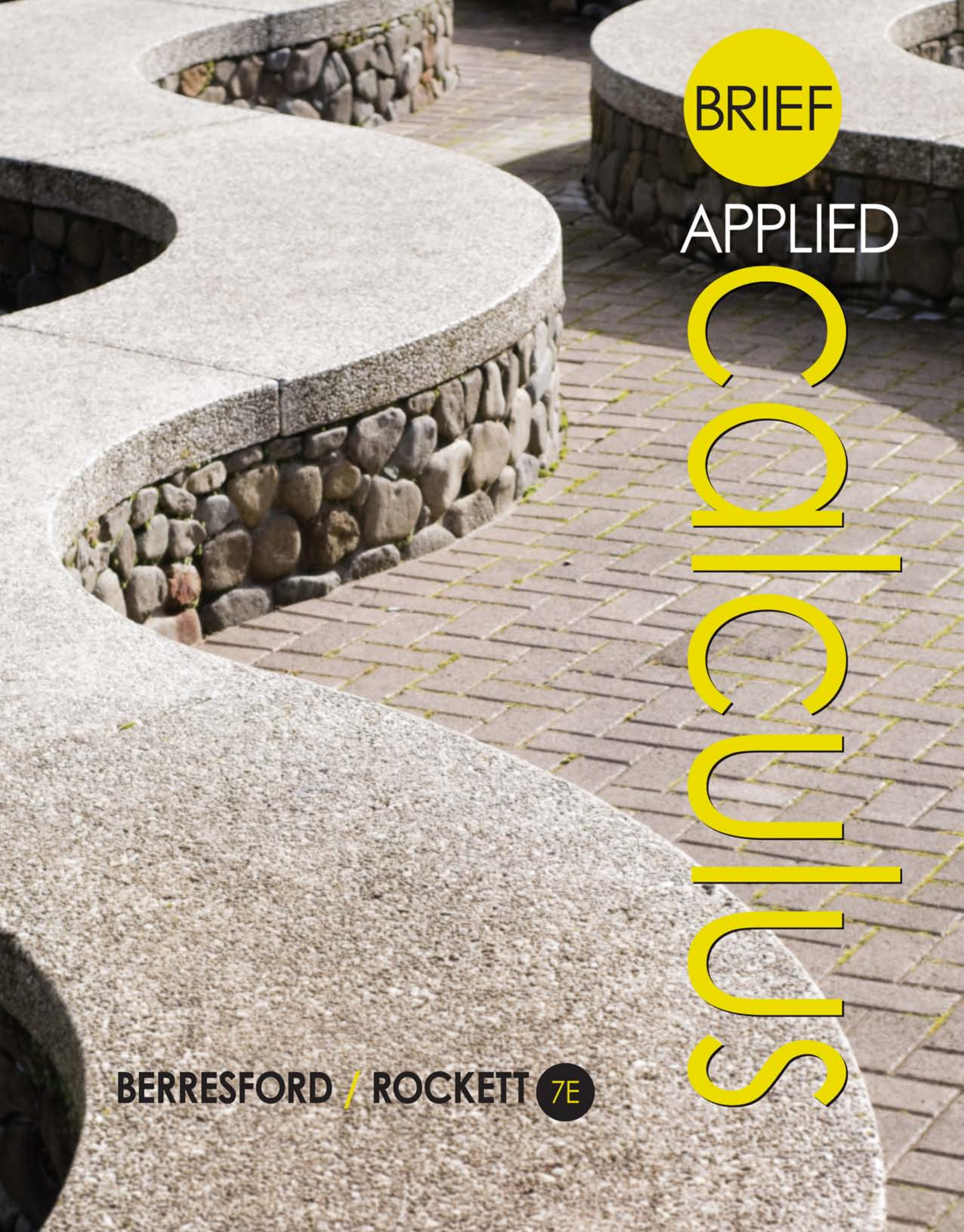
BRIEF
APPLIED
calculus
BERRESFORD / ROCKETT 7E

BRIEF

applied Calculus

Seventh Edition

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Long Island University

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CENGAGE Learning

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4 Exponential and Logarithmic Functions

5 Integration and Its Applications

6 Integration Techniques and Differential Equations

7 Calculus of Several Variables

Overview

A scientific study of yawning found that more yawns occurred in calculus class than anywhere else.* This book hopes to remedy that situation. Rather than being another dry recitation of standard results, our presentation exhibits many of the fascinating and useful applications of mathematics in business, the sciences, and everyday life. Even beyond its utility, however, there is a beauty to calculus, and we hope to convey some of its elegance and simplicity.

This book is an introduction to calculus and its applications to the management, social, behavioral, and biomedical sciences, and other fields. The seven-chapter *Brief Applied Calculus* contains more than enough material for a one-semester course, and the eleven-chapter *Applied Calculus* contains additional chapters on trignometry, differential equations, sequences and series, and probability for a two-semester course. The only prerequisites are some knowledge of algebra, functions, and graphing, which are reviewed in Chapter 1 and in greater detail in the Algebra Review appendix.

ACCURATE AND ACCESSIBLE

Our foremost goal in writing these books has been to make the content as accessible to as many students as possible. Over time, we have introduced various features to address the changing needs of students as they learn the essential techniques and fundamental concepts of calculus. In order maintain students' interest and provide them with the most accurate and engaging textbook, we have been guided by the following principles.

- ***Informal Proofs*** Because this book is applied rather than theoretical, we have preferred intuitive and geometric justifications to formal proofs. We provide a justification or proof for every important mathematical idea. When proofs are given, they are correct and mathematically honest.
- ***Integration of Mathematics and Applications*** Every section has applications to motivate the mathematics being developed (see, for example, pages 27–28 and 119–120). There are no "pure math" sections.
- ***Rapid Start*** When learning something, it is best to begin doing it as soon as possible. Therefore, we keep the preliminary material brief so that students begin calculus without delay (in Section 2.2). An early start allows more time for interesting applications throughout the course.
- ***Just-in-Time Review*** Review material is placed just before it is used, where it is more likely to be remembered, rather than in lengthy early chapters that "review" material that was never mastered in the first place. For example, exponential and logarithmic functions are reviewed just before they are differentiated in Section 4.3.
- ***Continual Algebra Reinforcement*** Since many of today's students have weak algebra skills, which impede their understanding of calculus, examples have blue annotations in the right margin giving brief explanations of the steps (see, for example, page 88). For extra support, we also offer a Diagnostic Test (appearing before Chapter 1) to help students identify skills that may

*Ronald Baenninger, "Some Comparative Aspects of Yawning in Betta splendens, Homo sapiens, Panthera leo, and Papoi spinx," *Journal of Comparative Psychology* **101** (4).

need review along with a supplementary Algebra Review appendix for additional reference.

CHANGES IN THE SEVENTH EDITION

New Content

- Section 3.7 *Differentials, Approximations, and Marginal Analysis* is new in the seventh edition. This section is optional and can be omitted without loss of continuity.
- An Algebra Review appendix is keyed to parts of the text (see, for example, page 49).
- A Diagnostic Test has been added to help students identify skills that may need review. This test appears before Chapter 1. Complete solutions are given in the Algebra Review appendix.
- New material on parallel and perpendicular lines has been added to Section 1.1, *Real Numbers, Inequalities, and Lines*.
- New exercises have been added and over 100 updated (including all of the Wall Street financial exercises) with current real-world data and sources. New *Explorations and Excursions* exercises give further details or theoretical underpinnings of the topics in the main text.
- A new "What You'll Explore" paragraph on the opening page of each chapter previews the ideas and applications to come.

Enhanced Learning Support

- Throughout the text there are now LOOKING AHEAD and LOOKING BACK marginal notes that show connections between current material and past or future developments to unify students' understanding of calculus.
- New Take Note marginal prompts provide observations that simplify or clarify ideas.
- Newly added FOR MORE HELP and FOR HELP GETTING STARTED prompts point students to Examples or parts of the Algebra Review appendix for additional help.

Graphing Calculator

- The graphing calculator screens throughout the book are now in color, based on the TI-84 Plus C Silver Edition, although students can still use the TI-83 or TI-84 (regular or Plus) calculators and follow instructions provided to get corresponding black-and-white graphs.
- References to the Internet are now given for graphing calculator programs from sites such as `ticalc.org`. The programs may be used for Riemann sums (page 332), trapezoidal approximation (page 418), Simpson's rule (page 421), and slope fields (pages 430, 432, and 450). The graphing calculator programs from earlier editions are now available on the Student and the Instructor Companion Sites.

User's Guide

To get the most out of this book, familiarize yourself with the following features—all designed to increase your understanding and mastery of the material. These learning aids, together with any help available through your college, should make your encounter with calculus both successful and enjoyable.

APPLICATIONS

From archaeological finds to physics, from social issues to politics, the applications show that calculus is more than just manipulation of abstract symbols. Rather, it is a powerful tool that can be used to help understand and manage both the natural world and our activities in it.

Application Preview

Following each chapter opener, an Application Preview offers a "mathematics in your world" application. A page with further information on the topic and a related exercise number are often given.

Functions

1

Moroccan runner Hicham El Guerrouj, current world record holder for the mile run, bested the record set 6 years earlier by 1.26 seconds.

What You'll Explore

To model how things change over time or to manage any complex enterprise, you will need a variety of ways to express relationships between important quantities. The functions introduced in this chapter will help you understand and predict quantities as diverse as populations, income, global energy, and even the world record times in the mile run. The techniques you learn in this chapter will serve as the basis for calculus in Chapter 2 and beyond.

1.1 Real Numbers, Inequalities, and Lines

1.2 Exponents

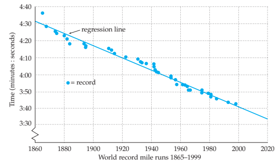
1.3 Functions: Linear and Quadratic

1.4 Functions: Polynomial, Rational, and Exponential

APPLICATION PREVIEW

World Record Mile Runs

The dots on the graph below show the world record times for the mile run from 1865 to the 1999 world record of 3 minutes 43.13 seconds, set by the Moroccan runner Hicham El Guerrouj. These points fall roughly along a line, called the **regression line.** In this section we will see how to use a graphing calculator to find a regression line (see Example 9 and Exercises 73–78), based on a method called **least squares,** whose mathematical basis will be explained in Chapter 7.



Notice that the times do not level off as you might expect but continue to decrease.

History of the Record for the Mile Run

Time	Year	Athlete	Time	Year	Athlete	Time	Year	Athlete
4:36.5	1865	Richard Webster	4:09.2	1931	Jules Ladoumegue	3:54.1	1964	Peter Snell
4:29.0	1868	William Chinnery	4:07.6	1933	Jack Lovelock	3:53.6	1965	Michel Jazy
4:28.8	1868	Walter Gibbs	4:06.8	1934	Glenn Cunningham	3:51.3	1966	Jim Ryun
4:26.0	1874	Walter Slade	4:06.4	1937	Sydney Wooderson	3:51.1	1967	Jim Ryun
4:24.5	1875	Walter Slade	4:06.2	1942	Gunder Hägg	3:51.0	1975	Filbert Bayi
4:23.2	1880	Walter George	4:06.2	1942	Arne Andersson	3:49.4	1975	John Walker
4:21.4	1882	Walter George	4:04.6	1942	Gunder Hägg	3:49.0	1979	Sebastian Coe
4:18.4	1884	Walter George	4:02.6	1943	Arne Andersson	3:48.8	1980	Steve Ovett
4:18.2	1894	Fred Bacon	4:01.6	1944	Arne Andersson	3:48.53	1981	Sebastian Coe
4:17.0	1895	Fred Bacon	4:01.4	1945	Gunder Hägg	3:48.40	1981	Steve Ovett
4:15.6	1895	Thomas Conneff	3:59.4	1954	Roger Bannister	3:47.33	1981	Sebastian Coe
4:15.4	1911	John Paul Jones	3:58.0	1954	John Landy	3:46.31	1985	Steve Cram
4:14.4	1913	John Paul Jones	3:57.2	1957	Derek Ibbotson	3:44.39	1993	Noureddine Morceli
4:12.6	1915	Norman Taber	3:54.5	1958	Herb Elliott	3:43.13	1999	Hicham El Guerrouj
4:10.4	1923	Paavo Nurmi	3:54.4	1962	Peter Snell			

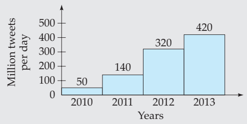
Source: USA Track & Field

Diverse Applications

Along with an emphasis on business and biomedical sciences, a variety of other fields are represented throughout the text. Applications based on contemporary real-world data are denoted with an icon

EXAMPLE 9 LINEAR REGRESSION USING A GRAPHING CALCULATOR

The following graph shows the average number of "tweets" per day sent on Twitter in recent years.



Source: Twitter

a. Use linear regression to fit a line to the data.

b. Interpret the slope of the line.

c. Use the regression line to predict the number of tweets per day in the year 2022.

GUIDED LEARNING SUPPORT

Annotations

To aid students' understanding of the solution steps within examples or to provide interpretations, blue annotations appear to the right of most mathematical formulas. Calculations presented within annotations provide explanations and justifications for the steps.

EXAMPLE 3 DEPRECIATING AN ASSET

A car worth \$30,000 depreciates in value by 40% each year. How much is it worth after 3 years?

Solution

The car loses 40% of its value each year, which is equivalent to an interest rate of *negative* 40%. The compound interest formula gives

$$30{,}000(1-0.40)^3 = \underbrace{30{,}000(0.60)^3 = \$6480}_{\text{Using a calculator}} \qquad P(1+r/m)^{mt} \text{ with } P = 30{,}000,\ r=-0.40,\ m=1, \text{ and } t=3$$

The exponential function $f(x) = 30{,}000(0.60)^x$, giving the value of the car after x years of depreciation, is graphed on the left. Notice that a yearly loss of 40% means that 60% of the value is retained each year.

Be Careful

The "Be Careful" icon marks places where the authors help students avoid common errors.

4.3 Differentiation of Logarithmic and Exponential Functions 279

Be Careful Do *not* take the derivative of e^x by the Power Rule,

$$\frac{d}{dx}x^n = nx^{n-1}$$

The Power Rule applies to x^n, *a variable to a constant power,* while e^x is *a constant to a variable power.* The two types of functions are quite different, as their graphs show.

Looking Ahead
Looking Back

New in the 7e! These notes appear in the margins and show connections between current material and previous or future developments to solidify and unify understanding of calculus topics.

Compound Interest

For P dollars invested at annual interest rate r compounded m times a year for t years,

$$\begin{pmatrix}\text{Value after}\\ t \text{ years}\end{pmatrix} = P\cdot\left(1+\frac{r}{m}\right)^{mt} \qquad \begin{aligned} r &= \text{annual rate}\\ m &= \text{periods per year}\\ t &= \text{number of years}\end{aligned}$$

LOOKING AHEAD On page 252 we will introduce a different kind of compound interest, where the compunding is done *continuously*.

For example, for monthly compounding we would use $m = 12$ and for daily compounding $m = 365$ (the number of days in the year).

EXAMPLE 3 DIFFERENTIATING A LOGARITHMIC FUNCTION

Find the derivative of $f(x) = \ln(x^4-1)^3$.

Solution

We need the rule for differentiating the natural logarithm of a function, together with the Generalized Power Rule [for differentiating $(x^4-1)^3$].

$$\frac{d}{dx}\ln(x^4-1)^3 = \frac{\frac{d}{dx}(x^4-1)^3}{(x^4-1)^3} \qquad \text{Using } \frac{d}{dx}\ln f = \frac{f'}{f}$$

$$= \frac{3(x^4-1)^2 4x^3}{(x^4-1)^3} \qquad \text{Using the Generalized Power Rule}$$

$$= \frac{12x^3}{x^4-1} \qquad \text{Dividing top and bottom by } (x^4-1)^2$$

FOR MORE HELP with simplifying expressions, see the Algebra Review appendix, pages B13–B14

Alternative Solution It is easier if we simplify first, using Property 8 of logarithms (see the inside back cover) to bring down the exponent 3:

$$\ln(x^4-1)^3 = 3\ln(x^4-1) \qquad \text{Using } \ln(M^P) = P\cdot\ln M$$

LOOKING BACK The properties of logarithms were stated on pages 262–263.

