An Applied Approach

BRIEF CALCULUS

with

CalcChat® & CalcView™

10e

RON LARSON

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Ron Larson
The Pennsylvania State University
The Behrend College

With the assistance of David C. Falvo
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The Behrend College

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Brief Calculus: An Applied Approach with CalcChat & CalcView
Tenth Edition

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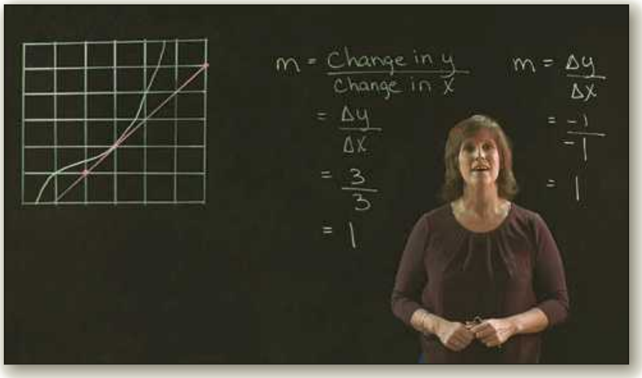
*Available at the text-specific website **CengageBrain.com**

Preface

Welcome to the Tenth Edition of *Brief Calculus: An Applied Approach with CalcChat & CalcView*! I am proud to present this new edition to you. As with all editions, I have been able to incorporate many useful comments from you, our user. In this edition, I introduce several new features and revise others. You will still find what you expect—a pedagogically sound, mathematically precise, and comprehensive textbook that includes a multitude of business and life sciences applications.

I am pleased and excited to offer you two brand new websites with this edition—**CalcView.com** and **LarsonAppliedCalculus.com.** Both websites were created with the goal of providing you with the resources needed to master Calculus. **CalcView.com** contains worked-out solution videos for selected exercises in the book, and **LarsonAppliedCalculus.com** offers multiple resources to supplement your learning experience. Best of all, these websites are completely *free*.



A theme throughout the book is **"IT'S ALL ABOUT YOU."** Please pay special attention to the study aids with a red **U**. These study aids will help you learn calculus, use technology, refresh your algebra skills, and prepare for tests. For an overview of these aids, check out CALCULUS & YOU on page 0. In each exercise set, quiz, and test, be sure to notice the reference to **CalcChat.com.** At this free site, you can download a step-by-step solution to any odd-numbered exercise. You can also work with a tutor, free of charge, during the hours posted at the site. Over the years, thousands of students have visited the site for help.

New To This Edition

CalcView™

The website **CalcView.com** contains video solutions of selected exercises. Calculus instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. You can use your smartphone's QR Code® reader to scan the code and go directly to a video solution. Or you can access the videos at **CalcView.com.**

NEW LarsonAppliedCalculus.com

This companion website offers multiple tools and resources to supplement your learning. Access to these features is *free*. Watch videos explaining concepts from the book, explore examples, take a diagnostic test, view solutions to the checkpoint problems, and much more.

QR Code is a registered trademark of Denso Wave Incorporated

NEW Data Spreadsheets

Download these editable spreadsheets from **LarsonAppliedCalculus.com** and use the data to solve exercises.

REVISED Exercise Sets

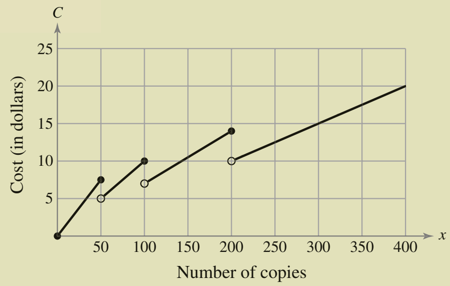
The exercise sets have been carefully and extensively examined to ensure they are rigorous, relevant, and cover all topics necessary to understand the fundamentals of Calculus. The exercises have been reorganized and titled so that you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations.

Trusted Features

HOW DO YOU SEE IT? Exercise

The *How Do You See It?* exercise in each section presents a real-life problem that you will solve by visual inspection using the concepts learned in the lesson.

76. HOW DO YOU SEE IT? The graph shows the cost C (in dollars) of making x photocopies at a copy shop.



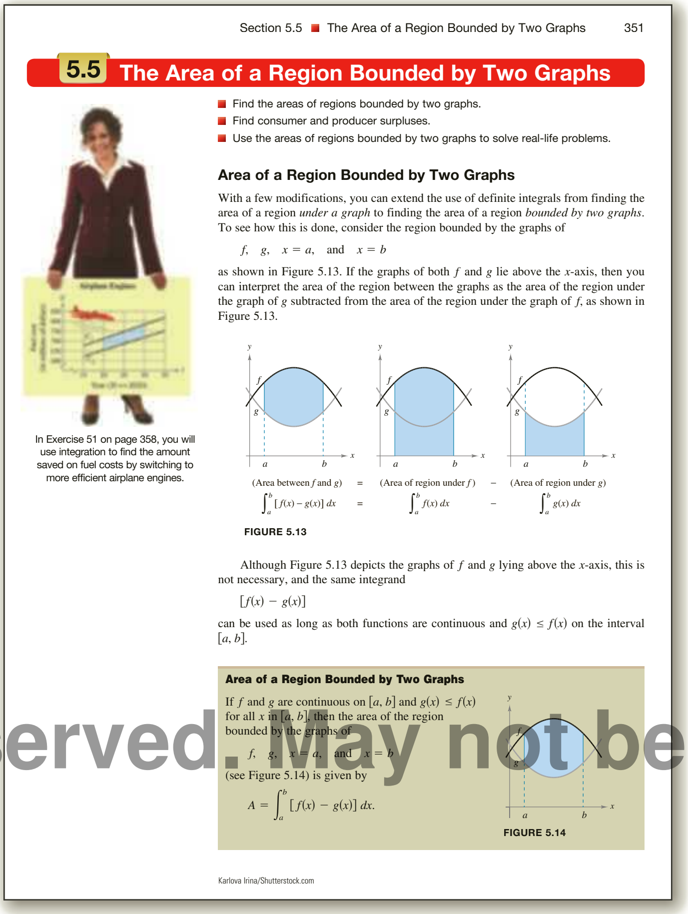
(a) Does $\lim_{x \to 50} C$ exist? Explain your reasoning.

(b) Does $\lim_{x \to 150} C$ exist? Explain your reasoning.

(c) You have to make 200 photocopies. Would it be better to make 200 or 201? Explain your reasoning.

CalcChat®

For the past several years, an independent website—**CalcChat.com**—has been maintained to provide free solutions to all odd-numbered problems in the text. Thousands of students have visited the site for practice and help with their homework from live tutors.


Section 5.5 ■ The Area of a Region Bounded by Two Graphs 351

5.5 The Area of a Region Bounded by Two Graphs

■ Find the areas of regions bounded by two graphs.
■ Find consumer and producer surpluses.
■ Use the areas of regions bounded by two graphs to solve real-life problems.

Area of a Region Bounded by Two Graphs

With a few modifications, you can extend the use of definite integrals from finding the area of a region *under a graph* to finding the area of a region *bounded by two graphs*. To see how this is done, consider the region bounded by the graphs of

$f, \quad g, \quad x = a, \quad \text{and} \quad x = b$

as shown in Figure 5.13. If the graphs of both f and g lie above the x-axis, then you can interpret the area of the region between the graphs as the area of the region under the graph of g subtracted from the area of the region under the graph of f, as shown in Figure 5.13.

