b$ be a linear position function. Sometimes, Always, or Never:

The average velocity for $f(x)$ from P to Q is the same as the instantaneous velocity at P.

Always. The tangent line and all secant lines are the same line: $y = mx + b$.

8) The distance that a runner is from the starting point after t seconds is given in this table:

t (seconds)	0	1	2	3	4
d (meters)	0	4	9	16	24

a) Find the average velocity over the time intervals [1, 4], [1, 3], [1, 2], and [0, 1].

Interval	Average Velocity
[1, 4]	$\frac{24-4}{4-1} = \frac{20}{3} = 6.67$ m/s
[1, 3]	$\frac{16-4}{3-1} = \frac{12}{2} = 6$ m/s
[1, 2]	$\frac{9-4}{2-1} = \frac{5}{1} = 5$ m/s
[0, 1]	$\frac{4-0}{1-0} = \frac{4}{1} = 4$ m/s

b) Estimate the instantaneous velocity at $t = 1$.

The intervals [0, 1] and [1, 2] give average velocities of 4 m/s and 5 m/s, respectively. We estimate the instantaneous velocity at $t = 1$ to be about 4.5 m/s, the average of the two average velocities.

Section 2.2 The Limit of a Function

This section gives an intuitive description of the concept of the limit of a function $y = f(x)$ at a point a—the number around which the values of $f(x)$ cluster for x-values near a. We will estimate the value of a limit by calculating several functional values or by observing trends in a graph. We will also look at limits from just one side of the limiting value—considering functional values only for x less than a, or only for x greater than a. Finally, we will see that some limits do not exist because the functional values get larger and larger without bound.

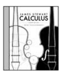
Concepts and Calculations

A. Limit of a function; Numerical estimation of a limit; Estimating a limit using a graph

B. Right- and left-hand limits; Relationships between limits and one-sided limits

C. Infinite limits; Vertical asymptotes

Summary and Focus Questions



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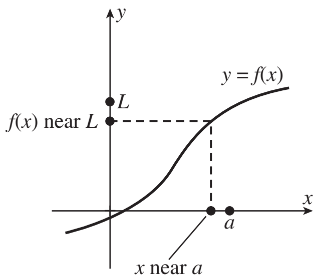


A. Suppose a function f is defined near a point a (that is, suppose $f(x)$ exists in an open interval about a, except perhaps at $x = a$).

$\boldsymbol{\lim_{x\to a} f(x) = L}$ means that as x gets closer and closer to the number a (but not equal to a), the corresponding $f(x)$ values get closer and closer to the number L.

In other words, $\lim_{x\to a} f(x) = L$ means we can force $f(x)$ to be arbitrarily close to L by restricting x to be sufficiently close (but never equal) to a. Thus $\lim_{x\to a} f(x) = L$ is like a guarantee: if you choose x close enough to a on the x-axis, $f(x)$ is guaranteed to be close to L on the y-axis.

Sometimes we will write "as $x \to a, f(x) \to L$" to mean $\lim_{x\to a} f(x) = L$.

Important: For now, ignore the value $f(a)$, regardless of whether it exists, in determining $\lim_{x\to a} f(x)$.

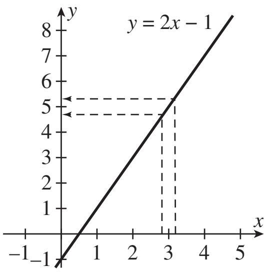
EXAMPLE. $\lim_{x\to 4} (x + 3) = 7$ because whenever x is near 4, then $x + 3$ will be near 7. ■

To say that $\boldsymbol{\lim_{x\to a} f(x)}$ **exists** means that there is some number L such that $\lim_{x\to a} f(x) = L$.

In this section we have three ways to determine a limit:

i. In some cases, we can intuitively "see" what the limit must be. For example, $\lim_{x\to 3}(x^2 - 1) = 8$. We reason that if x is near 3 then x^2 is near 9 and therefore $x^2 - 1$ is near 8.

ii. From the graph of $y = f(x)$, we can sometimes estimate the limit. For example, the graph of $f(x) = 2x - 1$ is given at the right. For x near 3, $f(x)$ is near 5.



iii. For $y = f(x)$, select several values of x near a, calculate $f(x)$ for each, observe a pattern in the $f(x)$ values and guess the limit. A computer or calculator may help. For example, from the table below, we estimate $\lim_{x\to 3} f(x) = 4$.

x	0	2	2.75	2.90	2.97	2.999
***f*(*x*)**	1.5	3.30	3.70	3.85	3.91	3.989

1) Find each:

a) $\lim_{x\to 9}(x - 5) =$ ________.

4. If x is near 9, then $x - 5$ is near 4.

b) $\lim_{x\to 4}(2x + 6) =$ ________.

14.

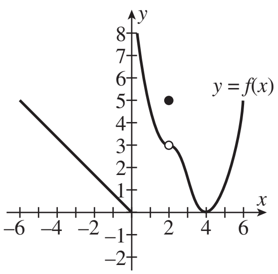
c) As $x \to 2$, $x^3 \to$ ________.

8.

d) $\lim_{x\to 3} 7 =$ ________.

7. For the constant function $f(x) = 7$, the limit is 7 because every function value is 7. The intuitive description "as x gets closer to 3, then 7 gets closer to 7" does not make much sense when taken literally. If x is near 3, then indeed the function values (7) are near (and actually equal to) 7.

2) Answer each using the graph below:



<For x-values near the limiting value on the x-axis, use the graph to see what the corresponding f(x) values are near on the y-axis.>

a) $\lim_{x\to-5} f(x) =$ ________.

4. (If x is near -5, then $f(x)$ is near 4.)

b) $\lim_{x\to 2} f(x) =$ ________.

3. (It does not matter that $f(2) = 5$.)

c) $\lim_{x\to 0} f(x) =$ ________.

Does not exist.
If x is near 0 and negative, $f(x)$ is near 0.
But if x is near 0 and positive then $f(x)$ is a large positive number.

d) $\lim_{x\to 4} f(x) =$ ________.

0.

3) a) Find $\lim_{x\to 2} |x-5|$ by making a table of values for x near 2.

x	$\lvert x-5\rvert$
3	2
2.1	2.9
1.99	3.01
2.0003	2.9997

We guess that $\lim_{x\to 2} |x-5| = 3$.

b) Let $f(x)=\begin{cases} 2x & \text{if } x<3 \\ 1-x & \text{if } x\geq 3 \end{cases}$.

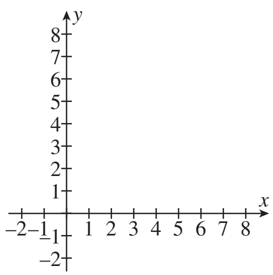
Find $\lim_{x\to 3} f(x)$ by making a table of values for x near 3.

Use $2x$ for $x < 3$ and $1 - x$ for $x > 3$.

x	$f(x)$
2	4
2.8	5.6
2.95	5.90
2.999	5.998
3.5	−2.5
3.01	−2.01
3.0001	−2.0001

The values of $f(x)$ do not cluster around a single number. $\lim_{x\to 3} f(x)$ does not exist.

4) Sketch three different functions, each of which has $\lim_{x\to 4} f(x) = 2$.



<Draw a graph so that if x is near 4 then $f(x)$ is near 2. The rest of the graph is unimportant.>