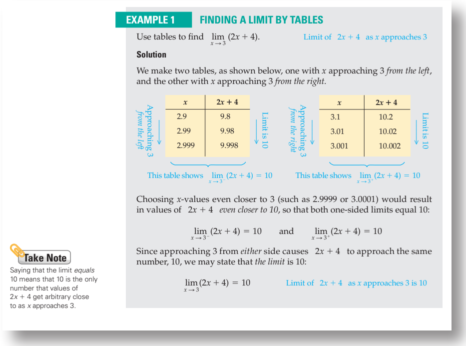
Now we differentiate the simplified expression:

$$\frac{d}{dx}3\ln(x^4-1) = 3\frac{4x^3}{x^4-1} = \frac{12x^3}{x^4-1} \qquad \text{Same answer as before}$$

GUIDED LEARNING SUPPORT

Take Note

New in the 7/e! Appearing in the margins, these prompts include observations to help simplify or clarify ideas in the text.



EXAMPLE 1 FINDING A LIMIT BY TABLES

Use tables to find $\lim\limits_{x \to 3} (2x + 4)$. Limit of $2x + 4$ as x approaches 3

Solution

We make two tables, as shown below, one with x approaching 3 *from the left*, and the other with x approaching 3 *from the right*.

x	$2x + 4$
2.9	9.8
2.99	9.98
2.999	9.998

Approaching 3 from the left — Limit is 10

This table shows $\lim\limits_{x \to 3^-} (2x + 4) = 10$

x	$2x + 4$
3.1	10.2
3.01	10.02
3.001	10.002

Approaching 3 from the right — Limit is 10

This table shows $\lim\limits_{x \to 3^+} (2x + 4) = 10$

Choosing x-values even closer to 3 (such as 2.9999 or 3.0001) would result in values of $2x + 4$ *even closer to 10*, so that both one-sided limits equal 10:

$$\lim_{x \to 3^-} (2x + 4) = 10 \quad \text{and} \quad \lim_{x \to 3^+} (2x + 4) = 10$$

Since approaching 3 from *either* side causes $2x + 4$ to approach the same number, 10, we may state that *the limit* is 10:

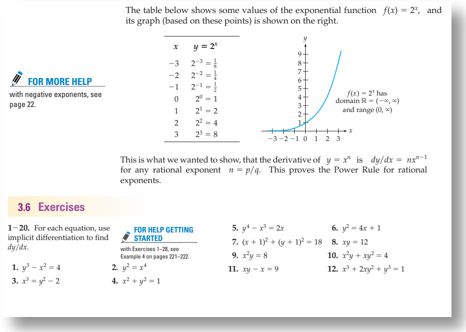
$$\lim_{x \to 3} (2x + 4) = 10$$ Limit of $2x + 4$ as x approaches 3 is 10

Take Note
Saying that the limit *equals* 10 means that 10 is the only number that values of $2x + 4$ get arbitrary close to as x approaches 3.

For More Help

For Help Getting Started

New in the 7/e! These prompts appear within the margins of the text and end-of-section exercises. They direct students to Examples from within the text or parts of the Algebra Review appendix, as a refresher.



The table below shows some values of the exponential function $f(x) = 2^x$, and its graph (based on these points) is shown on the right.

x	$y = 2^x$
−3	$2^{-3} = \frac{1}{8}$
−2	$2^{-2} = \frac{1}{4}$
−1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$

FOR MORE HELP with negative exponents, see page 22.

This is what we wanted to show, that the derivative of $y = x^n$ is $dy/dx = nx^{n-1}$ for any rational exponent $n = p/q$. This proves the Power Rule for rational exponents.

3.6 Exercises

1–20. For each equation, use implicit differentiation to find dy/dx.

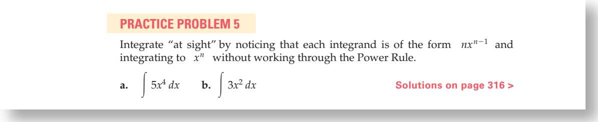
FOR HELP GETTING STARTED with Exercises 1–28, see Example 4 on pages 221–222.

1. $y^3 - x^2 = 4$ **2.** $y^2 = x^4$
3. $x^3 = y^2 - 2$ **4.** $x^2 + y^2 = 1$
5. $y^4 - x^3 = 2x$ **6.** $y^2 = 4x + 1$
7. $(x + 1)^2 + (y + 1)^2 = 18$ **8.** $xy = 12$
9. $x^2y = 8$ **10.** $x^2y + xy^2 = 4$
11. $xy - x = 9$ **12.** $x^3 + 2xy^2 + y^3 = 1$

PRACTICE AND PREPARE

Practice Problems

Students can check their understanding of a topic as they read the text or do homework by working out a Practice Problem. Complete solutions are found at the end of each section, just before the Section Summary.



PRACTICE PROBLEM 5

Integrate "at sight" by noticing that each integrand is of the form nx^{n-1} and integrating to x^n without working through the Power Rule.

a. $\int 5x^4\,dx$ **b.** $\int 3x^2\,dx$ Solutions on page 316 >

Exercises

The exercises that appear at the end of each section are graded from routine drills to significant applications. The *Applied Exercises* are labeled with general and specific titles so instructors can assign problems appropriate for the class. *Conceptual Exercises* develop intuitive insights to solve problems quickly and simply. *Explorations and Excursions* push students further. Just-in-time *Review Exercises* are found in selected sections. They recall skills previously learned that are relevant to content in an upcoming section (see, for example, page 355).

Although this may be used to *estimate* future costs (*about* \$15 for each additional unit), it does not mean that one additional unit will increase costs by exactly \$15, two more by exactly \$30, and so on, since the marginal rate usually changes as production increases. A marginal cost is only an *approximate* predictor of future costs.

2.5 Exercises

1–6. For each function, find:

a. $f'(x)$ **b.** $f''(x)$ **c.** $f'''(x)$ **d.** $f^{(4)}(x)$

1. $f(x) = x^4 - 2x^3 - 3x^2 + 5x - 7$

2. $f(x) = x^4 - 3x^3 + 2x^2 - 8x + 4$

3. $f(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5$

4. $f(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$

5. $f(x) = \sqrt{x^5}$ **6.** $f(x) = \sqrt{x^3}$

7–12. For each function, find: **a.** $f''(x)$ and **b.** $f''(3)$.

7. $f(x) = \dfrac{x-1}{x}$ **8.** $f(x) = \dfrac{x+2}{x}$

Applied Exercises

33. **GENERAL: Velocity** After t hours a freight train is $s(t) = 18t^2 - 2t^3$ miles due north of its starting point (for $0 \le t \le 9$).
- **a.** Find its velocity at time $t = 3$ hours.
- **b.** Find its velocity at time $t = 7$ hours.
- **c.** Find its acceleration at time $t = 1$ hour.

34. **GENERAL: Velocity** After t hours a passenger train is $s(t) = 24t^2 - 2t^3$ miles due west of its starting point (for $0 \le t \le 12$).
- **a.** Find its velocity at time $t = 4$ hours.
- **b.** Find its velocity at time $t = 10$ hours.
- **c.** Find its acceleration at time $t = 1$ hour.

35. **GENERAL: Velocity** A rocket can rise to a height of $h(t) = t^3 + 0.5t^2$ feet in t seconds. Find its velocity and acceleration 10 seconds after it is launched.

- **c.** Find the acceleration at any time t. (This number is called the *acceleration due to gravity*.)

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Conceptual Exercises

47–50. Suppose that the quantity described is represented by a function $f(t)$ where t stands for time. Based on the description:

a. Is the first derivative positive or negative?

b. Is the second derivative positive or negative?

47. The temperature is dropping increasingly rapidly.

48. The economy is growing, but more slowly.

49. The stock market is declining, but less rapidly.

50. The population is growing increasingly fast.

51. True or False: If $f(x)$ is a polynomial of degree n, then $f^{(n+1)}(x) = 0$.

Explorations and Excursions

The following problems extend and augment the material presented in the text.

More About Higher-Order Derivatives

55. Find $\dfrac{d^{100}}{dx^{100}}(x^{100} - 4x^{99} + 3x^{50} + 6)$.

[*Hint:* You may use the "factorial" notation: $n! = n(n-1) \cdots 1$. For example, $3! = 3 \cdot 2 \cdot 1 = 6$.]

56. Find a general formula for $\dfrac{d^n}{dx^n}x^{-1}$.

[*Hint:* Calculate the first few derivatives and look for a pattern. You may use the "factorial" notation: $n! = n(n-1) \cdots 1$. For example, $3! = 3 \cdot 2 \cdot 1 = 6$.]

57. Verify the following formula for the *second* derivative of a product, where f and g are differentiable functions of x:

$$\frac{d^2}{dx^2}(f \cdot g) = f'' \cdot g + 2f' \cdot g' + f \cdot g''$$

[*Hint:* Use the Product Rule repeatedly.]

58. Verify the following formula for the *third* derivative of a product, where f and g are differentiable functions of x:

$$\frac{d^3}{dx^3}(f \cdot g) = f''' \cdot g + 3f'' \cdot g' + 3f' \cdot g'' + f \cdot g'''$$

[*Hint:* Differentiate the formula in Exercise 57 by the Product Rule.]

PRACTICE AND PREPARE

Section Summary

Found at the end of every section, summaries briefly state the main ideas of the section and provide study tools or reminders for students

3.7 Section Summary

For an *independent* variable x, the differential dx is any nonzero number. For the *dependent* variable $y = f(x)$, the differential is $dy = f'(x)\,dx$. Values for both x and dx must be known before dy can be evaluated.

The best linear approximation of a differentiable function $y = f(x)$ near x is the *tangent line approximation* given by

$$f(x + \Delta x) \approx f(x) + f'(x)\,dx \qquad (\Delta x = dx)$$

since $\Delta y \approx dy$. This approximation becomes more accurate for values of $\Delta x = dx$ closer to zero.

For a *dependent* variable y, the *error* Δy resulting from a measurement error Δx is sometimes called the *absolute* error, and may be approximated by the differential, $\Delta y \approx dy$. The *relative* error $\Delta y/y \approx dy/y$ compares the absolute error to the actual value, and is usually written as a percentage. Errors are sometimes called "changes" depending on the situation.

Marginals can be used to find approximations of revenue, cost, and profit (see page 234), and indicate how these quantities vary near a particular level of production.

Chapter Summary

Found at the end of every chapter, the Chapter Summary with Hints and Suggestions review the important developments of the chapter and give insights to unify the material to help students prepare for tests and exams.

2 Chapter Summary with Hints and Suggestions

Reading the text and doing the exercises in this chapter have helped you to master the following concepts and skills, which are listed by section (in case you need to review them) and are keyed to particular Review Exercises. Answers for all Review Exercises are given at the back of the book, and full solutions can be found in the Student Solutions Manual.

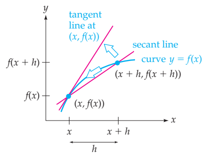
2.1 Limits and Continuity

- Find the limit of a function from tables. *(Review Exercises 1–2.)*
- Find left and right limits. *(Review Exercises 3–4.)*
- Find the limit of a function. *(Review Exercises 5–14.)*
- Determine whether a function is continuous or discontinuous. *(Review Exercises 15–22.)*

2.2 Rates of Change, Slopes, and Derivatives

- Find the derivative of a function from the *definition* of the derivative. *(Review Exercises 23–26.)*

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$



$$MC(x) = C'(x) \qquad MR(x) = R'(x) \qquad MP(x) = P'(x)$$

- Find and interpret the derivative of a learning curve. *(Review Exercise 35.)*
- Find and interpret the derivative of an area or volume formula. *(Review Exercises 36–37.)*

2.4 The Product and Quotient Rules

- Find the derivative of a function using the Product Rule or Quotient Rule. *(Review Exercises 38–48.)*

$$\frac{d}{dx}(f \cdot g) = f' \cdot g + f \cdot g'$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g \cdot f' - g' \cdot f}{g^2}$$

- Find the tangent line to a curve at a given point. *(Review Exercise 49.)*
- Use differentiation to solve an applied problem and interpret the answer. *(Review Exercises 50–52.)*

$$MAC(x) = \frac{C(x)}{x}$$

$$MAR(x) = \frac{R(x)}{x}$$

$$MAP(x) = \frac{P(x)}{x}$$

Review Exercises and Chapter Test

Following the Chapter Summary are the Review Exercises and a Chapter Test. Selected questions from the Review Exercises are specially color-coded to indicate that they may be used as a practice Chapter Test. Both even and odd answers are supplied in the back of the book for students to check their proficiency.

2 Review Exercises and Chapter Test

indicates a Chapter Test exercise.

2.1 Limits and Continuity

1–2. Complete the tables and use them to find each limit (or state that it does not exist). Round calculations to three decimal places.

1 **a.** $\lim_{x \to 2^-} (4x + 2)$

b. $\lim_{x \to 2^+} (4x + 2)$

c. $\lim_{x \to 2} (4x + 2)$

x	$4x + 2$
1.9	
1.99	
1.999	

x	$4x + 2$
2.1	
2.01	
2.001	

2 **a.** $\lim_{x \to 0^-} \frac{\sqrt{x+1} - 1}{x}$

b. $\lim_{x \to 0^+} \frac{\sqrt{x+1} - 1}{x}$

c. $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$

x	$\frac{\sqrt{x+1} - 1}{x}$
−0.1	
−0.01	
−0.001	

x	$\frac{\sqrt{x+1} - 1}{x}$
0.1	
0.01	
0.001	

Cumulative Review

Cumulative Review questions appear after every three to four chapters, with all answers supplied in the back of the book.

1–3 Cumulative Review for Chapters 1–3

The following exercises review some of the basic techniques that you learned in Chapters 1—3. Answers to all of these cumulative review exercises are given in the answer section at the back of the book.

1. Find an equation for the line through the points $(-4, 3)$ and $(6, -2)$. Write your answer in the form $y = mx + b$.

2. Simplify $\left(\frac{4}{25}\right)^{-1/2}$.

3. Find, correct to three decimal places: $\lim_{x \to 0} (1 + 3x)^{1/x}$.

4. For the function $f(x) = \begin{cases} 4x - 8 & \text{if } x < 3 \\ 7 - 2x & \text{if } x \ge 3 \end{cases}$

a. Draw its graph.

b. Find $\lim_{x \to 3^-} f(x)$.

c. Find $\lim_{x \to 3^+} f(x)$.

d. Find $\lim_{x \to 3} f(x)$.

e. Is $f(x)$ continuous or discontinuous, and if it is discontinuous, where?

5. Use the definition of the derivative, $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$, to find the derivative of $f(x) = 2x^2 - 5x + 7$.

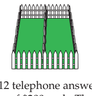
6. Find the derivative of $f(x) = 8\sqrt{x^3} - \frac{3}{x^2} + 5$.

14. Find the equation for the tangent line to the curve $y = \frac{4(x + 3)}{\sqrt{x^2 + 3}}$ at $x = -1$.

15. Make sign diagrams for the first and second derivatives and draw the graph of the function $f(x) = x^3 - 12x^2 - 60x + 400$. Show on your graph all relative extreme points and inflection points.

16. Make sign diagrams for the first and second derivatives and draw the graph of the function $f(x) = \sqrt[3]{x^2} - 1$. Show on your graph all relative extreme points and inflection points.

17. A homeowner wishes to use 600 feet of fence to enclose two identical adjacent pens, as in the diagram below. Find the largest total area that can be enclosed.



18. A store can sell 12 telephone answering machines per day at a price of $200 each. The manager estimates that for each $10 price reduction she can sell 2 more per day. The answering machines cost the store $80

TECHNOLOGY

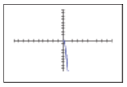
OPTIONAL! Using this book does not require a graphing calculator, but having one will enable you to do many problems more easily and as the same time deepen your understanding by allowing you to concentrate on concepts. The displays shown in the text are from the Texas Instruments TI-84 Plus *C* Silver Edition, except for a few from the TI-89, but any graphing calculator or computer may be used instead. For those who do not have a graphing calculator, the Explorations have been designed to be read for enrichment.

Similarly, if you have access to a computer, you may wish to do some of the Spreadsheet Explorations.

Graphing Calculator Explorations

To allow for optional use of the graphing calculator, these Explorations are boxed. Most can also be read simply for enrichment. Exercises and examples that are designed to be done with a graphing calculator are marked with an icon.

Graphing Calculator Exploration



on $[-10, 10]$ by $[-10, 10]$

The graph of the function from Example 5, $y_1 = 9x - 20x^{3/2}$ (written in x instead of t for ease of entry), is shown on the left on the standard window $[-10, 10]$ by $[-10, 10]$. This might lead you to believe, erroneously, that the function is maximized at the endpoint (0, 0).

a. Why does this graph not look like the graph at the end of the previous example? [*Hint:* Look at the scale.]