(Area between f and g) = (Area of region under f) − (Area of region under g)

$\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

FIGURE 5.13

Although Figure 5.13 depicts the graphs of f and g lying above the x-axis, this is not necessary, and the same integrand

$[f(x) - g(x)]$

can be used as long as both functions are continuous and $g(x) \le f(x)$ on the interval $[a, b]$.

Area of a Region Bounded by Two Graphs

If f and g are continuous on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of

$f, \quad g, \quad x = a, \quad \text{and} \quad x = b$

(see Figure 5.14) is given by

$A = \int_a^b [f(x) - g(x)]\,dx.$

FIGURE 5.14

In Exercise 51 on page 358, you will use integration to find the amount saved on fuel costs by switching to more efficient airplane engines.

Karlova Irina/Shutterstock.com

Chapter Opener

Each *Chapter Opener* highlights a real-life problem from an example in the chapter, showing a graph related to the data and describing the math concept used to solve the problem.

Section Opener

Each *Section Opener* highlights a real-life problem in the exercises, showing a graph for the situation with a description of how you will use the math of the section to solve the problem.

Section Objectives

A bulleted list of learning objectives provides you with the opportunity to preview what will be presented in the upcoming section.

Definitions and Theorems

All definitions and theorems are highlighted for emphasis and easy recognition.

Business Capsule

Susie Wang and Ric Kostick graduated in 2002 from the University of California at Berkeley with degrees in mathematics. Together they launched a cosmetics brand called 100% Pure, which uses fruit and vegetable pigments to color cosmetics and uses only organic ingredients for the purest skin care. The company grew quickly and now has annual sales of over $40 million. Wang and Kostick attribute their success to applying what they learned from their studies. "Mathematics teaches you logic, discipline, and accuracy, which help you with all aspects of daily life," says Ric Kostick.

49. Research Project Use your school's library, the Internet, or some other reference source to research the opportunity cost of attending graduate school for 2 years to receive a Masters of Business Administration (MBA) degree rather than working for 2 years with a bachelor's degree. Write a short paper describing these costs.

Checkpoint

Paired with every example, the *Checkpoint* problems encourage immediate practice and check your understanding of the concepts presented in the example. Answers to all *Checkpoint* problems appear at the back of the text to reinforce understanding of the skill sets learned.

Business Capsule

Business Capsules appear at the end of selected sections. These capsules and their accompanying research project highlight business situations related to the mathematical concepts covered in the chapter.

SUMMARIZE

The *Summarize* feature at the end of each section helps you organize the lesson's key concepts into a concise summary, providing you with a valuable study tool.

STUDY TIP

These hints and tips can be used to reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show alternative or additional steps to a solution of an example.

TECH TUTOR

The *Tech Tutor* gives suggestions for effectively using tools such as calculators, graphing calculators, and spreadsheet programs to help deepen your understanding of concepts, ease lengthy calculations, and provide alternate solution methods for verifying answers obtained by hand.

ALGEBRA TUTOR

The *Algebra Tutor* appears throughout each chapter and offers algebraic support at point of use. This support is revisited in a two-page algebra review at the end of the chapter, where additional details of example solutions with explanations are provided.

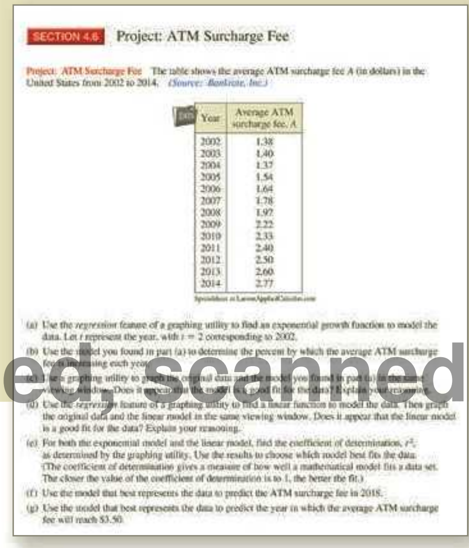
SKILLS WARM UP

The *Skills Warm Up* appears at the beginning of the exercise set for each section. These problems help you review previously learned skills that you will use in solving the section exercises.

Project

The projects at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These projects are offered online at **LarsonAppliedCalculus.com.**

47. Project: ATM Surcharge Fee For a project analyzing the average ATM surcharge fee in the United States from 2002 to 2014, visit this text's website at *LarsonAppliedCalculus.com.* *(Source: Bankrate, Inc.)*



SECTION 4.6 Project: ATM Surcharge Fee

Project: ATM Surcharge Fee The table shows the average ATM surcharge fee A (in dollars) in the United States from 2002 to 2014. *(Source: Bankrate, Inc.)*

Year	Average ATM surcharge fee, A
2002	1.38
2003	1.40
2004	1.37
2005	1.54
2006	1.64
2007	1.78
2008	1.97
2009	2.22
2010	2.33
2011	2.40
2012	2.50
2013	2.60
2014	2.77

Spreadsheet at LarsonAppliedCalculus.com

(a) Use the *regression* feature of a graphing utility to find an exponential growth function to model the data. Let t represent the year, with $t = 2$ corresponding to 2002.
(b) Use the model you found in part (a) to determine the percent by which the average ATM surcharge fee is increasing each year.
(c) Use a graphing utility to graph the original data and the model you found in part (a) in the same viewing window. Does it appear that the model is a good fit for the data? Explain your reasoning.
(d) Use the *regression* feature of a graphing utility to find a linear function to model the data. Then graph the original data and the linear model in the same viewing window. Does it appear that the linear model is a good fit for the data? Explain your reasoning.
(e) For both the exponential model and the linear model, find the coefficient of determination, r^2, as determined by the graphing utility. Use the results to choose which model best fits the data. (The coefficient of determination gives a measure of how well a mathematical model fits a data set. The closer the value of the coefficient of determination is to 1, the better the fit.)
(f) Use the model that best represents the data to predict the ATM surcharge fee in 2018.
(g) Use the model that best represents the data to predict the year in which the average ATM surcharge fee will reach $3.50.

Instructor Resources

Media

Complete Solutions Manual
The *Complete Solutions Manual* provides worked-out solutions for all exercises in the text, including Checkpoints, Quiz Yourself, Test Yourself, and Tech Tutors.

Turn the Light On with MindTap for Larson's *Calculus: An Applied Approach*
Through personalized paths of dynamic assignments and applications, MindTap is a digital learning solution and representation of your course that turns cookie cutter into cutting edge, apathy into engagement, and memorizers into higher-level thinkers.

> **The Right Content:** With MindTap's carefully curated material, you get the precise content and groundbreaking tools you need for every course you teach.
>
> **Personalization:** Customize every element of your course—from rearranging the Learning Path to inserting videos and activities.
>
> **Improved Workflow:** Save time when planning lessons with all of the trusted, most current content you need in one place in MindTap.
>
> **Tracking Students' Progress in Real Time:** Promote positive outcomes by tracking students in real time and tailoring your course as needed based on the analytics.

Learn more at **cengage.com/mindtap.**

Instructor Companion Site
Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online at **cengage.com/login.** Access and download PowerPoint presentations, images, solutions, videos, and more.

Cengage Learning Testing Powered by Cognero (ISBN: 978-1-3058-8230-0) is a flexible, online system that allows you to author, edit, and manage test bank content, create multiple test versions in an instant, and deliver tests from your LMS, your classroom, or wherever you want. This is available online at **cengage.com/login.**

Student Resources

Print

Student Solutions Manual

ISBN 13: 978-1-305-86099-5

The *Student Solutions Manual* provides complete worked-out solutions to all odd-numbered exercises in the text. In addition, the solutions of all Checkpoint, Quiz Yourself, Test Yourself, and Tech Tutor exercises are included.

Media

MindTap for Larson's *Calculus: An Applied Approach*
MindTap is a digital representation of your course that provides you with the tools you need to better manage your limited time, stay organized, and be successful. You can complete assignments whenever and wherever you are ready to learn with course material specially customized for you by your instructor and streamlined in one proven, easy-to-use interface. With an array of study tools, you will get a true understanding of course concepts, achieve better grades, and set the groundwork for your future courses.

Learn more at **cengage.com/mindtap.**

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To access additional course materials and companion resources, please visit **CengageBrain.com.** At the **CengageBrain.com** home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where free companion resources can be found.

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On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write to me. Over the past two decades I have received many useful comments from both instructors and students, and I value these comments very highly.

Ron Larson, Ph.D.
Professor of Mathematics
Penn State University
www.RonLarson.com

CALCULUS & YOU

Every feature in this text is designed to help you learn calculus. Whenever you see a red **U**, pay special attention to the study aid. These study aids represent years of experience in teaching students *just like you*. Ron Larson

STUDY TIP

The notation $\partial z/\partial x$ is read as "the partial derivative of z with respect to x," and $\partial z/\partial y$ is read as "the partial derivative of z with respect to y."

The *Study Tips* occur at point of use throughout the text. They represent **common questions** that students ask me, **insights** into understanding concepts, and **alternative ways to look at concepts**. For instance, the *Study Tip* at the left provides insight on how to read mathematical notation.

TECH TUTOR

If you have access to a symbolic integration utility, try using it to find antiderivatives.

The *Tech Tutors* give suggestions on how you can use various types of technology to help understand the material. This includes **graphing calculators**, **computer graphing programs**, and **spreadsheet programs** such as Excel. For instance, the *Tech Tutor* at the left points out that some calculators and some computer programs are capable of symbolic integration.

ALGEBRA TUTOR

Finding intercepts involves solving equations. For a review of some techniques for solving equations, see page 71.

Throughout years of teaching, I have found that the greatest stumbling block to success in calculus is a weakness in algebra. Each time you see an *Algebra Tutor*, please read it carefully. Then, flip ahead to the referenced page and give yourself a chance to enjoy a brief **algebra refresher**. It will be time well spent.

HOW DO YOU SEE IT?

The *How Do You See It?* question in each exercise set helps you **visually summarize concepts** without messy computations.

SUMMARIZE

The *Summarize* outline at the end of each section asks you to write each learning objective in **your own words**.

SKILLS WARM UP

The *Skills Warm Up* exercises that precede each exercise set will help you **review previously learned skills**.

SUMMARY AND STUDY STRATEGIES

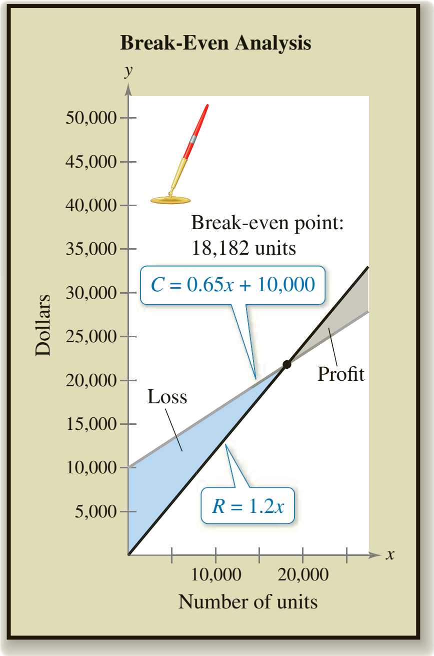
The *Summary and Study Strategies,* coupled with the Review Exercises are designed to help you organize your thoughts as you **prepare for a chapter test**.

QUIZ YOURSELF

The *Quiz Yourself* occurs midway in each chapter. Take each of these quizzes as you would **take a quiz in class**.

TEST YOURSELF

The *Test Yourself* occurs at the end of each chapter. All questions are answered so you can **check your progress**.



Example 5 on page 15 shows how the point of intersection of two graphs can be used to find the break-even point for a company manufacturing and selling a product.

1 Functions, Graphs, and Limits

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1.1 The Cartesian Plane and the Distance Formula

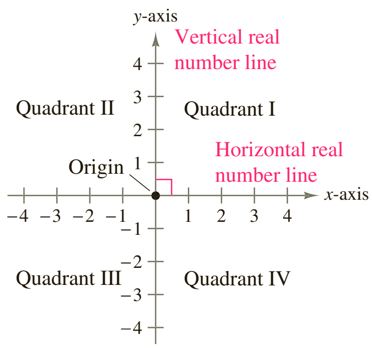
In Exercise 29 on page 9, you will use a line graph to estimate the Dow Jones Industrial Average.

- Plot points in a coordinate plane and represent data graphically.
- Find the distance between two points in a coordinate plane.
- Find the midpoint of a line segment connecting two points.
- Translate points in a coordinate plane.

The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system,** or the **Cartesian plane,** after the French mathematician René Descartes (1596–1650).

The Cartesian plane is formed by using two real number lines intersecting at right angles, as shown in Figure 1.1. The horizontal real number line is usually called the ***x*-axis,** and the vertical real number line is usually called the ***y*-axis.** The point of intersection of these two axes is the **origin,** and the two axes divide the plane into four parts called **quadrants.**



The Cartesian Plane

FIGURE 1.1

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y, called **coordinates** of the point. The ***x*-coordinate** represents the directed distance from the y-axis to the point, and the ***y*-coordinate** represents the directed distance from the x-axis to the point, as shown in Figure 1.2.

(x, y)

Directed distance from y-axis ↑ ↑ Directed distance from x-axis

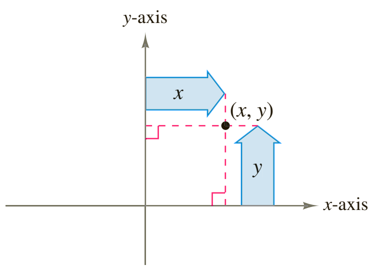


FIGURE 1.2

The notation (x, y) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

EXAMPLE 1 **Plotting Points in the Cartesian Plane**

Plot the points

$(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$.

SOLUTION To plot the point

$(-1, 2)$

x-coordinate ↑ ↑ y-coordinate

imagine a vertical line through -1 on the x-axis and a horizontal line through 2 on the y-axis. The intersection of these two lines is the point $(-1, 2)$. The other four points can be plotted in a similar way and are shown in Figure 1.3.

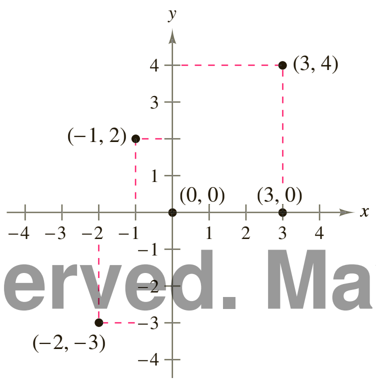


FIGURE 1.3

✓ **Checkpoint 1** *Worked-out solution available at LarsonAppliedCalculus.com*

Plot the points

$(-3, 2)$, $(4, -2)$, $(3, 1)$, $(0, -2)$, and $(-1, -2)$. ■

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Using a rectangular coordinate system allows you to visualize relationships between two variables. In Example 2, data are represented graphically by points plotted in a rectangular coordinate system. This type of graph is called a **scatter plot.**

EXAMPLE 2 Sketching a Scatter Plot

The numbers E (in millions of people) of private-sector employees in the United States from 2005 through 2013 are shown in the table, where t represents the year. Sketch a scatter plot of the data. *(Source: U.S. Bureau of Labor Statistics)*

DATA

t	2005	2006	2007	2008	2009	2010	2011	2012	2013
E	112	114	116	115	109	108	110	112	115

Spreadsheet at LarsonAppliedCalculus.com

SOLUTION To sketch a scatter plot of the data given in the table, represent each pair of values by an ordered pair

(t, E)

and plot the resulting points, as shown in Figure 1.4. For instance, the first pair of values is represented by the ordered pair

$(2005, 112)$.