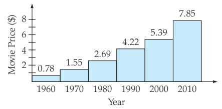
b. Can you find a window on which your graphing calculator will show a graph like the one at the end of the preceding solution?

This example illustrates one of the pitfalls of graphing calculators—the part of the curve where the "action" takes place may be entirely hidden in one pixel. Calculus, on the other hand, will *always* find the critical value, no matter where it is, and then a graphing calculator can be used to confirm your answer by showing the graph on an appropriate window.

Modeling

Selected application exercises feature regression capabilities of graphing calculators to fit curves to actual data.

90. BUSINESS: Movie Prices National average theater admissions prices for recent decades are shows in the following graph.



a. Number the bars with x-values 1–6 (so that x stands for *decades since 1950*) and use quadratic regression to fit a parabola to the data. State the regression function. [*Hint:* See Example 10.]

b. Use the regression function to predict movie prices in the years 2020 and 2030.

Source: Entertainment Weekly

Spreadsheet Explorations

Boxed for optional use, these explorations will enhance students' understanding of the material using Excel for those who prefer spreadsheet technology. See "Integrating Excel" on the next page for a list of exercises that can be done with Excel.

Spreadsheet Exploration

Another function that is not differentiable is $f(x) = x^{2/3}$. The following spreadsheet* calculates values of the difference quotient $\frac{f(x+h) - f(x)}{h}$ at $x = 0$ for this function. Since $f(0) = 0$, the difference quotient at $x = 0$ simplifies to:

$$\frac{f(x+h) - f(x)}{h} = \frac{f(0+h) - f(0)}{h} = \frac{f(h)}{h} = \frac{h^{2/3}}{h} = h^{-1/3}$$

For example, cell **B5** evaluates $h^{-1/3}$ at $h = \frac{1}{1000}$ obtaining $(\frac{1}{1000})^{-1/3} = 1000^{1/3} = \sqrt[3]{1000} = 10$. Column **B** evaluates this different quotient for the *positive* values of h in column **A**, while column **E** evaluates it for the corresponding negative values of h in column **D**.

B5 = =A5^(-1/3)

	A	B	C	D	E
1	h	(f(0+h)-f(0))/h		h	(f(0+h)-f(0))/h
2	1.0000000	1.0000000		-1.0000000	-1.0000000
3	0.1000000	2.1544347		-0.1000000	-2.1544347
4	0.0100000	4.6415888		-0.0100000	-4.6415888
5	0.0010000	10.0000000		-0.0010000	-10.0000000
6	0.0001000	21.5443469		-0.0001000	-21.5443469
7	0.0000100	46.4158883		-0.0000100	-46.4158883
8	0.0000010	100.0000000		-0.0000010	-100.0000000
9	0.0000001	215.4434690		-0.0000001	-215.4434690

becoming large — becoming small

Notice that the values in column **B** are becoming arbitrarily large, while the values in column **E** are becoming arbitrarily small, so the difference quotient does not approach a limit as $h \to 0$. This shows that the derivative of $f(x) = x^{2/3}$ at 0 *does not exist*, so the function $f(x) = x^{2/3}$ is *not differentiable* at $x = 0$.

INTEGRATING EXCEL

If you would like to use Excel or another spreadsheet software when working the exercises in this text, refer to the chart below. It lists exercises from many sections that you might find instructive to do with spreadsheet technology. If you would like help using Excel, please consider the *Excel Guide* available via CengageBrain.com.

Section	Suggested Exercises
1.1	59–78
1.2	103–110
1.3	69–82, 84–90
1.4	79–92
2.1	77–78, 81–82
2.2	9–16
2.3	47–50
2.4	61–64
2.5	45–46
2.6	65, 69
2.7	11–12
3.1	68–71, 85
3.2	61–64
3.3	23–40, 52–54
3.4	23–24
3.5	20
3.6	69–70
3.7	23–26
4.1	11–12, 47–51
4.2	31–50
4.3	97–99
4.4	38–39

Section	Suggested Exercises
5.1	41–42
5.2	45–46, 55–58
5.3	13–18, 83–88
5.4	32, 35–36, 61, 69
5.5	31–32
5.6	77–78
6.1	60–64
6.2	65, 66, 68
6.3	41–42
6.4	9–18, 27–37
6.5	71
6.6	54
7.1	29–30, 38–42
7.2	47–48, 53–56
7.3	29–32
7.4	13–18, 27–32
7.5	29–36
7.6	31–32, 35–36
7.7	41–42

SUPPLEMENTS

For the Student	For the Instructor
Student Solutions Manual ISBN: 978-1-305-10795-3 This manual contains fully worked-out solutions to all of the odd-numbered exercises in the text, giving students a way to check their answers and ensure that they took the correct steps to arrive at an answer.	**Complete Solutions Manual** This manual contains solutions to all exercises from the text including Chapter Review Exercises and Cumulative Reviews. It also contains two chapter-level tests for each chapter, one short-answer and one multiple choice, along with answers to each. This manual can be found on the Instructor Companion Site.
CengageBrain.com To access additional course materials, please visit **www.cengagebrain.com.** At the **CengageBrain.com** home page, search for the ISBN (from the back cover of your book) of your title using the search box at the top of the page. This will take you to the product page where these resources can be found.	**Instructor Companion Site** Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online via **www.cengage.com/login.** Access and download PowerPoint® presentations, images, solutions manual, and more.
Enhanced WebAssign® Instant Access Code: 978-1-285-85761-9 Printed Access Card: 978-1-285-85758-9 Enhanced WebAssign combines exceptional mathematics content with the most powerful online homework solution, WebAssign. It now includes QuickPrep content to review key precalculus content, available as a CoursePack of prebuilt assignments to assign at the beginning of the course or where needed most. Enhanced WebAssign engages students with immediate feedback, rich tutorial content, and an interactive, fully customizable eBook, the Cengage YouBook, helping students to develop a deeper conceptual understanding of their subject matter.	**Enhanced WebAssign®** Instant Access Code: 978-1-285-85761-9 Printed Access Card: 978-1-285-85758-9 Enhanced WebAssign combines exceptional mathematics content with the most powerful online homework solution, WebAssign. It now includes QuickPrep content to review key precalculus content, available as a CoursePack of prebuilt assignments to assign at the beginning of the course or where needed most. Enhanced WebAssign engages students with immediate feedback, rich tutorial content, and an interactive, fully customizable eBook, the Cengage YouBook, helping students to develop a deeper conceptual understanding of their subject matter. Visit **www.cengage.com/ewa** to learn more.
	Cengage Learning Testing Powered by Cognero® Instant Access Code: 978-1-305-11229-2 Cognero is a flexible, online system that allows you to author, edit, and manage test bank content, create multiple test versions in an instant, and deliver tests from your LMS, your classroom or wherever you want. This is available online via **www.cengage.com/login**.

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Finally, and most importantly, we thank our wives, Barbara and Kathryn, for their encouragement and support.

COMMENTS WELCOMED

With the knowledge that any book can always be improved, we welcome corrections, constructive criticisms, and suggestions from every reader.

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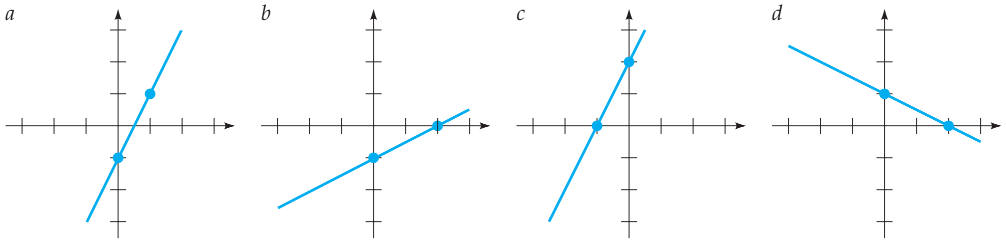
DIAGNOSTIC TEST

Are you ready to study calculus?

Algebra is the language in which we express the ideas of calculus. Therefore, to understand calculus and express its ideas with precision, you need to know some algebra.

If you are comfortable with the algebra covered in the following problems, you are ready to begin your study of calculus. If not, turn to the *Algebra Review* appendix beginning on page B1 and review the *Complete Solutions* to these problems, and continue reading the other parts of the Appendix that cover anything that you do not know.

Problems	Answers
1. True or False? $\frac{1}{2} < -3$	False
2. Express $\{x \mid -4 < x \leq 5\}$ in interval notation.	$(-4, 5]$
3. What is the slope of the line through the points $(6, -7)$ and $(9, 8)$?	5
4. On the line $y = 3x + 4$, what value of Δy corresponds to $\Delta x = 2$?	6
5. Which sketch shows the graph of the line $y = 2x - 1$?	*a*



Problems	Answers
6. True of False? $\left(\frac{\sqrt{x}}{y}\right)^{-2} = \frac{y^2}{x}$	True
7. Find the zeros of the function $f(x) = 9x^2 - 6x - 1$.	$x = \frac{1 \pm \sqrt{2}}{3}$
8. Expand and simplify $x(8 - x) - (3x + 7)$.	$-x^2 + 5x - 7$
9. What is the domain of $f(x) = \frac{x^2 - 3x + 2}{x^3 + x^2 - 6x}$?	$\{x \mid x \neq -3, x \neq 0, x \neq 2\}$
10. Find the difference quotient $\frac{f(x+h) - f(x)}{h}$ for $f(x) = x^2 - 5x$.	$2x - 5 + h$

Functions

1

ZUMA/ ZUMA Press, Inc/Alamy

Moroccan runner Hicham El Guerrouj, current world record holder for the mile run, bested the record set 6 years earlier by 1.26 seconds.

What You'll Explore

To model how things change over time or to manage any complex enterprise, you will need a variety of ways to express relationships between important quantities. The functions introduced in this chapter will help you understand and predict quantities as diverse as populations, income, global energy, and even the world record times in the mile run. The techniques you learn in this chapter will serve as the basis for calculus in Chapter 2 and beyond.

1.1 Real Numbers, Inequalities, and Lines

1.2 Exponents

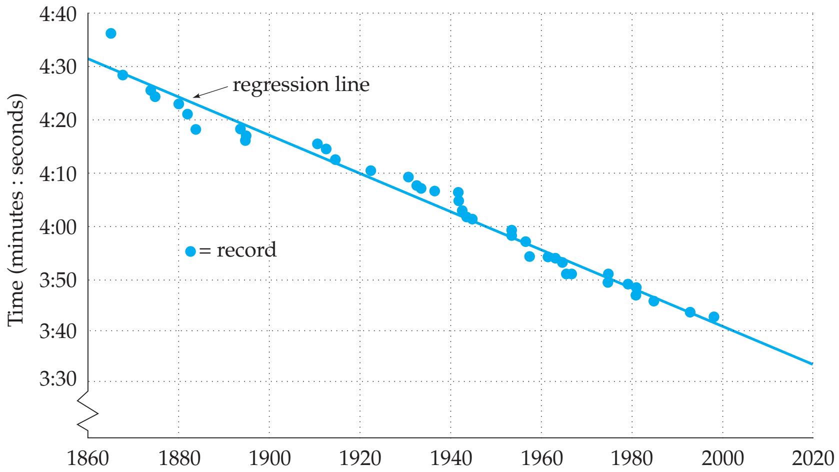
1.3 Functions: Linear and Quadratic

1.4 Functions: Polynomial, Rational, and Exponential

APPLICATION PREVIEW

World Record Mile Runs

The dots on the graph below show the world record times for the mile run from 1865 to the 1999 world record of 3 minutes 43.13 seconds, set by the Moroccan runner Hicham El Guerrouj. These points fall roughly along a line, called the **regression line.** In this section we will see how to use a graphing calculator to find a regression line (see Example 9 and Exercises 73–78), based on a method called **least squares,** whose mathematical basis will be explained in Chapter 7.



World record mile runs 1865–1999

Notice that the times do not level off as you might expect but continue to decrease.

History of the Record for the Mile Run

Time	Year	Athlete	Time	Year	Athlete	Time	Year	Athlete
4:36.5	1865	Richard Webster	4:09.2	1931	Jules Ladoumegue	3:54.1	1964	Peter Snell
4:29.0	1868	William Chinnery	4:07.6	1933	Jack Lovelock	3:53.6	1965	Michel Jazy
4:28.8	1868	Walter Gibbs	4:06.8	1934	Glenn Cunningham	3:51.3	1966	Jim Ryun
4:26.0	1874	Walter Slade	4:06.4	1937	Sydney Wooderson	3:51.1	1967	Jim Ryun
4:24.5	1875	Walter Slade	4:06.2	1942	Gunder Hägg	3:51.0	1975	Filbert Bayi
4:23.2	1880	Walter George	4:06.2	1942	Arne Andersson	3:49.4	1975	John Walker
4:21.4	1882	Walter George	4:04.6	1942	Gunder Hägg	3:49.0	1979	Sebastian Coe
4:18.4	1884	Walter George	4:02.6	1943	Arne Andersson	3:48.8	1980	Steve Ovett
4:18.2	1894	Fred Bacon	4:01.6	1944	Arne Andersson	3:48.53	1981	Sebastian Coe
4:17.0	1895	Fred Bacon	4:01.4	1945	Gunder Hägg	3:48.40	1981	Steve Ovett
4:15.6	1895	Thomas Conneff	3:59.4	1954	Roger Bannister	3:47.33	1981	Sebastian Coe
4:15.4	1911	John Paul Jones	3:58.0	1954	John Landy	3:46.31	1985	Steve Cram
4:14.4	1913	John Paul Jones	3:57.2	1957	Derek Ibbotson	3:44.39	1993	Noureddine Morceli
4:12.6	1915	Norman Taber	3:54.5	1958	Herb Elliott	3:43.13	1999	Hicham El Guerrouj
4:10.4	1923	Paavo Nurmi	3:54.4	1962	Peter Snell			

Source: USA Track & Field

The equation of the regression line is $y = -0.356x + 257.44$, where x represents years after 1900 and y is the time in seconds. The regression line can be used to predict the world mile record in future years. Notice that the most recent world record would have been predicted quite accurately by this line, since the rightmost dot falls almost exactly on the line.

Linear trends, however, must not be extended too far. The downward slope of this line means that it will eventually "predict" mile runs in a fraction of a second, or even in *negative* time (see Exercises 59 and 60 on pages 17–18). *Moral:* In the real world, linear trends do not continue indefinitely. This and other topics in "linear" mathematics will be developed in Section 1.1.

1.1 Real Numbers, Inequalities, and Lines

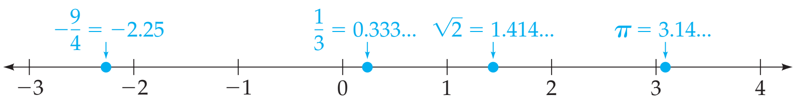
Introduction

Quite simply, *calculus is the study of rates of change*. We will use calculus to analyze rates of inflation, rates of learning, rates of population growth, and rates of natural resource consumption.

In this first section we will study **linear** relationships between two variable quantities—that is, relationships that can be represented by **lines.** In later sections we will study **nonlinear** relationships, which can be represented by **curves.**

Real Numbers and Inequalities

In this book the word "number" means **real number,** a number that can be represented by a point on the number line (also called the **real line**).



The *order* of the real numbers is expressed by **inequalities.** For example, $a < b$ means "a is to the *left* of b" or, equivalently, "b is to the *right* of a."

Inequalities

Inequality	In Words	Brief Examples
$a < b$	a is less than (smaller than) b	$3 < 5$
$a \leq b$	a is less than or equal to b	$-5 \leq -3$
$a > b$	a is greater than (larger than) b	$\pi > 3$
$a \geq b$	a is greater than or equal to b	$2 \geq 2$

The inequalities $a < b$ and $a > b$ are called **strict inequalities,** and $a \leq b$ and $a \geq b$ are called **nonstrict inequalities.**

IMPORTANT NOTE Throughout this book are many **Practice Problems**—short questions designed to check your understanding of a topic before moving on to new material. Full solutions are given at the end of the section. Solve the following Practice Problem and then check your answer.