Note that the break in the t-axis indicates that the numbers between 0 and 2005 have been omitted, and the break in the E-axis indicates that the numbers between 0 and 104 have been omitted.

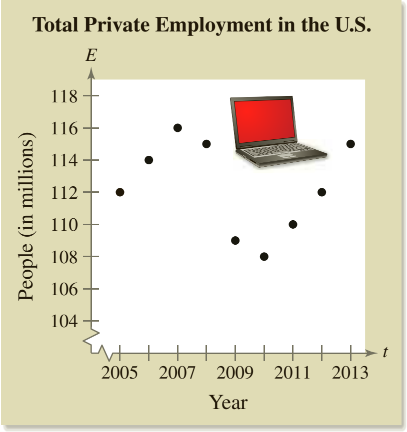


FIGURE 1.4

✓ ***Checkpoint 2*** *Worked-out solution available at LarsonAppliedCalculus.com*

The numbers E (in thousands of people) of employees in the consumer lending industry in the United States from 2005 through 2013 are shown in the table, where t represents the year. Sketch a scatter plot of the data. *(Source: U.S. Bureau of Labor Statistics)*

DATA

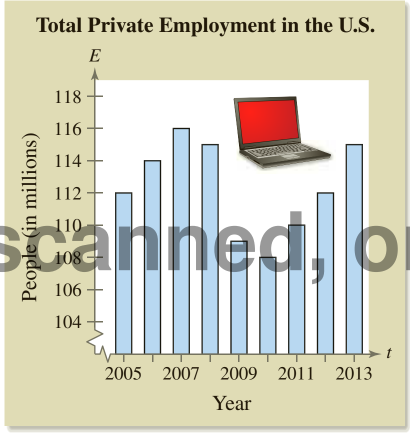
t	2005	2006	2007	2008	2009	2010	2011	2012	2013
E	113	118	119	110	97	91	87	91	95

Spreadsheet at LarsonAppliedCalculus.com

In Example 2, $t = 1$ could have been used to represent the year 2005. In that case, the horizontal axis would not have been broken, and the tick marks would have been labeled 1 through 9 (instead of 2005 through 2013).

The scatter plot in Example 2 is one way to represent the given data graphically. Another technique, a *bar graph,* is shown in the figure at the right. If you have access to a graphing utility, try using it to represent the data given in Example 2 graphically.

Another way to represent data is with a *line graph* (see Exercise 29).



The Distance Formula

Recall from the Pythagorean Theorem that, for a right triangle with hypotenuse of length c and sides of lengths a and b, you have

$$a^2 + b^2 = c^2 \qquad \text{Pythagorean Theorem}$$

as shown in Figure 1.5. Note that the converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Suppose you want to determine the distance d between two points

$$(x_1, y_1) \quad \text{and} \quad (x_2, y_2)$$

in the plane. These two points can form a right triangle, as shown in Figure 1.6. The length of the vertical side of the triangle is

$$|y_2 - y_1|$$

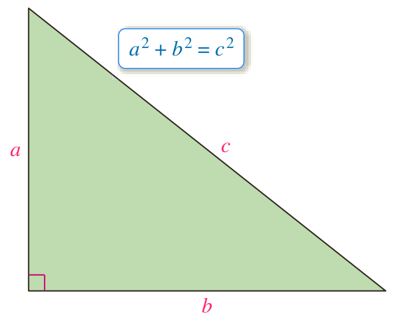
and the length of the horizontal side is

$$|x_2 - x_1|.$$

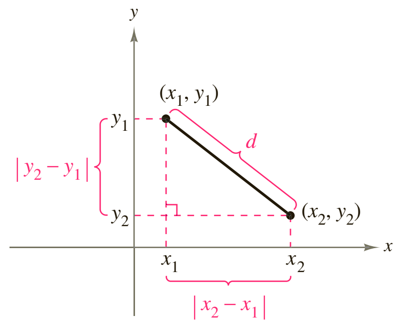
By the Pythagorean Theorem, you can write

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This result is the **Distance Formula.**



Pythagorean Theorem
FIGURE 1.5



Distance Between Two Points
FIGURE 1.6

> **The Distance Formula**
>
> The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is
>
> $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 3 Finding a Distance

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

SOLUTION Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula as shown.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{Distance Formula}$$
$$= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} \qquad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$
$$= \sqrt{(5)^2 + (3)^2} \qquad \text{Simplify.}$$
$$= \sqrt{34} \qquad \text{Simplify.}$$
$$\approx 5.83 \qquad \text{Use a calculator.}$$

So, the distance between the points is about 5.83 units. Note in Figure 1.7 that a distance of 5.83 looks about right.

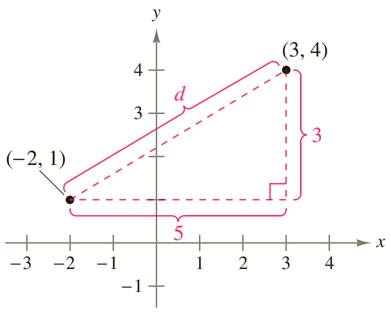


FIGURE 1.7

✓ ***Checkpoint 3*** *Worked-out solution available at LarsonAppliedCalculus.com*

Find the distance between the points $(-2, 1)$ and $(2, 4)$. ■

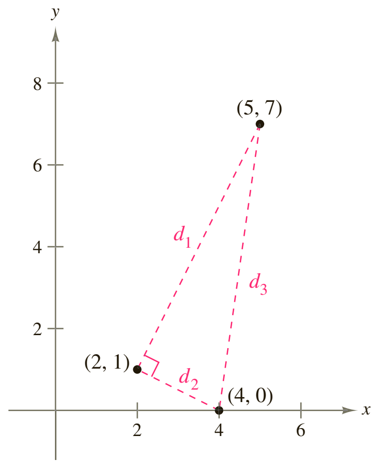


FIGURE 1.8

EXAMPLE 4 Verifying a Right Triangle

Use the Distance Formula to show that the points

$(2, 1), \quad (4, 0), \quad \text{and} \quad (5, 7)$

are vertices of a right triangle.

SOLUTION The three points are plotted in Figure 1.8. Using the Distance Formula, you can find the lengths of the three sides as shown below.

$$d_1 = \sqrt{(5-2)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45}$$
$$d_2 = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$$
$$d_3 = \sqrt{(5-4)^2 + (7-0)^2} = \sqrt{1+49} = \sqrt{50}$$

Because

$$d_1^2 + d_2^2 = 45 + 5 = 50 = d_3^2$$

you can apply the converse of the Pythagorean Theorem to conclude that the triangle must be a right triangle.

✓ Checkpoint 4 *Worked-out solution available at LarsonAppliedCalculus.com*

Use the Distance Formula to show that the points $(2, -1)$, $(5, 5)$, and $(6, -3)$ are vertices of a right triangle. ■

The figures provided with Examples 3 and 4 were not really essential to the solution. *Nevertheless*, it is strongly recommended that you develop the habit of including sketches with your solutions—even when they are not required.

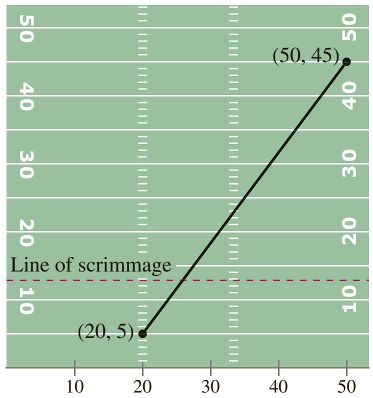


FIGURE 1.9

EXAMPLE 5 Finding the Length of a Pass

In a football game, a quarterback throws a pass from the 5-yard line, 20 yards from one sideline. The pass is caught by a wide receiver on the 45-yard line, 50 yards from the same sideline, as shown in Figure 1.9. How long is the pass?

SOLUTION You can find the length of the pass by finding the distance between the points (20, 5) and (50, 45).

$$d = \sqrt{(50-20)^2 + (45-5)^2} \qquad \text{Distance Formula}$$
$$= \sqrt{900 + 1600} \qquad \text{Simplify.}$$
$$= 50 \qquad \text{Simplify.}$$

So, the pass is 50 yards long.

✓ Checkpoint 5 *Worked-out solution available at LarsonAppliedCalculus.com*

A quarterback throws a pass from the 10-yard line, 10 yards from one sideline. The pass is caught by a wide receiver on the 30-yard line, 25 yards from the same sideline. How long is the pass? ■

STUDY TIP

In Example 5, the scale along the goal line showing distance from the sideline does not normally appear on a football field. However, when you use coordinate geometry to solve real-life problems, you are free to place the coordinate system in any way that is convenient for the solution of the problem.

The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, find the average values of the respective coordinates of the two endpoints.

> **The Midpoint Formula**
>
> The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is
>
> $$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

EXAMPLE 6 Finding the Midpoint of a Line Segment

Find the midpoint of the line segment joining the points

$(-5, -3)$ and $(9, 3)$.

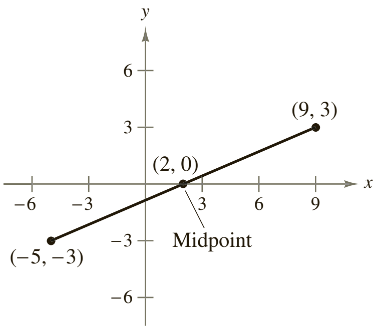


FIGURE 1.10

SOLUTION Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \text{Midpoint Formula}$$

$$= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2}\right) \quad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$

$$= (2, 0) \quad \text{Simplify.}$$

The midpoint of the line segment is $(2, 0)$, as shown in Figure 1.10.

✓ ***Checkpoint 6*** *Worked-out solution available at LarsonAppliedCalculus.com*

Find the midpoint of the line segment joining the points

$(-6, 2)$ and $(2, 8)$.

EXAMPLE 7 Estimating Annual Revenues

McDonald's Corporation had annual revenues of about \$27.0 billion in 2011 and about \$28.1 billion in 2013. Without knowing any additional information, estimate the 2012 annual revenues. *(Source: McDonald's Corp.)*

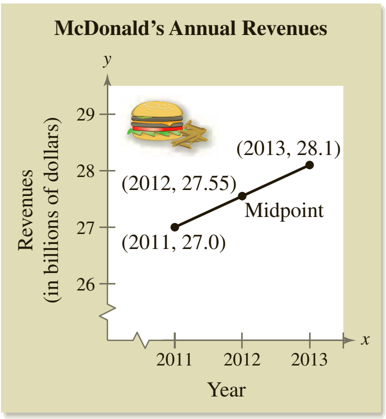


FIGURE 1.11

SOLUTION One solution to the problem is to assume that revenues followed a linear pattern. Then you can estimate the 2012 revenues by finding the midpoint of the line segment connecting the points (2011, 27.0) and (2013, 28.1).

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \text{Midpoint Formula}$$

$$= \left(\frac{2011 + 2013}{2}, \frac{27.0 + 28.1}{2}\right) \quad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$

$$= (2012, 27.55) \quad \text{Simplify.}$$

So, you can estimate that the 2012 revenues were about \$27.55 billion, as shown in Figure 1.11. (The actual 2012 revenues were about \$27.6 billion.)

✓ ***Checkpoint 7*** *Worked-out solution available at LarsonAppliedCalculus.com*

Kellogg Company had annual sales of about \$13.2 billion in 2011 and about \$14.8 billion in 2013. Without knowing any additional information, estimate the 2012 annual sales. *(Source: Kellogg Co.)* ■

Many movies now use extensive computer graphics, much of which consists of transformations of points in two- and three-dimensional space. The photo above is from *The Amazing Spider-Man*. The movie's animators used computer graphics to design the scenery, characters, motion, and even the lighting throughout much of the film.

Translating Points in the Plane

Much of computer graphics consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 8. Other types of transformations include reflections, rotations, and stretches.

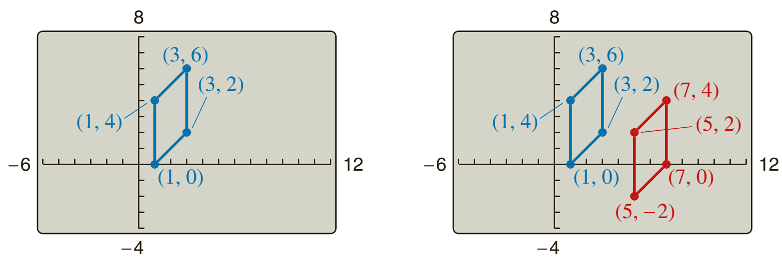
EXAMPLE 8 Translating Points in the Plane

Figure 1.12(a) shows the vertices of a parallelogram. Find the vertices of the parallelogram after it has been translated four units to the right and two units down.

SOLUTION To translate each vertex four units to the right, add 4 to each x-coordinate. To translate each vertex two units down, subtract 2 from each y-coordinate.

Original Point	*Translated Point*
$(1, 0)$	$(1 + 4, 0 - 2) = (5, -2)$
$(3, 2)$	$(3 + 4, 2 - 2) = (7, 0)$
$(3, 6)$	$(3 + 4, 6 - 2) = (7, 4)$
$(1, 4)$	$(1 + 4, 4 - 2) = (5, 2)$

The translated parallelogram is shown in Figure 1.12(b).



(a) (b)

FIGURE 1.12

✓ ***Checkpoint 8*** ***Worked-out solution available at LarsonAppliedCalculus.com***

Find the vertices of the parallelogram in Example 8 after it has been translated two units to the left and four units down. ■

SUMMARIZE (Section 1.1)

1. Describe the Cartesian plane *(page 2)*. For an example of plotting points in the Cartesian plane, see Example 1.
2. Describe a scatter plot *(page 3)*. For an example of a scatter plot, see Example 2.
3. State the Distance Formula *(page 4)*. For examples of using the Distance Formula, see Examples 3, 4, and 5.
4. State the Midpoint Formula *(page 6)*. For an example of using the Midpoint Formula, see Example 6.
5. Describe a real-life example of how the Midpoint Formula can be used to estimate annual revenues *(page 6, Example 7)*.
6. Describe how to translate points in the Cartesian plane *(page 7)*. For an example of translating points in the Cartesian plane, see Example 8.