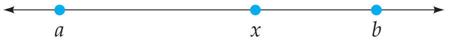
PRACTICE PROBLEM 1

Which number is smaller: $\frac{1}{100}$ or $-1{,}000{,}000$? Solution on page 15 >

Multiplying or dividing both sides of an inequality by a negative number reverses the direction of the inequality:

$$-3 < 2 \quad \text{but} \quad 3 > -2$$ Multiplying by -1

A **double inequality,** such as $a < x < b$, means that *both* the inequalities $a < x$ and $x < b$ hold. The inequality $a < x < b$ can be interpreted graphically as "x is between a and b."



$a < x < b$

LOOKING AHEAD

Sets and intervals will be important on page 33 when we define *domains* of functions.

Sets and Intervals

Braces { } are read "the set of all" and a **vertical bar** | is read "such that."

EXAMPLE 1 INTERPRETING SETS

The set of all

a. $\{ x \mid x > 3 \}$ means "the set of all x such that x is greater than 3."

Such that

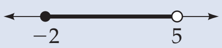
b. $\{ x \mid -2 < x < 5 \}$ means "the set of all x such that x is between -2 and 5."

PRACTICE PROBLEM 2

a. Write in set notation "the set of all x such that x is greater than or equal to -7."
b. Express in words: $\{ x \mid x < -1 \}$. Solution on page 15 >

The set $\{ x \mid 2 \le x \le 5 \}$ can be expressed in **interval notation** by enclosing the endpoints 2 and 5 in **square brackets,** [2, 5], to indicate that the endpoints are *included.* The set $\{ x \mid 2 < x < 5 \}$ can be written with **parentheses,** (2, 5), to indicate that the endpoints 2 and 5 are *excluded.* An interval is **closed** if it includes both endpoints, and **open** if it includes neither endpoint. The four types of intervals are shown below: a **solid dot** ● on the graph indicates that the point is *included* in the interval; a **hollow dot** ○ indicates that the point is *excluded.*

Finite Intervals

Interval Notation	Set Notation	Graph	Type	Brief Examples	
$[a, b]$	$\{ x \mid a \le x \le b \}$		Closed (includes endpoints)	$[-2, 5]$	
(a, b)	$\{ x \mid a < x < b \}$		Open (excludes endpoints)	$(-2, 5)$	
$[a, b)$	$\{ x \mid a \le x < b \}$		Half-open or half-closed	$[-2, 5)$	
$(a, b]$	$\{ x \mid a < x \le b \}$			$(-2, 5]$	

An interval may extend infinitely far to the *right* (indicated by the symbol ∞ for **infinity**) or infinitely far to the *left* (indicated by $-\infty$ for **negative infinity**). Note that ∞ and $-\infty$ are not numbers but are merely symbols to indicate that the interval extends endlessly in that direction. The infinite intervals in the following box are said to be **closed** or **open** depending on whether they *include* or *exclude* their single endpoint.

Infinite Intervals

Interval Notation	Set Notation	Graph	Type	Brief Examples	
$[a, \infty)$	$\{ x \mid x \geq a \}$	a	Closed	$[3, \infty)$	3
(a, ∞)	$\{ x \mid x > a \}$	a	Open	$(3, \infty)$	3
$(-\infty, a]$	$\{ x \mid x \leq a \}$	a	Closed	$(-\infty, 5]$	5
$(-\infty, a)$	$\{ x \mid x < a \}$	a	Open	$(-\infty, 5)$	5

We use *parentheses* rather than square brackets with ∞ and $-\infty$ since they are not actual numbers.

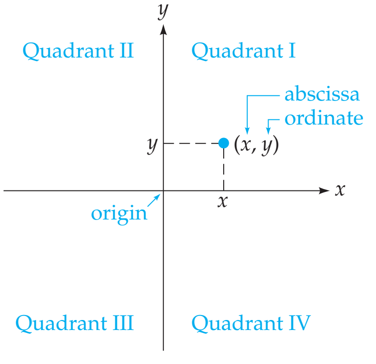
The interval $(-\infty, \infty)$ extends infinitely far in *both* directions (meaning the entire real line) and is also denoted by $\mathbb{R}$ (the set of all real numbers).

$\mathbb{R} = (-\infty, \infty)$

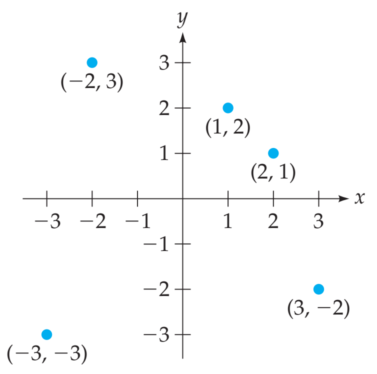
Cartesian Plane

Two real lines or **axes,** one horizontal and one vertical, intersecting at their zero points, define the **Cartesian plane.*** The point where they meet is called the **origin.** The axes divide the plane into four **quadrants,** I through IV, as shown below.

Any point in the Cartesian plane can be specified uniquely by an ordered pair of numbers (x, y); x, called the **abscissa** or ***x*-coordinate,** is the number on the horizontal axis corresponding to the point; y, called the **ordinate** or ***y*-coordinate,** is the number on the vertical axis corresponding to the point.



The Cartesian plane



The Cartesian plane with several points.
Order matters: (1, 2) is not the same as (2, 1)

LOOKING AHEAD

We will use the Δ notation again on page 95.

Lines and Slopes

The symbol Δ (read "delta," the Greek letter D) means "the change in." For any two points (x_1, y_1) and (x_2, y_2) we define

*So named because it was originated by the French philosopher and mathematician René Descartes (1596–1650). Following the custom of the day, Descartes signed his scholarly papers with his Latin name Cartesius, hence "Cartesian" plane.

$\Delta x = x_2 - x_1$ — The change in x is the difference in the x-coordinates

$\Delta y = y_2 - y_1$ — The change in y is the difference in the y-coordinates

Any two distinct points determine a line. A nonvertical line has a **slope** that measures the *steepness* of the line, and is defined as *the change in y divided by the change in x* for any two points on the line.

Take Note

One of the main purposes of calculus is to extend the concept of slope from lines to *curves*.

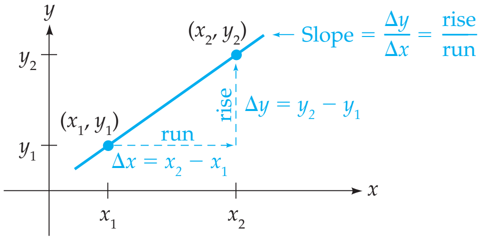
Slope of Line Through (x_1, y_1) and (x_2, y_2)

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope is the change in y over the change in x ($x_2 \neq x_1$)

Be Careful In slope, the x-values go in the *denominator*.

The changes Δy and Δx are often called, respectively, the "rise" and the "run," with the understanding that a negative "rise" means a "fall." Slope is then "rise over run."



EXAMPLE 2 FINDING SLOPES AND GRAPHING LINES

Find the slope of the line through each pair of points, and graph the line.

a. (2, 1), (3, 4) **b.** (2, 4), (3, 1)

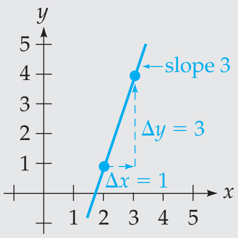
c. (−1, 3), (2, 3) **d.** (2, −1), (2, 3)

Solution

We use the slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ for each pair $(x_1, y_1), (x_2, y_2)$.

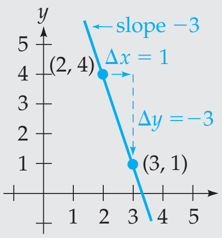
a. For (2, 1) and (3, 4) the slope is

$$\frac{4 - 1}{3 - 2} = \frac{3}{1} = 3.$$

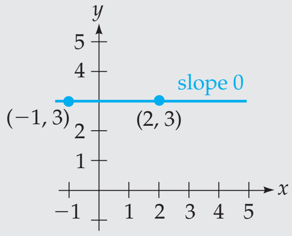


b. For (2, 4) and (3, 1) the slope is

$$\frac{1 - 4}{3 - 2} = \frac{-3}{1} = -3.$$

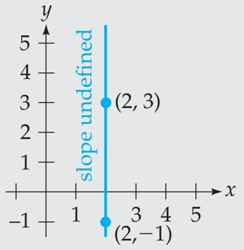


c. For $(-1, 3)$ and $(2, 3)$ the slope is $\dfrac{3-3}{2-(-1)} = \dfrac{0}{3} = 0.$



d. For $(2, -1)$ and $(2, 3)$ the slope is *undefined:*

$$\frac{3-(-1)}{2-2} = \frac{4}{0}.$$



Notice in the preceding graphs that when the x-coordinates are the same [as in part (d)], the line is *vertical*, and when the y-coordinates are the same [as in part (c)], the line is *horizontal*.

If $\Delta x = 1$, as in Examples 2a and 2b, then the slope is just the "rise," giving an alternative definition for slope:

$$\text{Slope} = \left(\begin{array}{c}\text{Amount that the line rises}\\ \text{when } x \text{ increases by 1}\end{array}\right)$$

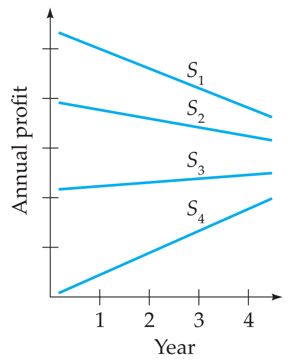
PRACTICE PROBLEM 3

A company president is considering four different business strategies, called S_1, S_2, S_3, and S_4, each with different projected future profits. The graph on the right shows the annual projected profit for the first few years for each of the strategies.

Which strategy yields:

a. the highest projected profit in year 1?
b. the highest projected profit in the long run?

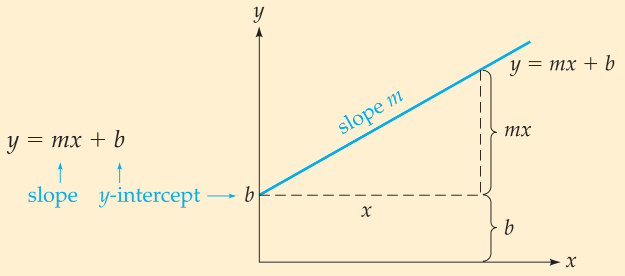
Solutions on page 15 >



Equations of Lines

The point where a nonvertical line crosses the y-axis is called the ***y*-intercept** of the line. The y-intercept can be given either as the y-coordinate b or as the point $(0, b)$. Such a line can be expressed very simply in terms of its slope and y-intercept, representing the points by variable coordinates (or "variables") x and y.

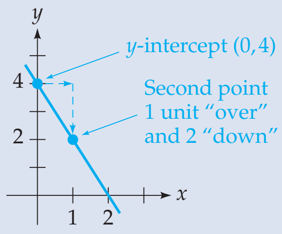
Slope-Intercept From of a Line

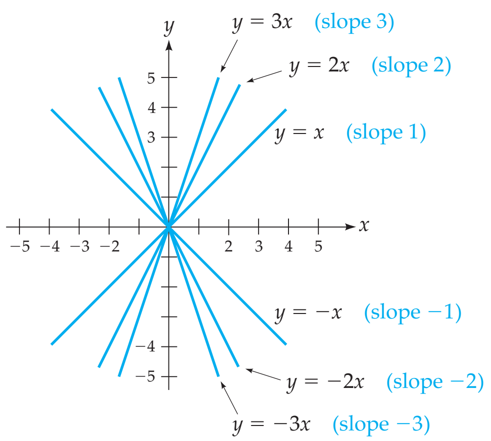


Brief Example

For the line with slope -2 and y-intercept 4:

$$y = -2x + 4$$





For lines through the origin, the equation takes the particularly simple form, $y = mx$ (since $b = 0$), as illustrated on the left.

The most useful equation for a line is the *point-slope form*.

Point-Slope Form of a Line

$$y - y_1 = m(x - x_1)$$

(x_1, y_1) = point on the line
m = slope

This form comes directly from the slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ by replacing x_2 and y_2 by x and y, and then multiplying each side by $(x - x_1)$. It is most useful when you know the slope of the line and a point on it.

EXAMPLE 3 USING THE POINT-SLOPE FORM

Find an equation of the line through $(6, -2)$ with slope $-\frac{1}{2}$.

Solution

$$y - (-2) = -\frac{1}{2}(x - 6) \qquad y - y_1 = m(x - x_1) \text{ with } y_1 = -2,\ m = -\tfrac{1}{2}, \text{ and } x_1 = 6$$

$$y + 2 = -\frac{1}{2}x + 3 \qquad \text{Eliminating parentheses}$$

$$y = -\frac{1}{2}x + 1 \qquad \text{Subtracting 2 from each side}$$

Alternatively, we could have found this equation using $y = mx + b$, replacing m by the given slope $-\frac{1}{2}$, and then substituting the given $x = 6$ and $y = -2$ to evaluate b.

EXAMPLE 4 FINDING AN EQUATION FOR A LINE THROUGH TWO POINTS

Find an equation for the line through the points $(4, 1)$ and $(7, -2)$.

Solution

The slope is not given, so we calculate it from the two points.

$$m = \frac{-2 - 1}{7 - 4} = \frac{-3}{3} = -1 \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \text{ with } (4, 1) \text{ and } (7, -2)$$

Then we use the point-slope formula with this slope and either of the two points.

$$y - 1 = -1(x - 4) \qquad y - y_1 = m(x - x_1) \text{ with slope } -1 \text{ and point } (4, 1)$$

$$y - 1 = -x + 4 \qquad \text{Eliminating parentheses}$$

$$y = -x + 5 \qquad \text{Adding 1 to each side}$$

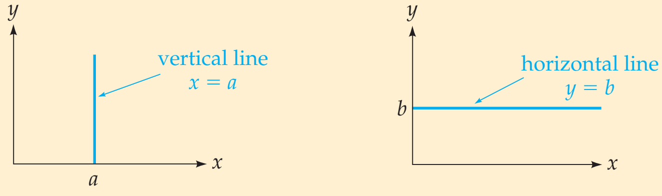
PRACTICE PROBLEM 4

Find the slope-intercept form of the line through the points $(2, 1)$ and $(4, 7)$.

Solution on page 15 >

Vertical and horizontal lines have particularly simple equations: a variable equaling a constant.

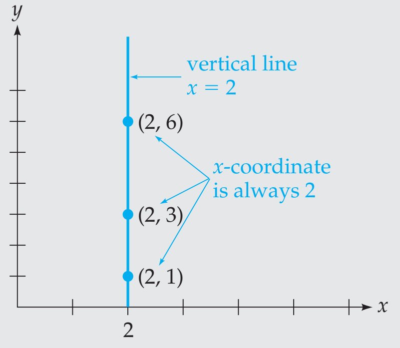
Vertical Line	***Horizontal Line***
$x = a$	$y = b$

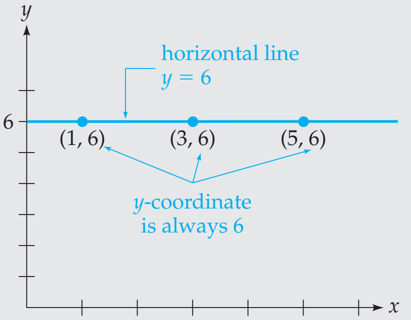


EXAMPLE 5 GRAPHING VERTICAL AND HORIZONTAL LINES

Graph the lines $x = 2$ and $y = 6$.

Solution





EXAMPLE 6 FINDING EQUATIONS OF VERTICAL AND HORIZONTAL LINES

a. Find an equation for the *vertical* line through (3, 2).

b. Find an equation for the *horizontal* line through (3, 2).

Solution

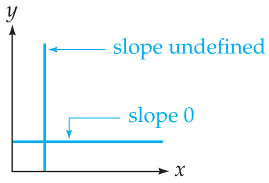
a. Vertical line $\quad x = 3 \quad$ $x = a$, with a being the x-coordinate from (3, 2)

b. Horizontal line $\quad y = 2 \quad$ $y = b$, with b being the y-coordinate from (3, 2)

PRACTICE PROBLEM 5

Find an equation for the vertical line through $(-2, 10)$.

Solution on page 15 >



In a vertical line, the x-coordinate does not change, so $\Delta x = 0$, making the slope $m = \Delta y/\Delta x$ *undefined*. Therefore, distinguish carefully between slopes of vertical and horizontal lines:

Vertical line: Slope is *undefined.*

Horizontal line: Slope *is* defined, and is *zero.*

There is one form that covers *all* lines, vertical and nonvertical.