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iStockphoto.com/PhotoEuphoria

SKILLS WARM UP 1.1

The following warm-up exercises involve skills that were covered in a previous course. You will use these skills in the exercise set for this section. For additional help, review Appendix A.3.

In Exercises 1–6, simplify the expression.

1. $\dfrac{5+(-4)}{2}$

2. $\dfrac{-3+(-1)}{2}$

3. $\sqrt{(3-6)^2+[1-(-5)]^2}$

4. $\sqrt{(-2-0)^2+[-7-(-3)]^2}$

5. $\sqrt{27}+\sqrt{12}$

6. $\sqrt{8}-\sqrt{18}$

In Exercises 7–10, solve for *x* or *y*.

7. $\dfrac{x+(-5)}{2}=7$

8. $\dfrac{-7+y}{2}=-3$

9. $\sqrt{(3-x)^2+(7-4)^2}=\sqrt{45}$

10. $\sqrt{(6-2)^2+(-2-y)^2}=\sqrt{52}$

Exercises 1.1

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.

Plotting Points in the Cartesian Plane In Exercises 1 and 2, plot the points in the Cartesian plane. *See Example 1.*

1. $(-5, 3), (1, -1), (-2, -4), (2, 0), (1, 4)$

2. $(0, -4), (5, 1), (-3, 5), (2, -2), (-6, -1)$

Finding a Distance and the Midpoint of a Line Segment In Exercises 3–12, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points. *See Examples 1, 3, and 6.*

3. $(3, 1), (5, 5)$

4. $(-3, 2), (3, -2)$

5. $(-3, 7), (1, -1)$

6. $(2, 2), (4, 14)$

7. $(2, -12), (8, -4)$

8. $(-5, -2), (7, 3)$

9. $\left(\frac{1}{2}, 1\right), \left(-\frac{3}{2}, -5\right)$

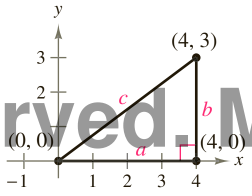
10. $\left(\frac{2}{3}, -\frac{1}{3}\right), \left(\frac{5}{6}, 1\right)$

11. $(0, -4.8), (0.5, 6)$

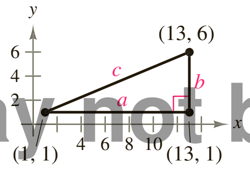
12. $(5.2, 6.4), (-2.7, 1.8)$

Verifying a Right Triangle In Exercises 13–16, (a) find the length of each side of the right triangle and (b) show that these lengths satisfy the Pythagorean Theorem. *See Example 4.*

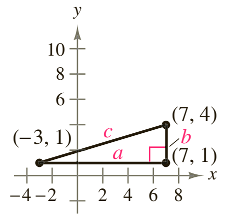
13.



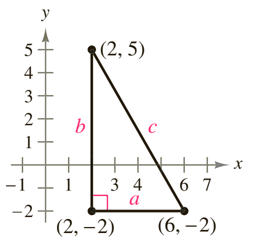
14.



15.



16.



Verifying a Polygon In Exercises 17–20, show that the points form the vertices of the indicated polygon. (A rhombus is a quadrilateral whose sides have the same length.)

17. Right triangle: $(0, 1), (3, 7), (4, -1)$

18. Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$

19. Rhombus: $(0, 0), (1, 2), (2, 1), (3, 3)$

20. Parallelogram: $(0, 1), (3, 7), (4, 4), (1, -2)$

Finding Values In Exercises 21 and 22, find the value(s) of *x* such that the distance between the points is 5.

21. $(1, 0), (x, -4)$

22. $(2, -1), (x, 2)$

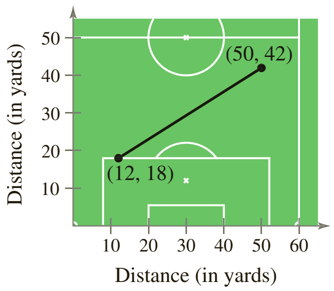
Finding Values In Exercises 23 and 24, find the value(s) of *y* such that the distance between the points is 8.

23. $(-3, 0), (-5, y)$

24. $(4, -6), (4, y)$

The symbol [QR] and a red exercise number indicates that a video solution can be seen at *CalcView.com*.

25. Sports A soccer player passes the ball from a point that is 18 yards from an endline and 12 yards from a sideline. The pass is received by a teammate who is 42 yards from the same endline and 50 yards from the same sideline, as shown in the figure. How long is the pass?



26. Sports The first soccer player in Exercise 25 passes the ball to another teammate who is 37 yards from the same endline and 33 yards from the same sideline. How long is the pass?

Graphing Data In Exercises 27 and 28, use a graphing utility to graph a scatter plot, a bar graph, and a line graph to represent the data. Describe any trends that appear.

27. Consumer Trends The numbers (in billions) of individuals using the Internet in the world for 2006 through 2013 are shown in the table. *(Source: International Telecommunications Union)*

DATA

Year	2006	2007	2008	2009
Individuals	1.151	1.365	1.561	1.751

Year	2010	2011	2012	2013
Individuals	2.032	2.271	2.510	2.710

Spreadsheet at LarsonAppliedCalculus.com

28. Consumer Trends The numbers (in millions) of cellular telephone subscribers in the United States for 2006 through 2013 are shown in the table. *(Source: CTIA-The Wireless Association)*

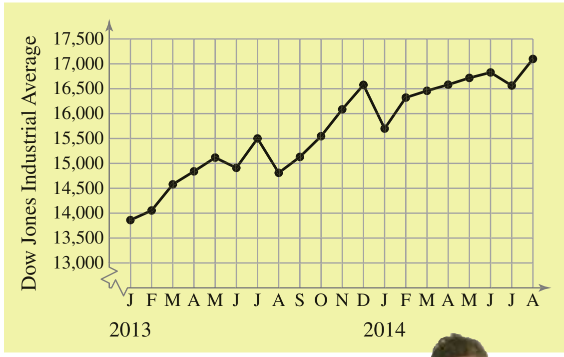
DATA

Year	2006	2007	2008	2009
Subscribers	233.0	255.4	270.3	285.6

Year	2010	2011	2012	2013
Subscribers	296.3	316.0	326.5	335.7

Spreadsheet at LarsonAppliedCalculus.com

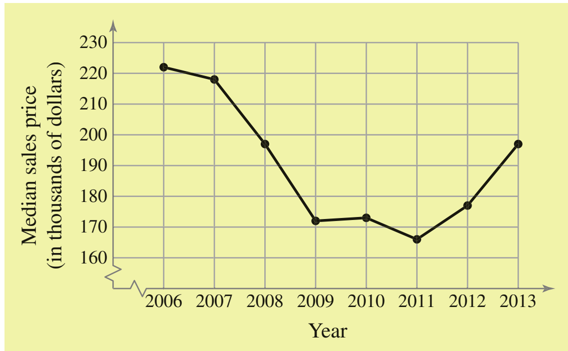
29. Dow Jones Industrial Average The graph shows the Dow Jones Industrial Average for common stocks. *(Source: S&P Dow Jones Indices LLC)*



(a) Estimate the Dow Jones Industrial Average for March 2013, July 2013, and July 2014.

(b) Estimate the percent increase or decrease in the Dow Jones Industrial Average from December 2013 to January 2014.