General Linear Equation

$$ax + by = c$$

For constants a, b, c, with a and b not both zero

Any equation that can be written in this form is called a **linear equation,** and the variables are said to **depend linearly** on each other.

EXAMPLE 7 FINDING THE SLOPE AND THE Y-INTERCEPT FROM A LINEAR EQUATION

Find the slope and y-intercept of the line $2x + 3y = 12$.

Solution

We write the line in slope-intercept form. Solving for y:

$$3y = -2x + 12$$

Subtracting $2x$ from both sides of $2x + 3y = 12$

$$y = -\frac{2}{3}x + 4$$

Dividing each side by 3 gives the slope-intercept form $y = mx + b$

Therefore, the slope is $-\frac{2}{3}$ and the y-intercept is $(0, 4)$.

PRACTICE PROBLEM 6

Find the slope and y-intercept of the line $x - \dfrac{y}{3} = 2$.

Solution on page 15 >

Parallel and Perpendicular Lines

Slope can be used to determine whether two lines are parallel or perpendicular.

Slopes of Parallel and Perpendicular Lines

Two distinct lines are *parallel* if they have the *same* slope:

$$m_1 = m_2$$

Two lines are *perpendicular* if their slopes are *negative reciprocals*:

$$m_1 = -\frac{1}{m_2}$$

Brief Examples

The following pairs of numbers are *negative reciprocals* (reciprocals with opposite signs):

$$2 \quad \text{and} \quad -\frac{1}{2}$$

$$-\frac{3}{4} \quad \text{and} \quad \frac{4}{3}$$

$$\frac{1}{5} \quad \text{and} \quad -5$$

EXAMPLE 8 FINDING PARALLEL AND PERPENDICULAR LINES

Find an equation for the line through the point $(6, -3)$ that is (a) parallel to the line $2x + 3y = 12$ and (b) perpendicular to the line $2x + 3y = 12$.

Solution

a. Ordinarily, we would now find the slope of the line $2x + 3y = 12$. However, in Example 7 we found that the slope of this line is $-\frac{2}{3}$. Therefore, we want the line with slope $m = -\frac{2}{3}$ that passes through $(6, -3)$. We use the slope-intercept form with the above slope and point:

$$y - (-3) = -\frac{2}{3}(x-6) \qquad y - y_1 = m(x - x_1) \text{ with } m = -\tfrac{2}{3},\ x_1 = 6, \text{ and } y_1 = -3$$

$$y + 3 = -\frac{2}{3}x + 4 \qquad \text{Multiplying out and simplifying}$$

$$y = -\frac{2}{3}x + 1 \qquad \text{Answer (after subtracting 3)}$$

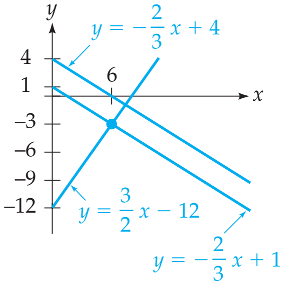
b. The *perpendicular* line will have slope that is the *negative reciprocal* of $-\frac{2}{3}$, which is $m = \frac{3}{2}$. With this slope and the same point, the slope-intercept form gives

$$y - (-3) = \frac{3}{2}(x - 6) \qquad y - y_1 = m(x - x_1) \text{ with } m = \tfrac{3}{2},\ x_1 = 6, \text{ and } y_1 = -3$$

$$y + 3 = \frac{3}{2}x - 9 \qquad \text{Multiplying out and simplifying}$$

$$y = \frac{3}{2}x - 12 \qquad \text{Answer (after subtracting 3)}$$

The graphs of the three lines are shown on the left.



PRACTICE PROBLEM 7

Find an equation for the line through the point (9, 2) that is perpendicular to the line $x - \frac{y}{3} = 2$. [*Hint*: Use your answer to Practice Problem 6.]

Solution on page 16 >

Linear Regression

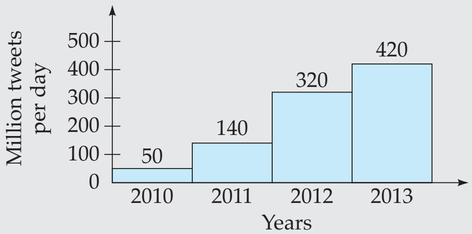
Take Note

You don't need to know about regression to read most of this book.

Given two points, we can find a line through them, as in Example 4. However, some real-world situations involve *many* data points, which may lie approximately but not exactly on a line. How can we find the line that, in some sense, *lies closest* to the points or *best approximates* the points? The most widely used technique is called **linear regression** or **least squares,** and its mathematical basis will be explained in Section 7.4. Even before studying its mathematical basis, however, we can easily find the regression line using a graphing calculator (or spreadsheet or other computer software).

EXAMPLE 9 LINEAR REGRESSION USING A GRAPHING CALCULATOR

The following graph shows the average number of "tweets" per day sent on Twitter in recent years.



Source: Twitter

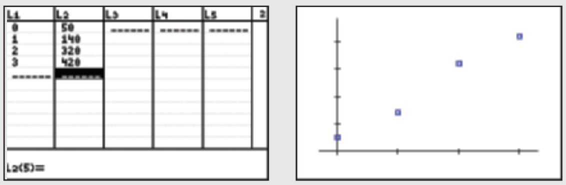
a. Use linear regression to fit a line to the data.

b. Interpret the slope of the line.

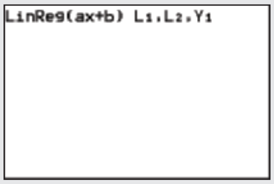
c. Use the regression line to predict the number of tweets per day in the year 2022.

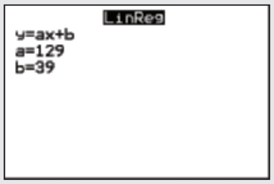
Solution

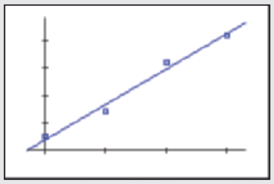
a. We number the years with x-values 0−3, so x stands for *years since 2010* (we could choose other x-values instead). We enter the data into lists, as shown in the first screen below (as explained in the appendix *Graphing Calculator Basics—Entering Data* on page A3), and use *ZoomStat* to graph the data points.



Then (using STAT, CALC, and LinReg) graph the regression along with the data points.





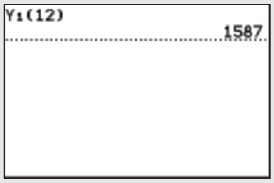


The regression line, which clearly fits the points quite well, is

$$y = 129x + 39$$

b. Since x is in years, the slope 129 means that the number of tweets per day increases by about 129 million each year.

c. To predict the number of tweets per day in the year 2022, we evaluate Y1 at 12 (since $x = 12$ corresponds to 2022). From the screen on the right, if the current trend continues, about 1.6 billion tweets per day will be sent in 2022.



Solutions TO PRACTICE PROBLEMS

1. $-1{,}000{,}000$ [the negative sign makes it less than (to the left of) the positive number $\frac{1}{100}$]

2. a. $\{ x \mid x \geq -7 \}$

b. The set of all x such that x is less than -1

3. a. S_1 **b.** S_4

4. $m = \dfrac{7-1}{4-2} = \dfrac{6}{2} = 3$ From points (2, 1) and (4, 7)

$y - 1 = 3(x - 2)$ Using the point-slope form with $(x_1, y_1) = (2, 1)$

$y - 1 = 3x - 6$

$y = 3x - 5$

5. $x = -2$

6. $x - \dfrac{y}{3} = 2$

$-\dfrac{y}{3} = -x + 2$ Subtracting x from each side

$y = 3x - 6$ Multiplying each side by -3

Slope is $m = 3$ and y-intercept is $(0, -6)$.

7. $m = -\frac{1}{3}$ The negative reciprocal of the slope found in Practice Problem 6

$y - 2 = -\frac{1}{3}(x - 6)$ $y - y_1 = m(x - x_1)$ with $m = -\frac{1}{3}$, $x_1 = 6$, and $y_1 = 2$

$y = -\frac{1}{3}x + 4$ After simplifying

1.1 Section Summary

An **interval** is a set of real numbers corresponding to a section of the real line. The interval is **closed** if it contains all of its endpoints, and **open** if it contains none of its endpoints.

The nonvertical line through two points (x_1, y_1) and (x_2, y_2) has **slope**

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_1 \neq x_2$$

The slope of a *vertical* line is *undefined* or, equivalently, *does not exist.*

There are five **equations** or **forms** for lines:

$y = mx + b$ Slope-intercept form
m = slope, b = y-intercept

$y - y_1 = m(x - x_1)$ Point-slope form
(x_1, y_1) = point, m = slope

$x = a$ Vertical line (slope undefined)
a = x-intercept

$y = b$ Horizontal line (slope zero)
b = y-intercept

$ax + by = c$ General linear equation

A graphing calculator can find the regression line for a set of points, which can then be used to predict future trends.

1.1 Exercises

1–4. Write each interval in set notation and graph it on the real line.

1. $[0, 6)$ **2.** $(-3, 5]$ **3.** $(-\infty, 2]$ **4.** $[7, \infty)$

5. Given the equation $y = 5x - 12$, how will y change if x:
a. Increases by 3 units?
b. Decreases by 2 units?

6. Given the equation $y = -2x + 7$, how will y change if x:
a. Increases by 5 units?
b. Decreases by 4 units?

7–14. Find the slope (if it is defined) of the line determined by each pair of points.

7. $(2, 3)$ and $(4, -1)$ **8.** $(3, -1)$ and $(5, 7)$
9. $(-4, 0)$ and $(2, 2)$ **10.** $(-1, 4)$ and $(5, 1)$
11. $(0, -1)$ and $(4, -1)$ **12.** $(-2, \frac{1}{2})$ and $(5, \frac{1}{2})$
13. $(2, -1)$ and $(2, 5)$ **14.** $(6, -4)$ and $(6, -3)$

15–32. For each equation, find the slope m and y-intercept $(0, b)$ (when they exist) and draw the graph.

15. $y = 3x - 4$ **16.** $y = 2x$
17. $y = -\frac{1}{2}x$ **18.** $y = -\frac{1}{3}x + 2$
19. $y = 4$ **20.** $y = -3$
21. $x = 4$ **22.** $x = -3$
23. $2x - 3y = 12$ **24.** $3x + 2y = 18$
25. $x + y = 0$ **26.** $x = 2y + 4$
27. $x - y = 0$ **28.** $y = \frac{2}{3}(x - 3)$
29. $y = \frac{x + 2}{3}$ **30.** $\frac{x}{2} + \frac{y}{3} = 1$

31. $\dfrac{2x}{3} - y = 1$

32. $\dfrac{x+1}{2} + \dfrac{y+1}{2} = 1$

33–46. Write an equation of the line satisfying the following conditions. If possible, write your answer in the form $y = mx + b$.

33. Slope -2.25 and y-intercept 3

34. Slope $\frac{2}{3}$ and y-intercept -8

35. Slope 5 and passing through the point $(-1, -2)$

36. Slope -1 and passing through the point $(4, 3)$

37. Horizontal and passing through the point $(1.5, -4)$

38. Horizontal and passing through the point $(\frac{1}{2}, \frac{3}{4})$

39. Vertical and passing through the point $(1.5, -4)$

40. Vertical and passing through the point $(\frac{1}{2}, \frac{3}{4})$

41. Passing through the points $(5, 3)$ and $(7, -1)$

42. Passing through the points $(3, -1)$ and $(6, 0)$

43. Passing through the points $(1, -1)$ and $(5, -1)$

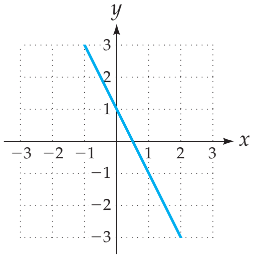
44. Passing through the points $(2, 0)$ and $(2, -4)$

45. Passing through the point $(12, 2)$ that is (a) parallel to the line $4y - 3x = 5$ and (b) perpendicular to the line $4y - 3x = 5$

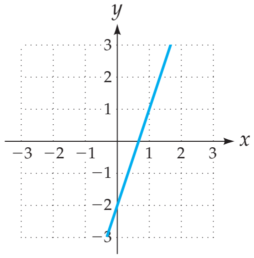
46. Passing through the point $(-6, 5)$ that is (a) parallel to the line $x + 3y = 7$ (b) perpendicular to the line $x + 3y = 7$

47–50. Write an equation of the form $y = mx + b$ for each line in the following graphs. [*Hint:* Either find the slope and y-intercept or use any two points on the line.]

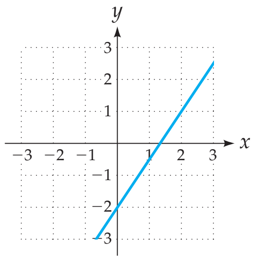
47.



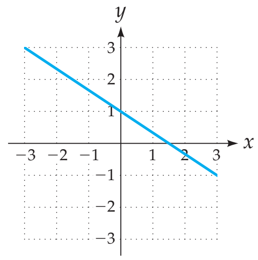
48.



49.

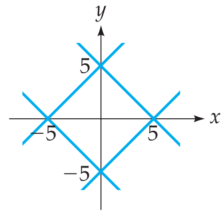


50.

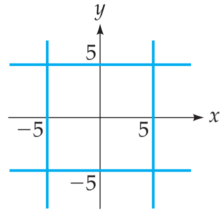


51–52. Write equations for the lines determining the four sides of each figure.

51.



52.



53. Show that $y - y_1 = m(x - x_1)$ simplifies to $y = mx + b$ if the point (x_1, y_1) is the y-intercept $(0, b)$.

54. Show that the linear equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ has x-intercept $(a, 0)$ and y-intercept $(0, b)$. (The x-intercept is the point where the line crosses the x-axis.)

55. **a.** Graph the lines $y_1 = -x$, $y_2 = -2x$, and $y_3 = -3x$ on the window $[-5, 5]$ by $[-5, 5]$. Observe how the coefficient of x changes the slope of the line.

b. Predict how the line $y = -9x$ would look, and then check your prediction by graphing it.

56. **a.** Graph the lines $y_1 = x + 2$, $y_2 = x + 1$, $y_3 = x$, $y_4 = x - 1$, and $y_5 = x - 2$ on the window $[-5, 5]$ by $[-5, 5]$. Observe how the constant changes the position of the line.

b. Predict how the lines $y = x + 4$ and $y = x - 4$ would look, and then check your prediction by graphing them.

Applied Exercises

57. **BUSINESS: Energy Usage** A utility considers demand for electricity "low" if it is below 8 mkW (million kilowatts), "average" if it is at least 8 mkW but below 20 mkW, "high" if it is at least 20 mkW but below 40 mkW, and "critical" if it is 40 mkW or more. Express these demand levels in interval notation. [*Hint:* The interval for "low" is [0, 8).]

58. **GENERAL: Grades** If a grade of 90 through 100 is an A, at least 80 but less than 90 is a B, at least 70 but less than 80 a C, at least 60 but less than 70 a D, and below 60 an F, write these grade levels in interval form (ignoring rounding). [*Hint:* F would be [0, 60).]

59. **ATHLETICS: Mile Run** Read the Application Preview on pages 3–4.

a. Use the regression line $y = -0.356x + 257.44$ to predict the world record in the year 2020. [*Hint:* If x represents years after 1900, what value of x corresponds to the year 2020? The resulting y will be in seconds, and should be converted to minutes and seconds.]

b. According to this formula, when will the record be 3 minutes 30 seconds? [*Hint:* Set the formula equal to 210 seconds and solve. What year corresponds to this x-value?]

60. ATHLETICS: Mile Run Read the Application Preview on pages 3–4. Evaluate the regression line $y = -0.356x + 257.44$ at $x = 720$ and at $x = 722$ (corresponding to the years 2620 and 2622). Does the formula give reasonable times for the mile record in these years? [*Moral:* Linear trends may not continue indefinitely.]

61. BUSINESS: U.S. Computer Sales Recently, tablet computer sales in the United States have been growing approximately linearly. In 2011 sales were 70 million units, and in 2013 sales were 146 million units.

a. Use the two (year, sales) data points (1, 70) and (3, 146) to find the linear relationship $y = mx + b$ between $x =$ years since 2010 and $y =$ sales (in millions).
b. Interpret the slope of the line.
c. Use the linear relationship to predict sales in the year 2020.

Note: Tablet computers include iPads, Kindles, and Nooks.