30. Home Sales The graph shows the median sales prices (in thousands of dollars) of existing one-family homes sold in the United States from 2006 through 2013. *(Source: National Association of Realtors)*



(a) Estimate the median sales prices of existing one-family homes for 2007, 2009, and 2012.

(b) Estimate the percent increase or decrease in the median value of existing one-family homes from 2011 to 2012.

The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by use of appropriate technology.

31. Revenue and Profit The revenues and profits of Buffalo Wild Wings for 2011 and 2013 are shown in the table. (a) Use the Midpoint Formula to estimate the revenue and profit in 2012. (b) Then use your school's library, the Internet, or some other reference source to find the actual revenue and profit for 2012. (c) Did the revenue and profit increase in a linear pattern from 2011 to 2013? Explain your reasoning. (d) What were the expenses during each of the given years? (e) How would you rate the growth of Buffalo Wild Wings from 2011 to 2013? *(Source: Buffalo Wild Wings, Inc.)*

Year	2011	2012	2013
Revenue (millions of $)	784.5		1266.7
Profit (millions of $)	50.4		71.6

32. Revenue and Profit The revenues and profits of Walt Disney Company for 2011 and 2013 are shown in the table. (a) Use the Midpoint Formula to estimate the revenue and profit in 2012. (b) Then use your school's library, the Internet, or some other reference source to find the actual revenue and profit for 2012. (c) Did the revenue and profit increase in a linear pattern from 2011 to 2013? Explain your reasoning. (d) What were the expenses during each of the given years? (e) How would you rate the growth of Walt Disney Company from 2011 to 2013? *(Source: Walt Disney Company)*

Year	2011	2012	2013
Revenue (billions of $)	40.9		45.0
Profit (billions of $)	4.8		6.1

33. Economics The table shows the numbers of ear infections treated by doctors at HMO clinics of three different sizes: small, medium, and large.

DATA

Number of doctors	0	1	2	3	4
Cases per small clinic	0	20	28	35	40
Cases per medium clinic	0	30	42	53	60
Cases per large clinic	0	35	49	62	70

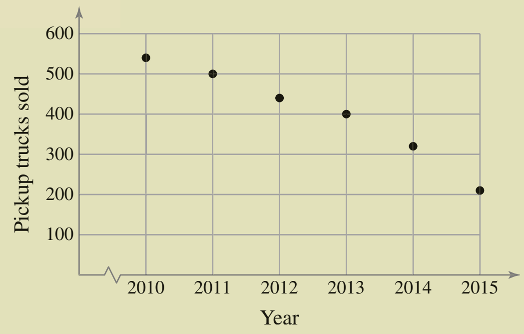
Spreadsheet at LarsonAppliedCalculus.com

(a) On the same coordinate plane, show the relationship between doctors and treated ear infections using *three* line graphs, where the number of doctors is on the horizontal axis and the number of ear infections treated is on the vertical axis.

(b) Compare the three relationships.

(Source: Adapted from Taylor, Economics, *Fifth Edition)*

34. HOW DO YOU SEE IT? The scatter plot shows the numbers of pickup trucks sold in a city from 2010 to 2015.



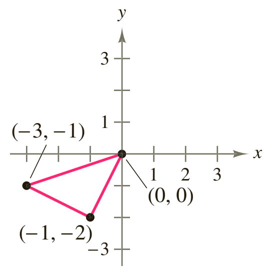
(a) In what year were 500 pickup trucks sold?

(b) About how many pickup trucks were sold in 2013?

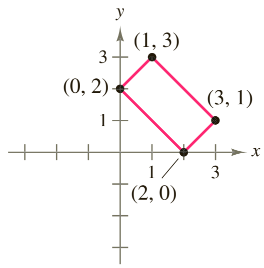
(c) Describe the pattern shown by the data.

Translating Points in the Plane In Exercises 35 and 36, use the translation and the graph to find the vertices of the figure after it has been translated. *See Example 8.*

35. 3 units left and 5 units down



36. 2 units right and 4 units up



37. Using the Midpoint Formula Use the Midpoint Formula repeatedly to find the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four equal parts.

38. Using the Midpoint Formula Use Exercise 37 to find the points that divide the line segment joining the given points into four equal parts.

(a) $(1, -2), (4, -1)$ (b) $(-2, -3), (0, 0)$

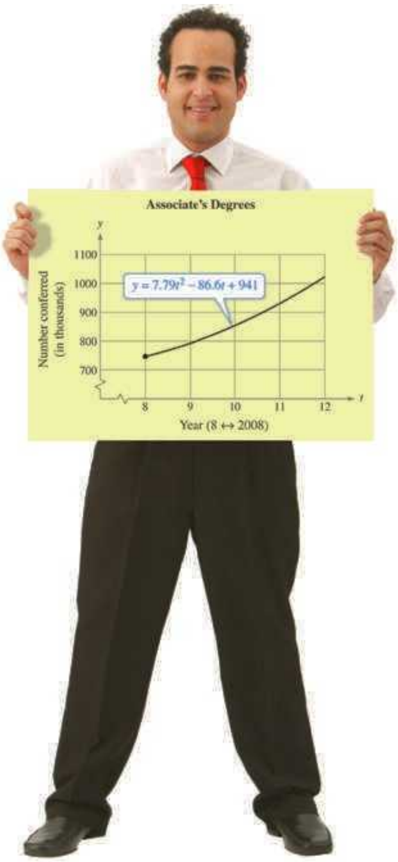
39. Using the Midpoint Formula Show that $\left(\frac{1}{3}[2x_1 + x_2], \frac{1}{3}[2y_1 + y_2]\right)$ is one of the points of trisection of the line segment joining (x_1, y_1) and (x_2, y_2). Then, find the second point of trisection by finding the midpoint of the line segment joining

$$\left(\frac{1}{3}[2x_1 + x_2], \frac{1}{3}[2y_1 + y_2]\right) \text{ and } (x_2, y_2).$$

40. Using the Midpoint Formula Use Exercise 39 to find the points of trisection of the line segment joining the given points.

(a) $(1, -2), (4, 1)$ (b) $(-2, -3), (0, 0)$

1.2 Graphs of Equations



In Exercise 61 on page 21, you will use a mathematical model to analyze the number of associate's degrees conferred in the United States.

- Sketch graphs of equations by hand.
- Find the x- and y-intercepts of graphs of equations.
- Write the standard forms of equations of circles.
- Find the points of intersection of two graphs.
- Use mathematical models to model and solve real-life problems.

The Graph of an Equation

In Section 1.1, you used a coordinate system to represent graphically the relationship between two quantities. There, the graphical picture consisted of a collection of points in a coordinate plane (see Example 2 in Section 1.1).

Frequently, a relationship between two quantities is expressed as an equation. For instance, degrees on the Fahrenheit scale are related to degrees on the Celsius scale by the equation

$$F = \frac{9}{5}C + 32.$$

In this section, you will study some basic procedures for sketching the graphs of such equations. The **graph** of an equation is the set of all points that are solutions of the equation.

EXAMPLE 1 Sketching the Graph of an Equation

Sketch the graph of $y = 7 - 3x$.