Source: Standard & Poor's

62. ECONOMICS: Per Capita Personal Income In the short run, per capita personal income (PCPI) in the United States grows approximately linearly. In 2009 PCPI was 38.6, and in 2012 it had grown to 42.8 (both in thousands of dollars).

a. Use the two (year, PCPI) data points (1, 38.6) and (4, 42.8) to find the linear relationship $y = mx + b$ between $x =$ years since 2008 and $y =$ PCPI.
b. Interpret the slope of the line.
c. Use your linear relationship to predict PCPI in 2020.

Source: Bureau of Economic Analysis

63. GENERAL: Temperature On the Fahrenheit temperature scale, water freezes at 32° and boils at 212°. On the Celsius (centigrade) scale, water freezes at 0° and boils at 100°.

a. Use the two (Celsius, Fahrenheit) data points (0, 32) and (100, 212) to find the linear relationship $y = mx + b$ between $x =$ Celsius temperature and $y =$ Fahrenheit temperature.
b. Find the Fahrenheit temperature that corresponds to 20° Celsius.

64. BUSINESS: Financial Engineering Salaries Starting salaries in the United States for associates with master's degrees in financial engineering (FE) have been rising approximately linearly, from \$74,800 in 2009 to \$89,800 in 2013.

a. Use the two (year, salary) data points (0, 74.8) and (4, 89.8) to find the linear relationship $y = mx + b$ between $x =$ years since 2009 and $y =$ salary in thousands of dollars.
b. Use your formula to predict a new FE associate's salary in 2021. [*Hint:* Since x is years after 2009, what x-value corresponds to 2021?]

Source: salaryquest.com

65–66. BUSINESS: Straight-Line Depreciation
Straight-line depreciation is a method for estimating the value of an asset (such as a piece of machinery) as it loses value ("depreciates") through use. Given the original *price* of an asset, its *useful lifetime*, and its *scrap value* (its value at the end of its useful lifetime), the value of the asset after t years is given by the formula:

$$\text{Value} = (\text{Price}) - \left(\frac{(\text{Price}) - (\text{Scrap value})}{(\text{Useful lifetime})}\right)\cdot t$$

$$\text{for} \quad 0 \le t \le (\text{Useful lifetime})$$

65. a. A farmer buys a harvester for \$50,000 and estimates its useful life to be 20 years, after which its scrap value will be \$6000. Use the formula above to find a formula for the value V of the harvester after t years, for $0 \le t \le 20$.
b. Use your formula to find the value of the harvester after 5 years.
c. Graph the function found in part (a) on a graphing calculator on the window [0, 20] by [0, 50,000]. [*Hint:* Use x instead of t.]

66. a. A newspaper buys a printing press for \$800,000 and estimates its useful life to be 20 years, after which its scrap value will be \$60,000. Use the formula above Exercise 65 to find a formula for the value V of the press after t years, for $0 \le t \le 20$.
b. Use your formula to find the value of the press after 10 years.
c. Graph the function found in part (a) on a graphing calculator on the window [0, 20] by [0, 800,000]. [*Hint:* Use x instead of t.]

67–68. BUSINESS: Isocost Lines An *isocost line* (*iso* means "same") shows the different combinations of labor and capital (the value of factory buildings, machinery, and so on) a company may buy for the same total cost. An isocost line has equation

$$wL + rK = C \qquad \text{for} \qquad L \ge 0, \quad K \ge 0$$

where L is the units of labor costing w dollars per unit, K is the units of capital purchased at r dollars per unit, and C is the total cost. Since both L and K must be non-negative, an isocost line is a line segment in just the first quadrant.

67. a. Write the equation of the isocost line with $w = 10$, $r = 5$, $C = 1000$, and graph it in the first quadrant.
b. Verify that the following (L, K) pairs all have the same total cost.

$$(100, 0), \quad (75, 50), \quad (20, 160), \quad (0, 200)$$

68. a. Write the equation of the isocost line with $w = 8$, $r = 6$, $C = 15{,}000$, and graph it in the first quadrant.

b. Verify that the following (L, K) pairs all have the same total cost.

$(1875, 0)$, $(1200, 900)$, $(600, 1700)$, $(0, 2500)$

69. SOCIAL SCIENCE: Age at First Marriage Americans are marrying later and later. Based on data for the years 2000 to 2011, the median age at first marriage for men is $y_1 = 0.18x + 26.7$, and for women it is $y_2 = 0.14x + 25$, where x is the number of years since 2000.

a. Graph these lines on the window [0, 30] by [0, 35].

b. Use these lines to predict the median marriage ages for men and women in the year 2020. [*Hint:* Which x-value corresponds to 2020?]

c. Predict the median marriage ages for men and women in the year 2030.

Source: U.S. Census Bureau

70. SOCIAL SCIENCE: Equal Pay for Equal Work Women's pay has often lagged behind men's, although Title VII of the Civil Rights Act requires equal pay for equal work. Based on data from 2000–2011, women's annual earnings as a percent of men's can be approximated by the formula $y = 0.36x + 77$, where x is the number of years since 2000. (For example, $x = 10$ gives $y = 80.6$, so in 2010 women's wages were about 80.6% of men's wages.)

a. Graph this line on the window [0, 30] by [0, 100].

b. Use this line to predict the percentage in the year 2020. [*Hint:* Which x-value corresponds to 2020?]

c. Predict the percentage in the year 2025.

Source: U.S. Department of Labor—Women's Bureau

71. SOCIAL SCIENCES: Smoking and Income Based on a recent study, the probability that someone is a smoker decreases with the person's income. If someone's family income is x thousand dollars, then the probability (expressed as a percentage) that the person smokes is approximately $y = -0.31x + 40$ (for $10 \le x \le 100$).

a. Graph this line on the window [0, 100] by [0, 50].

b. What is the probability that a person with a family income of $40,000 is a smoker? [*Hint:* Since x is in thousands of dollars, what x-value corresponds to $40,000?]

c. What is the probability that a person with a family income of $70,000 is a smoker?

Round your answers to the nearest percent.

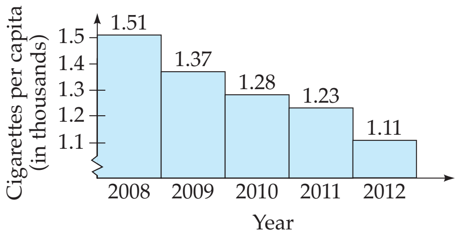
Source: Journal of Risk and Uncertainty 21(2/3)

72. ECONOMICS: Does Money Buy Happiness? Several surveys in the United States and Europe have asked people to rate their happiness on a scale of 3 = "very happy," 2 = "fairly happy," and 1 = "not too happy," and then tried to correlate the answer with the person's income. For those in one income group (making $25,000 to $55,000) it was found that their "happiness" was approximately given by $y = 0.065x - 0.613$. Find the reported "happiness" of a person with the following incomes (rounding your answers to one decimal place).

a. $25,000 **b.** $35,000 **c.** $45,000

Source: Review of Economics and Statistics 85(4)

73. BUSINESS: Cigarettes The following graph gives the number of cigarettes per capita sold to adults in the United States in recent years.



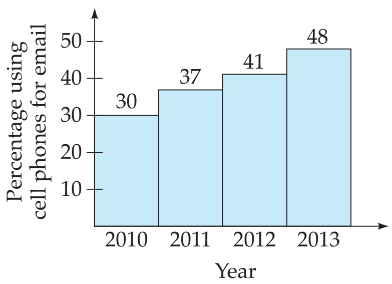
a. Number the years (bars) with x-values 1–5 so that x stands for *years since 2007*, and use linear regression to fit a line to the data. State the regression formula. [*Hint:* See Example 9.]

b. Interpret the slope of the line.

c. Use the regression line to predict the number of cigarettes per capita sold in the year 2020.

Source: Centers for Disease Control

74. SOCIAL SCIENCES Email on Cell Phones Although 91% of American adults own a cell phone, not all use it to check their email, many preferring to use computers instead. The percentages of cell phone owners who use their phones for email in recent years are shown in the following graph.



a. Number the years (bars) with x-values 1–4 so that x stands for *years since 2009*, and use linear regression to fit a line to the data. State the regression formula. [*Hint:* See Example 9.]

b. Interpret the slope of the line.

c. Use the regression formula to predict the percentage of Americans who will use their cell phones for email in 2020.

Source: Pew Internet

75–76. BIOMEDICAL SCIENCES: Life Expectancy The following tables give the life expectancy for a newborn child born in the indicated year. (Exercise 75 is for males, Exercise 76 for females.)

75.

Birth Year	1970	1980	1990	2000	2010
Life Expectancy (male)	67.1	70.0	71.8	74.1	75.7

76.

Birth Year	1970	1980	1990	2000	2010
Life Expectancy (female)	74.7	77.4	78.8	79.3	80.8

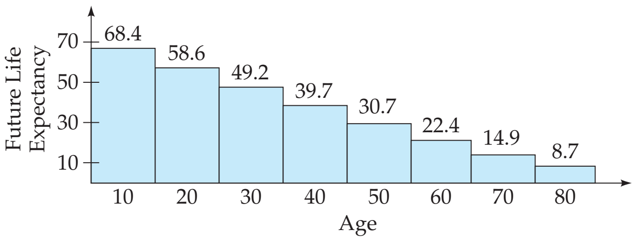
a. Number the data columns with x-values 1–5 and use linear regression to fit a line to the data. State the regression formula. [*Hint:* See Example 9.]

b. Interpret the slope of the line. From your answer, what is the *yearly* change in life expectancy?

c. Use the regression line to predict life expectancy for a child born in 2025.

Source: U.S. Census Bureau

77. BIOMEDICAL SCIENCES: Future Life Expectancy Clearly, as people age they should expect to live for fewer additional years. The following graph gives the future life expectancy for Americans of different ages.



a. Letting $x = \text{Age}$, use linear regression to fit a line to the data. State the regression formula. [*Hint:* See Example 9.]

b. Interpret the slope of the line.

c. Use the regression line to estimate future longevity at age 25.

d. Would it make sense to use the regression line to estimate longevity at age 90? What future longevity would the line predict?

Source: National Center for Health Statistics

78. GENERAL: Seat Belt Use Because of driver education programs and stricter laws, seat belt use has increased steadily over recent decades. The following table gives the percentage of automobile occupants using seat belts in selected years.

Year	1995	2000	2005	2010
Seat Belt Use (%)	60	71	81	85

a. Number the data columns with x-values 1–4 and use linear regression to fit a line to the data. State the regression formula. [*Hint:* See Example 9.]

b. Interpret the slope of the line. From your answer, what is the *yearly* increase?

c. Use the regression line to predict seat belt use in 2017. [*Hint:* What (decimal) x-value corresponds to 2017?]

d. Would it make sense to use the regression line to predict seat belt use in 2025? What percentage would you get?

Source: National Highway Traffic Safety Administration

Conceptual Exercises

79. True or False: ∞ is the largest number.

80. True or False: All negative numbers are smaller than all positive numbers.

81. Give two definitions of *slope.*

82. Fill in the missing words: If a line slants downward as you go to the right, then its __________ is ____________.

83. True or False: A vertical line has slope 0.

84. True or False: Every line has a slope.

85. True or False: Every line can be expressed in the form $ax + by = c$.

86. True or False: $x = 3$ is a vertical line.

87. True or False: The slope of a line is $\dfrac{x_2 - x_1}{y_2 - y_1}$.

88. True or False: A vertical line can be expressed in slope-intercept form.

89. A 5-foot-long board is leaning against a wall so that it meets the wall at a point 4 feet above the floor. What is the slope of the board? [*Hint:* Draw a picture.]

90. A 5-foot-long ramp is to have a slope of 0.75. How high should the upper end be elevated above the lower end? [*Hint:* Draw a picture.]

Explorations and Excursions

The following problems extend and augment the material presented in the text.

More About Linear Equations

91. Find the x-intercept $(a, 0)$ where the line $y = mx + b$ crosses the x-axis. Under what condition on m will a single x-intercept exist?

92. i. Show that the general linear equation $ax + by = c$ with $b \neq 0$ can be written as $y = -\dfrac{a}{b}x + \dfrac{c}{b}$ which is the equation of a line in slope-intercept form.

ii. Show that the general linear equation $ax + by = c$ with $b = 0$ but $a \neq 0$ can be written as $x = \frac{c}{a}$, which is the equation of a vertical line.

[*Note:* Since these steps are reversible, parts (i) and (ii) together show that the general linear equation $ax + by = c$ (for a and b not both zero) includes vertical and nonvertical lines.]

Beverton-Holt Recruitment Curve

93–94. Some organisms exhibit a density-dependent mortality from one generation to the next. Let $R > 1$ be the net reproductive rate (that is, the number of surviving offspring per parent), let $x > 0$ be the density of parents and y be the density of surviving offspring. The *Beverton-Holt recruitment curve* is

$$y = \frac{Rx}{1 + \left(\frac{R-1}{K}\right)x}$$

where $K > 0$ is the *carrying capacity* of the environment. Notice that if $x = K$, then $y = K$.

93. Show that if $x < K$, then $x < y < K$. Explain what this means about the population size over successive generations if the initial population is smaller than the carrying capacity of the environment.

94. Show that if $x > K$, then $K < y < x$. Explain what this means about the population size over successive generations if the initial population is larger than the carrying capacity of the environment.

1.2 Exponents

Introduction

Not all variables are related linearly. In this section we will discuss exponents, which will enable us to express many *nonlinear* relationships.

Positive Integer Exponents

Numbers may be expressed with exponents, as in $2^3 = 2 \cdot 2 \cdot 2 = 8$. More generally, for any positive integer n, x^n means the product of n x's.

$$x^n = \overbrace{x \cdot x \cdot \cdots \cdot x}^{n}$$

The number being raised to the power is called the **base** and the power is the **exponent:**

$$x^n$$

Exponent or power

Base

There are several *properties of exponents* for simplifying expressions. The first three are known, respectively, as the addition, subtraction, and multiplication properties of exponents.

Properties of Exponents

		Brief Examples
$x^m \cdot x^n = x^{m+n}$	To *multiply* powers of the same base, *add* the exponents	$x^2 \cdot x^3 = x^5$
$\frac{x^m}{x^n} = x^{m-n}$	To *divide* powers of the same base, *subtract* the exponents (top exponent minus bottom exponent)	$\frac{x^5}{x^3} = x^2$
$(x^m)^n = x^{m \cdot n}$	To raise a power to a power, *multiply* the powers	$(x^2)^3 = x^6$

$(xy)^n = x^n \cdot y^n$	To raise a product to a power, raise *each factor* to the power	$(2x)^3 = 2^3 \cdot x^3 = 8x^3$
$\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$	To raise a fraction to a power, raise the numerator *and* denominator to the power	$\left(\dfrac{x}{5}\right)^3 = \dfrac{x^3}{5^3} = \dfrac{x^3}{125}$

PRACTICE PROBLEM 1

Simplify: **a.** $\dfrac{x^5 \cdot x}{x^2}$ **b.** $[(x^3)^2]^2$ **c.** $[2x^2 y^4]^3$

Solutions on page 28 >

Remember: For exponents in the form $x^2 \cdot x^3 = x^5$, *add* exponents.
For exponents in the form $(x^2)^3 = x^6$, *multiply* exponents.

Zero and Negative Exponents

Any number except zero can be raised to a negative or zero power.

Zero and Negative Integer Exponents

For $x \neq 0$		Brief Examples
$x^0 = 1$	x to the power 0 is one	$5^0 = 1$
$x^{-1} = \dfrac{1}{x}$	x to the power -1 is one over x	$7^{-1} = \dfrac{1}{7}$
$x^{-2} = \dfrac{1}{x^2}$	x to the power -2 is one over x squared	$3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$
$x^{-n} = \dfrac{1}{x^n}$	x to a negative power is one over x to the positive power	$(-2)^{-3} = \dfrac{1}{(-2)^3} = \dfrac{1}{-8} = -\dfrac{1}{8}$

Note that 0^0 and 0^{-3} are *undefined*.

The definitions of x^0 and x^{-n} are motivated by the following calculations.

$$1 = \frac{x^2}{x^2} = x^{2-2} = x^0$$

The subtraction property of exponents leads to $x^0 = 1$

$$\frac{1}{x^n} = \frac{x^0}{x^n} = x^{0-n} = x^{-n}$$

$x^0 = 1$ and the subtraction property of exponents lead to $x^{-n} = \dfrac{1}{x^n}$

PRACTICE PROBLEM 2

Evaluate: **a.** 2^0 **b.** 2^{-4}

Solutions on page 28 >

A fraction to a negative power means *division* by the fraction, so we "invert and multiply."

$$\left(\frac{x}{y}\right)^{-1} = \frac{1}{\frac{x}{y}} = 1 \cdot \frac{y}{x} = \frac{y}{x}$$

Reciprocal of the original fraction

Therefore, for $x \neq 0$ and $y \neq 0$,

$$\left(\frac{x}{y}\right)^{-1} = \frac{y}{x}$$

A fraction to the power -1 is the reciprocal of the fraction

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^{n}$$

A fraction to the negative power is the reciprocal of the fraction to the positive power

EXAMPLE 1 SIMPLIFYING FRACTIONS TO NEGATIVE EXPONENTS

a. $\left(\frac{3}{2}\right)^{-1} = \frac{2}{3}$

Reciprocal of the original

b. $\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^{3} = \frac{2^3}{1^3} = 8$

PRACTICE PROBLEM 3

Simplify: $\left(\frac{2}{3}\right)^{-2}$

Solution on page 28 >

Roots and Fractional Exponents

We may take the square root of any *nonnegative* number, and the cube root of *any* number.

EXAMPLE 2 EVALUATING ROOTS

a. $\sqrt{9} = 3$

b. $\sqrt{-9}$ is undefined

Square root of negative numbers are not defined

c. $\sqrt[3]{8} = 2$

d. $\sqrt[3]{-8} = -2$

Cube roots of negative numbers *are* defined

e. $\sqrt[3]{\frac{27}{8}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$

There are *two* square roots of 9, namely 3 and -3, but the radical sign $\sqrt{}$ means just the *positive* one (the "principal" square root).

$\sqrt[n]{a}$ means the principal nth root of a.

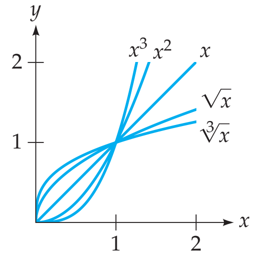
Principal means the positive root if there are two

In general, we may take *odd* roots of *any* number, but *even* roots only if the number is positive or zero.

EXAMPLE 3 **EVALUATING ROOTS OF POSITIVE AND NEGATIVE NUMBERS**

Odd roots of negative numbers *are* defined

a. $\sqrt[4]{81} = 3$ **b.** $\sqrt[5]{-32} = -2$ Since $(-2)^5 = -32$



The diagram on the left shows the graphs of some powers and roots of x for $x > 0$. Which of these would *not* be defined for $x < 0$? [*Hint:* See Example 2.]

Fractional Exponents

Fractional exponents are defined as follows:

Powers of the Form $\frac{1}{n}$

		Brief Examples
$x^{\frac{1}{2}} = \sqrt{x}$	Power $\frac{1}{2}$ means the principal square root	$9^{\frac{1}{2}} = \sqrt{9} = 3$
$x^{\frac{1}{3}} = \sqrt[3]{x}$	Power $\frac{1}{3}$ means the cube root	$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$
$x^{\frac{1}{n}} = \sqrt[n]{x}$	Power $\frac{1}{n}$ means the principal nth root (for a positive integer n)	$(-32)^{\frac{1}{5}} = \sqrt[5]{-32} = -2$

The definition of $x^{\frac{1}{2}}$ is motivated by the multiplication property of exponents:

$$\left(x^{\frac{1}{2}}\right)^2 = x^{\frac{1}{2}\cdot 2} = x^1 = x$$

Taking square roots of each side of $\left(x^{\frac{1}{2}}\right)^2 = x$ gives

$$x^{\frac{1}{2}} = \sqrt{x}$$

x to the half power means the square root of x

EXAMPLE 4 **EVALUATING FRACTIONAL EXPONENTS**

a. $\left(\dfrac{4}{25}\right)^{\frac{1}{2}} = \sqrt{\dfrac{4}{25}} = \dfrac{\sqrt{4}}{\sqrt{25}} = \dfrac{2}{5}$

b. $\left(-\dfrac{27}{8}\right)^{\frac{1}{3}} = \sqrt[3]{-\dfrac{27}{8}} = -\dfrac{\sqrt[3]{27}}{\sqrt[3]{8}} = -\dfrac{3}{2}$

PRACTICE PROBLEM 4

Evaluate: **a.** $(-27)^{\frac{1}{3}}$ **b.** $\left(\dfrac{16}{81}\right)^{\frac{1}{4}}$

Solutions on page 28 >

To define $x^{\frac{m}{n}}$ for positive integers m and n, the exponent $\frac{m}{n}$ must be fully reduced (for example, $\frac{4}{6}$ must be reduced to $\frac{2}{3}$). Then

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(x^m\right)^{\frac{1}{n}}$$

Since in both cases the exponents multiply to $\frac{m}{n}$

Therefore, we define:

Fractional Exponents

$$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m}$$

$x^{m/n}$ means the mth power of the nth root, or equivalently, the nth root of the mth power

LOOKING AHEAD

Fractional exponents will be important when we use the *power rule* on page 100 and beyond.

Both expressions, $\left(\sqrt[n]{x}\right)^m$ and $\sqrt[n]{x^m}$, will give the same answer. In either case *the numerator determines the power* and *the denominator determines the root*.

$$x^{\frac{m}{n}} \quad \begin{matrix} \leftarrow \text{Power} \\ \leftarrow \text{Root} \end{matrix}$$

Power over root

EXAMPLE 5 EVALUATING FRACTIONAL EXPONENTS

a. $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$ First the power, then the root

b. $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$ First the root, then the power

same

c. $25^{3/2} = \left(\sqrt{25}\right)^3 = (5)^3 = 125$

d. $\left(\frac{-27}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{-27}{8}}\right)^2 = \left(\frac{-3}{2}\right)^2 = \frac{9}{4}$

PRACTICE PROBLEM 5

Evaluate: **a.** $16^{3/2}$ **b.** $(-8)^{2/3}$

Solutions on page 28 >

EXAMPLE 6 EVALUATING NEGATIVE FRACTIONAL EXPONENTS

a. $8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{\left(\sqrt[3]{8}\right)^2} = \frac{1}{2^2} = \frac{1}{4}$

A negative exponent means the reciprocal of the number to the positive exponent, which is then evaluated as before

b. $\underbrace{\left(\frac{9}{4}\right)^{-3/2}}_{\text{Negative exponent}} = \underbrace{\left(\frac{4}{9}\right)^{3/2}}_{\text{Reciprocal to the positive exponent}} = \underbrace{\left(\sqrt{\frac{4}{9}}\right)^3}_{\text{Interpreting the power 3/2}} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

PRACTICE PROBLEM 6

Evaluate: **a.** $25^{-3/2}$ **b.** $\left(\frac{1}{4}\right)^{-1/2}$ **c.** $5^{1.3}$ [*Hint:* Use a calculator.]

Solutions on page 28 >

Be Careful While the square root of a product *is* equal to the product of the square roots,

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

the corresponding statement for *sums* is *not* true:

$$\sqrt{a + b} \quad \text{is } \textit{not equal to} \quad \sqrt{a} + \sqrt{b}$$

For example,

$$\underbrace{\sqrt{9 + 16}}_{\sqrt{25}} \neq \underbrace{\sqrt{9}}_{3} + \underbrace{\sqrt{16}}_{4}$$

The two sides are not equal: one is 5 and the other is 7

Therefore, do not "simplify" $\sqrt{x^2 + 9}$ into $x + 3$. The expression $\sqrt{x^2 + 9}$ *cannot be simplified*. Similarly,

$$(x + y)^2 \quad \text{is } \textit{not} \text{ equal to} \quad x^2 + y^2$$

The expression $(x + y)^2$ means $(x + y)$ times itself:

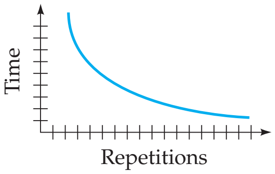
$$(x + y)^2 = (x + y)(x + y) = x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$$

This result is worth remembering, since we will use it frequently in Chapter 2.

$$(x + y)^2 = x^2 + 2xy + y^2$$

$(x + y)^2$ is the first number squared plus twice the product of the numbers plus the second number squared

Learning Curves in Airplane Production



It is a truism that the more you practice a task, the faster you can do it. Successive repetitions generally take less time, following a "learning curve" like that on the left. Learning curves are used in industrial production. For example, it took 150,000 work-hours to build the first Boeing 707 airliner, while later planes $(n = 2, 3, \ldots, 300)$ took less time.*

$$\begin{pmatrix}\text{Time to build}\\ \text{plane number } n\end{pmatrix} = 150\, n^{-0.322} \text{ thousand work-hours}$$

The time for the 10th Boeing 707 is found by substituting $n = 10$:

$$\begin{pmatrix}\text{Time to build}\\ \text{plane } 10\end{pmatrix} = 150(10)^{-0.322}$$

$150n^{-0.322}$ with $n = 10$

$$\approx 71.46 \text{ thousand work-hours}$$

Using a calculator

This shows that building the 10th Boeing 707 took about 71,460 work-hours, which is less than half of the 150,000 work-hours needed for the first. For the 100th 707:

$$\begin{pmatrix}\text{Time to build}\\ \text{plane } 100\end{pmatrix} = 150(100)^{-0.322}$$

$150n^{-0.322}$ with $n = 100$

$$\approx 34.05 \text{ thousand work-hours}$$

*A work-hour is the amount of work that a person can do in 1 hour. For further information on learning curves in industrial production, see J. M. Dutton et al., "The History of Progress Functions as a Managerial Technology," *Business History Review* **58**.

or about 34,050 work-hours, which is less than the half time needed to build the 10th. Such learning curves are used for determining the cost of a contract to build several planes.

Notice that the learning curve graphed on the previous page decreases less steeply as the number of repetitions increases. This means that while construction time continues to decrease, it does so more slowly for later planes. This behavior, called **diminishing returns,** is typical of learning curves.

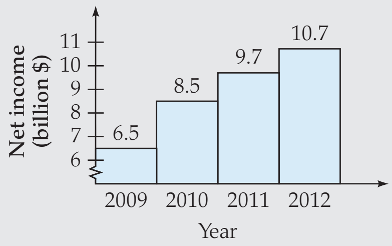
Power Regression (Optional)

Just as we used *linear* regression to fit a *line* to data points, we can use **power regression** to fit a *power curve* like those shown on page 24 to data points. The procedure is easily accomplished using a graphing calculator (or spreadsheet or other computer software), as in the following Example.

When do you use power regression instead of linear regression (or some other type)? You should look at the data and see if they lie more along a *curve* like those shown on page 24 rather than along a line. Furthermore, sometimes there are *theoretical* reasons to prefer a curve. For example, sales of a product may increase linearly for a short time, but then usually grow more slowly because of market saturation or competition, and so are best modeled by a curve.

EXAMPLE 7 POWER REGRESSION USING A GRAPHING CALCULATOR

The following graph shows Google's net income in billions of dollars for recent years.



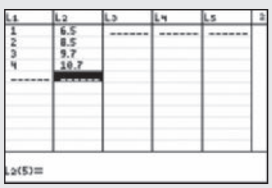
Source: Standard & Poor's

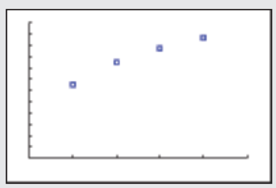
a. Use power regression to fit a power curve to the data and state the regression formula.

b. Use the regression formula to predict Google's net income in 2020.

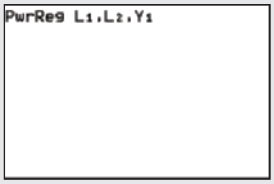
Solution

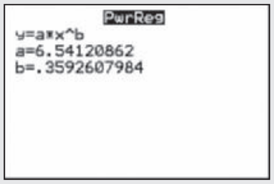
a. We number the years with x-values $1-4$, so x stands for *years since 2008* (we could choose other x-values instead, but with power regression the x-values must all be *positive*). We enter the data into lists, as shown in the first screen below (as explained in the appendix *Graphing Calculator Basics—Entering Data* on page A3) and use *ZoomStat* to graph the data points.

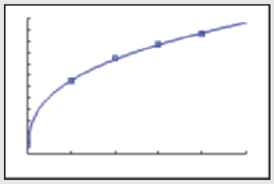




Then (using STAT, CALC, and PwrReg) we graph the regression curve along with the data points.



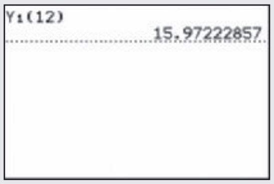




The regression curve, which fits the points reasonably well, is

$$y = 6.54x^{0.359}$$ Rounded

b. To predict income in 2020, we evaluate Y1 at 12 (since $x = 12$ corresponds to 2020). From the screen below, if the current trend continues, net income in 2020 will be approximately \$16 billion.



Solutions TO PRACTICE PROBLEMS

1. a. $\dfrac{x^5 \cdot x}{x^2} = \dfrac{x^6}{x^2} = x^4$

b. $[(x^3)^2]^2 = x^{3 \cdot 2 \cdot 2} = x^{12}$

c. $[2x^2y^4]^3 = 2^3(x^2)^3(y^4)^3 = 8x^6y^{12}$

2. a. $2^0 = 1$

b. $2^{-4} = \dfrac{1}{2^4} = \dfrac{1}{16}$

3. $\left(\dfrac{2}{3}\right)^{-2} = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

4. a. $(-27)^{1/3} = \sqrt[3]{-27} = -3$

b. $\left(\dfrac{16}{81}\right)^{1/4} = \sqrt[4]{\dfrac{16}{81}} = \dfrac{\sqrt[4]{16}}{\sqrt[4]{81}} = \dfrac{2}{3}$

5. a. $16^{3/2} = \left(\sqrt{16}\right)^3 = 4^3 = 64$

b. $(-8)^{2/3} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = 4$

6. a. $25^{-3/2} = \dfrac{1}{25^{3/2}} = \dfrac{1}{\left(\sqrt{25}\right)^3} = \dfrac{1}{5^3} = \dfrac{1}{125}$

b. $\left(\dfrac{1}{4}\right)^{-1/2} = \left(\dfrac{4}{1}\right)^{1/2} = \sqrt{4} = 2$ **c.** $5^{1.3} \approx 8.103$

1.2 Section Summary

We defined zero, negative, and fractional exponents as follows:

$$x^0 = 1 \qquad \text{for } x \neq 0$$

$$x^{-n} = \frac{1}{x^n} \qquad \text{for } x \neq 0$$

$$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m} \qquad m > 0, \quad n > 0, \quad \frac{m}{n} \text{ fully reduced}$$

With these definitions, the following properties of exponents hold for *all* exponents, whether integral or fractional, positive or negative.

$$x^m \cdot x^n = x^{m+n} \qquad (x^m)^n = x^{m \cdot n} \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$\frac{x^m}{x^n} = x^{m-n} \qquad (xy)^n = x^n \cdot y^n$$

1.2 Exercises

1–48. Evaluate each expression *without* using a calculator.

1. $(2^2 \cdot 2)^2$ **2.** $(5^2 \cdot 4)^2$ **3.** 2^{-4}

4. 3^{-3} **5.** $\left(\frac{1}{2}\right)^{-3}$ **6.** $\left(\frac{1}{3}\right)^{-2}$

7. $\left(\frac{5}{8}\right)^{-1}$ **8.** $\left(\frac{3}{4}\right)^{-1}$ **9.** $4^{-2} \cdot 2^{-1}$

10. $3^{-2} \cdot 9^{-1}$ **11.** $\left(\frac{3}{2}\right)^{-3}$ **12.** $\left(\frac{2}{3}\right)^{-3}$

13. $\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}$ **14.** $\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-2}$

15. $\left[\left(\frac{2}{3}\right)^{-2}\right]^{-1}$ **16.** $\left[\left(\frac{2}{5}\right)^{-2}\right]^{-1}$

17. $25^{1/2}$ **18.** $36^{1/2}$ **19.** $25^{3/2}$

20. $16^{3/2}$ **21.** $16^{3/4}$ **22.** $27^{2/3}$

23. $(-8)^{2/3}$ **24.** $(-27)^{2/3}$ **25.** $(-8)^{5/3}$

26. $(-27)^{5/3}$ **27.** $\left(\frac{25}{36}\right)^{3/2}$ **28.** $\left(\frac{16}{25}\right)^{3/2}$

29. $\left(\frac{27}{125}\right)^{2/3}$ **30.** $\left(\frac{125}{8}\right)^{2/3}$ **31.** $\left(\frac{1}{32}\right)^{2/5}$

32. $\left(\frac{1}{32}\right)^{3/5}$ **33.** $4^{-1/2}$ **34.** $9^{-1/2}$

35. $4^{-3/2}$ **36.** $9^{-3/2}$ **37.** $8^{-2/3}$

38. $16^{-3/4}$ **39.** $(-8)^{-1/3}$ **40.** $(-27)^{-1/3}$

41. $(-8)^{-2/3}$ **42.** $(-27)^{-2/3}$ **43.** $\left(\frac{25}{16}\right)^{-1/2}$

44. $\left(\frac{16}{9}\right)^{-1/2}$ **45.** $\left(\frac{25}{16}\right)^{-3/2}$ **46.** $\left(\frac{16}{9}\right)^{-3/2}$

47. $\left(-\frac{1}{27}\right)^{-5/3}$ **48.** $\left(-\frac{1}{8}\right)^{-5/3}$

49–52. Use a calculator to evaluate each expression. Round answers to two decimal places.

49. $7^{0.39}$ **50.** $5^{0.47}$ **51.** $8^{2.7}$ **52.** $5^{3.9}$

53–56. Use a graphing calculator to evaluate each expression.

53. $[(0.1)^{0.1}]^{0.1}$ **54.** $\left(1 + \frac{1}{1000}\right)^{1000}$

55. $\left(1 - \frac{1}{1000}\right)^{-1000}$ **56.** $(1 + 10^{-6})^{10^6}$

57–70. Write each expression in power form ax^b for numbers a and b.

57. $\frac{4}{x^5}$ **58.** $\frac{6}{2x^3}$

59. $\frac{4}{\sqrt[3]{8x^4}}$ **60.** $\frac{6}{\sqrt{4x^3}}$

61. $\frac{24}{(2\sqrt{x})^3}$ **62.** $\frac{18}{(3\sqrt[3]{x})^2}$

63. $\sqrt{\dfrac{9}{x^4}}$

64. $\sqrt[3]{\dfrac{8}{x^6}}$

65. $\dfrac{5x^2}{\sqrt{x}}$

66. $\dfrac{3\sqrt{x}}{x}$

67. $\dfrac{12\sqrt[3]{x^2}}{3x^2}$

68. $\dfrac{10\sqrt{x}}{2\sqrt[3]{x}}$

69. $\dfrac{\sqrt{36x}}{2x}$

70. $\dfrac{\sqrt[3]{8x^2}}{4x}$

71–86. Simplify.

71. $(x^3 \cdot x^2)^2$ **72.** $(x^4 \cdot x^3)^2$ **73.** $[z^2(z \cdot z^2)^2 z]^3$

74. $[z(z^3 \cdot z)^2 z^2]^2$ **75.** $[(x^2)^2]^2$

76. $[(x^3)^3]^3$ **77.** $(3x^2y^5z)^3$ **78.** $(2x^4yz^6)^4$

79. $\dfrac{(ww^2)^3}{w^3w}$ **80.** $\dfrac{(ww^3)^2}{w^3w^2}$ **81.** $\dfrac{(5xy^4)^2}{25x^3y^3}$

82. $\dfrac{(4x^3y)^2}{8x^2y^3}$ **83.** $\dfrac{(9xy^3z)^2}{3(xyz)^2}$ **84.** $\dfrac{(5x^2y^3z)^2}{5(xyz)^2}$

85. $\dfrac{(2u^2vw^3)^2}{4(uw^2)^2}$ **86.** $\dfrac{(u^3vw^2)^2}{9(u^2w)^2}$

Applied Exercises

87–88. ALLOMETRY: Dinosaurs The study of size and shape is called "allometry," and many allometric relationships involve exponents that are fractions or decimals. For example, the body measurements of most four-legged animals, from mice to elephants, obey (approximately) the following power law:

$$\begin{pmatrix}\text{Average body}\\ \text{thickness}\end{pmatrix} = 0.4\ (\text{hip-to-shoulder length})^{3/2}$$

where body thickness is measured vertically and all measurements are in feet. Assuming that this same relationship held for dinosaurs, find the average body thickness of the following dinosaurs, whose hip-to-shoulder length can be measured from their skeletons:

87. Diplodocus, whose hip-to-shoulder length was 16 feet.

88. Triceratops, whose hip-to-shoulder length was 14 feet.

89–90. BUSINESS: The Rule of .6 Many chemical and refining companies use "the rule of point six" to estimate the cost of new equipment. According to this rule, if a piece of equipment (such as a storage tank) originally cost C dollars, then the cost of similar equipment that is x times as large will be approximately $x^{0.6}C$ dollars. For example, if the original equipment cost C dollars, then new equipment with twice the capacity of the old equipment $(x = 2)$ would cost $2^{0.6}C = 1.516C$ dollars—that is, about 1.5 times as much. Therefore, to increase capacity by 100% costs only about 50% more.*

89. Use the rule of .6 to find how costs change if a company wants to quadruple $(x = 4)$ its capacity.

*Although the rule of .6 is only a rough "rule of thumb," it can be somewhat justified on the basis that the equipment of such industries consists mainly of containers, and the cost of a container depends on its surface area (square units), which increases more slowly than its capacity (cubic units).

90. Use the rule of .6 to find how costs change if a company wants to triple $(x = 3)$ its capacity.

91–92. BUSINESS: Phillips Curves Unemployment and inflation are inversely related, with one rising as the other falls, and an equation giving the relation is called a *Phillips curve* after the economist A.W. Phillips (1914–1975).

91. Phillips used data from 1861 to 1957 to establish that in the United Kingdom the unemployment rate x and the wage inflation rate y were related by

$$y = 9.638x^{-1.394} - 0.900$$

where x and y are both percents. Use this relation to estimate the inflation rate when the unemployment rate was

a. 2 percent **b.** 5 percent

Source: Economica **25**

92. Between 2000 and 2010, the Phillips curve for the U.S. unemployment rate x and Consumer Price Index (CPI) inflation rate y was

$$y = 45.4x^{-1.54} - 1$$

where x and y are both percents. Use this relation to estimate the inflation rate when the unemployment rate is

a. 3 percent **b.** 8 percent

Source: Bureau of Labor Statistics

93–94. ALLOMETRY: Heart Rate It is well known that the hearts of smaller animals beat faster than the hearts of larger animals. The actual relationship is approximately

$$(\text{Heart rate}) = 250(\text{Weight})^{-1/4}$$

where the heart rate is in beats per minute and the weight is in pounds. Use this relationship to estimate the heart rate of:

93. A 16-pound dog.

94. A 625-pound grizzly bear.

Source: Biology Review **41**

95–96. BUSINESS: Learning Curves in Airplane Production Recall (pages 26–27) that the learning curve for the production of Boeing 707 airplanes is $150n^{-0.322}$ (thousand work-hours). Find how many work-hours it took to build:

95. The 50th Boeing 707.

96. The 250th Boeing 707.

97–98. GENERAL: Earthquakes The sizes of major earthquakes are measured on the *Moment Magnitude Scale*, or MMS, although the media often still refer to the outdated *Richter* scale. The MMS measures the total *energy released* by an earthquake, in units denoted M_W (W for the *work* accomplished). An increase of 1 M_W means the energy increased by a factor of 32, so an increase from A to B means the energy increased by a factor of 32^{B-A}. Use this formula to find the increase in energy between the following earthquakes:

97. The 1994 Northridge, California, earthquake that measured 6.7 M_W and the 1906 San Francisco earthquake that measured 7.8 M_W. (The San Francisco earthquake resulted in 3000 deaths and a 3-day fire that destroyed 4 square miles of San Francisco.)

98. The 2001 earthquake in India that measured 7.7 M_W and the 2011 earthquake in Japan that measured 9.0 M_W. (The earthquake in Japan generated a 28-foot tsunami wave that traveled six miles inland, killing 24,000 and causing an estimated $300 billion in damage, making it the most expensive natural disaster ever recorded.)

Source for M_W: U.S. Geological Survey

99–100. BUSINESS: Isoquant Curves An *isoquant curve* (*iso* means "same" and *quant* is short for "quantity") shows the various combinations of labor and capital (the invested value of factory buildings, machinery, and raw materials) a company could use to achieve the same total production level. For a given production level, an isoquant curve can be written in the form $K = aL^b$ where K is the amount of capital, L is the amount of labor, and a and b are constants. For each isoquant curve, find the value of K corresponding to the given value of L.

99. $K = 3000L^{-1/2}$ and $L = 225$

100. $K = 4000L^{-2/3}$ and $L = 125$

101–102. GENERAL: Waterfalls Water falling from a waterfall that is x feet high will hit the ground with speed $\frac{60}{11}x^{0.5}$ miles per hour (neglecting air resistance).

101. Find the speed of the water at the bottom of the highest waterfall in the world, Angel Falls in Venezuela (3281 feet high).

102. Find the speed of the water at the bottom of the highest waterfall in the United States, Ribbon Falls in Yosemite, California (1650 feet high).

103–104. ENVIRONMENTAL SCIENCE: Biodiversity It is well known that larger land areas can support larger numbers of species. According to one study, multiplying the land area by a factor of x multiplies the number of species by a factor of $x^{0.239}$. Use a graphing calculator to graph $y = x^{0.239}$. Use the window [0, 100] by [0, 4].

Source: Robert H. MacArthur and Edward O. Wilson, *The Theory of Island Biogeography*

103. Find the multiple x for the land area that leads to *double* the number of species. That is, find the value of x such that $x^{0.239} = 2$. [*Hint:* Either use TRACE or find where $y_1 = x^{0.239}$ INTERSECTs $y_2 = 2$.]

104. Find the multiple x for the land area that leads to triple the number of species. That is, find the value of x such that $x^{0.239} = 3$. [*Hint:* Either use TRACE or find where $y_1 = x^{0.239}$ INTERSECTs $y_2 = 3$.]

105–106. GENERAL: Speed and Skidmarks Police or insurance investigators often want to estimate the speed of a car from the skidmarks it left while stopping. A study found that for standard tires on dry asphalt, the speed (in mph) is given approximately by $y = 9.4x^{0.37}$, where x is the length of the skidmarks in feet. (This formula takes into account the deceleration that occurs even *before* the car begins to skid.) Estimate the speed of a car if it left skidmarks of:

105. 150 feet. **106.** 350 feet.

Source: Accident Analysis and Prevention **36**

107. BUSINESS: Semiconductor Sales The following table shows worldwide sales for semiconductors used in cell phones and laptop computers for recent years.

Year	2011	2012	2013
Sales (billions $)	80.2	87.1	93.6

a. Number the data columns with x-values 1–3 (so that x stands for *years since 2010*), use power regression to fit a power curve to the data, and state the regression formula. [*Hint:* See Example 7.]

b. Use the regression formula to predict sales in 2020.
[*Hint:* What x-value corresponds to 2020?]

Source: Standard and Poor's

108. SOCIAL SCIENCE: Alcohol and Tobacco Expenditures The following table gives the per capita expenditures for alcohol and tobacco for Americans in recent years.

Year	2009	2010	2011	2012
Expenditures (in $)	607	646	667	685

a. Number the data columns with x-values 1–4 (so that x stands for *years since 2008*), use power regression to fit a power curve to the data, and state the regression formula. [*Hint:* See Example 7.]

b. Use the regression formula to predict these expenditures in the year 2020. [*Hint:* What x-value corresponds to 2020?]

Source: Consumer Americas 2013

109. BUSINESS: Nevada Gambling Winnings The following table shows the winnings in Nevada casinos for recent years.

Year	2010	2011	2012
Winnings (million $)	41.6	42.8	43.4

a. Number the data columns with x-values 1–3 (so that x stands for *years since 2009*), use power regression to fit a power curve to the data, and state the regression formula. [*Hint:* See Example 7.]

b. Use the regression formula to predict Nevada casino winnings in the year 2020.

Source: Standard & Poor's Industry Surveys

110. BUSINESS: American Express Operating Revenues The following table shows the operating revenues (in billions of dollars) for American Express.

Year	2009	2010	2011	2012
Operating revenues (billion $)	26.5	30.2	32.3	33.8

a. Number the data columns with x-values 1–4 (so that x stands for *years since 2008*), use power regression to fit a power curve to the data, and state the regression formula. [*Hint:* See Example 7.]

b. Use the regression formula to predict American Express operating revenues in the year 2020.

Source: Standard & Poor's

Conceptual Exercises

111. Should $\sqrt{9}$ be evaluated as 3 or ± 3?

112–114. For each statement, either state that it is True (and find a property in the text that shows this) or state that it is False (and give an example to show this).

112. $x^m \cdot x^n = x^{m \cdot n}$ **113.** $\dfrac{x^m}{x^n} = x^{m/n}$ **114.** $(x^m)^n = x^{m^n}$

115–117. For each statement, state *in words* the values of x for which each exponential expression is defined.

115. $x^{1/2}$ **116.** $x^{1/3}$ **117.** x^{-1}

118. When defining $x^{m/n}$, why did we require that the exponent $\frac{m}{n}$ be fully reduced?

[*Hint:* $(-1)^{2/3} = \left(\sqrt[3]{-1}\right)^2 = 1$, but with an equal but *unreduced* exponent you get $(-1)^{4/6} = \left(\sqrt[6]{-1}\right)^4$. Is this defined?]

1.3 Functions: Linear and Quadratic

Introduction

In the previous section we saw that the time required to build a Boeing 707 airliner varies, depending on the number that have already been built. Mathematical relationships such as this, in which one number depends on another, are called *functions* and are central to the study of calculus. In this section we define and give some applications of functions.

Functions

A **function** is a rule or procedure for finding, from a given number, a new number.* If the function is denoted by f and the given number by x, then the resulting number is written $\boldsymbol{f(x)}$ (read "f of x") and is called *the value of the function f at x*. We emphasize that $f(x)$ must be a *single* number.

*In this chapter the word "function" will mean *function of one variable*. In Chapter 7 we will discuss functions of more than one variable.

The set of numbers x for which a function f is defined is called the **domain** of f, and the set of all resulting function values $f(x)$ is called the **range** of f.

Function

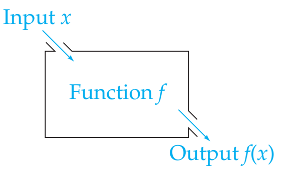
A *function* f is a rule that assigns to each number x in a set exactly one number $f(x)$.

The set of all allowable values of x is called the *domain*.

The set of all values $f(x)$ for x in the domain is called the *range*.

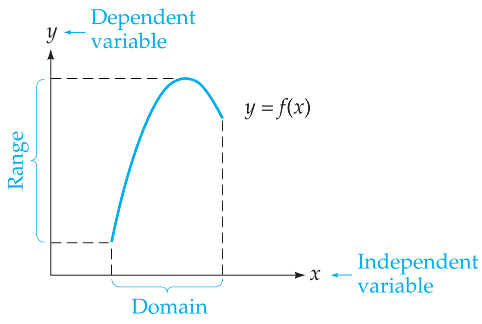
For example, recording the temperature at a given location throughout a particular day would define a *temperature* function:

$$f(x) = \begin{pmatrix}\text{Temperature at}\\ \text{time } x \text{ hours}\end{pmatrix} \qquad \text{Domain would be } [0, 24)$$



A function f may be thought of as a numerical procedure or "machine" that takes an "input" number x and produces an "output" number $f(x)$, as shown on the left. The permissible input numbers form the domain, and the resulting output numbers form the range.

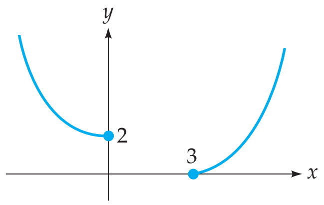
We will be mostly concerned with functions that are defined by *formulas* for calculating $f(x)$ from x. If the domain of such a function is not stated, then it is always taken to be the *largest* set of numbers for which the function is defined, called the **natural domain** of the function. To **graph** a function f, we plot all points (x, y) such that x is in the domain and $y = f(x)$. We call x the **independent variable** and y the **dependent variable,** since y *depends on* (is calculated from) x. The domain and range can be illustrated graphically.



The domain of a function $y = f(x)$ is the set of all allowable x-values, and the range is the set of all corresponding y-values.

PRACTICE PROBLEM 1

Find the domain and range of the function graphed below.



Solution on page 43 >