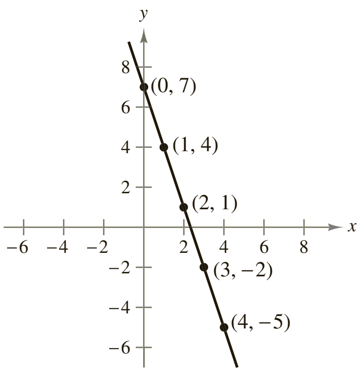
SOLUTION One way to sketch the graph of an equation is the *point-plotting method*. With this method, you construct a table of values that consists of several solution points of the equation, as shown in the table below. For instance, when $x = 0$,

$$y = 7 - 3(0) = 7$$

which implies that $(0, 7)$ is a solution point of the equation.

x	0	1	2	3	4
$y = 7 - 3x$	7	4	1	-2	-5

From the table, it follows that $(0, 7)$, $(1, 4)$, $(2, 1)$, $(3, -2)$, and $(4, -5)$ are solution points of the equation. After plotting these points, you can see that they appear to lie on a line, as shown in Figure 1.13. The graph of the equation is the line that passes through the five plotted points.



Solution Points for $y = 7 - 3x$
FIGURE 1.13

Checkpoint 1 Worked-out solution available at LarsonAppliedCalculus.com

Sketch the graph of $y = 2x - 1$. ■

STUDY TIP

Even though the sketch shown in Figure 1.13 is referred to as the graph of $y = 7 - 3x$, it actually represents only a *portion* of the graph. The entire graph is a line that would extend off the page.

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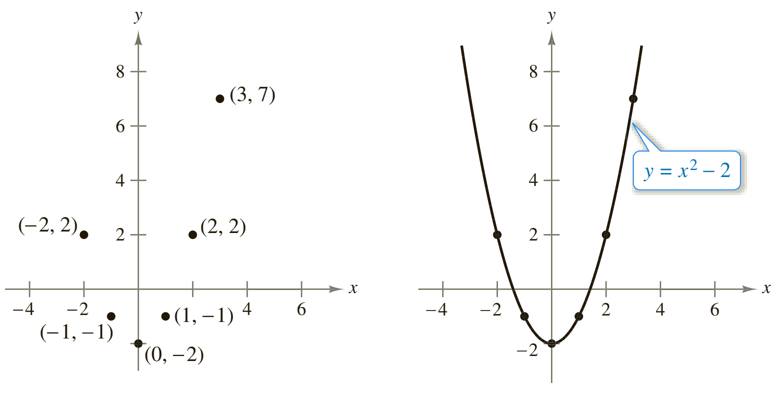
EXAMPLE 2 Sketching the Graph of an Equation

Sketch the graph of $y = x^2 - 2$.

SOLUTION Begin by constructing a table of values, as shown below.

x	-2	-1	0	1	2	3
$y = x^2 - 2$	2	-1	-2	-1	2	7

Next, plot the points given in the table, as shown in Figure 1.14(a). Finally, connect the points with a smooth curve, as shown in Figure 1.14(b).



(a) (b)

FIGURE 1.14

✓ Checkpoint 2

Worked-out solution available at LarsonAppliedCalculus.com

Sketch the graph of $y = x^2 - 4$. ■

The graph shown in Example 2 is a **parabola.** The graph of any second-degree equation of the form

$$y = ax^2 + bx + c, \quad a \neq 0$$

has a similar shape. If $a > 0$, then the parabola opens upward, as shown in Figure 1.14(b), and if $a < 0$, then the parabola opens downward.

Note that the point-plotting technique demonstrated in Examples 1 and 2 has some shortcomings. With too few solution points, you can badly misrepresent the graph of a given equation. For instance, how would you connect the four points in Figure 1.15? Without further information, any one of the three graphs in Figure 1.16 would be reasonable.

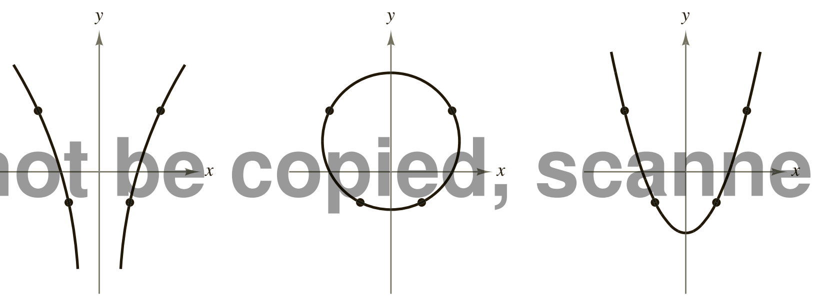


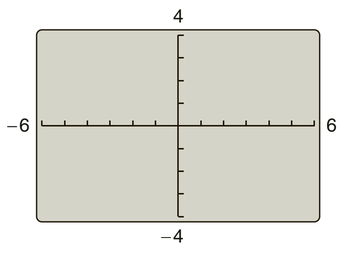
FIGURE 1.16

TECH TUTOR

On most graphing utilities, the display screen is two-thirds as high as it is wide. On such screens, you can obtain a graph with a true geometric perspective by using a *square setting*—one in which

$$\frac{\text{Ymax} - \text{Ymin}}{\text{Xmax} - \text{Xmin}} = \frac{2}{3}.$$

One such setting is shown below. Notice that the x and y tick marks are equally spaced on a square setting, but not on a *nonsquare* setting.



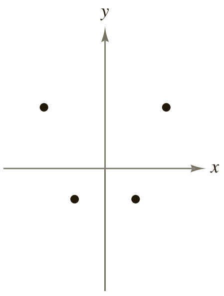


FIGURE 1.15

ALGEBRA TUTOR

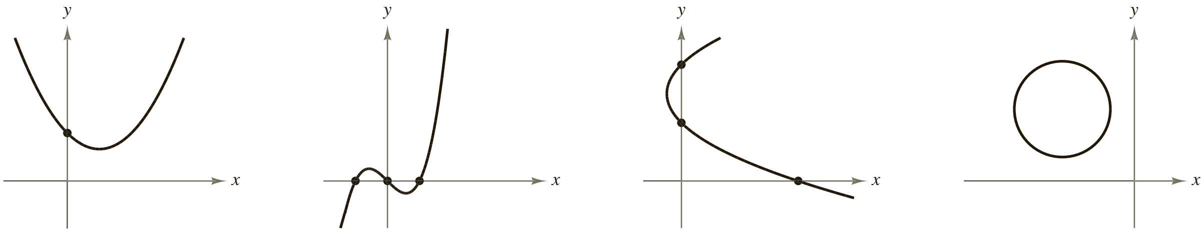
Finding intercepts involves solving equations. For a review of some techniques for solving equations, see page 71.

Intercepts of a Graph

Some solution points have zero as either the x-coordinate or the y-coordinate. These points are called **intercepts** because they are the points at which the graph intersects the x- or y-axis.

Some texts denote the x-intercept as simply the x-coordinate of the point $(a, 0)$ rather than the point itself. Likewise, some texts denote the y-intercept as the y-coordinate of the point $(0, b)$. Unless it is necessary to make a distinction, the term *intercept* will refer to either the point or the coordinate.

It is possible for a graph to have no intercepts, one intercept, or several intercepts, as shown in Figure 1.17.



No x-intercept
One y-intercept

Three x-intercepts
One y-intercept

One x-intercept
Two y-intercepts

No intercepts

FIGURE 1.17

TECH TUTOR

Some graphing utilities have a built-in program that can find the x-intercepts of a graph. If your graphing utility has this feature, try using it to find the x-intercepts of the graph of the equation in Example 3. (Your utility may call this the *root* or *zero* feature.)

Finding Intercepts

1. To find ***x*-intercepts,** let y be zero and solve the equation for x.
2. To find ***y*-intercepts,** let x be zero and solve the equation for y.

EXAMPLE 3 Finding *x*- and *y*-Intercepts

Find the x- and y-intercepts of the graph of $y = x^3 - 4x$.

SOLUTION To find the x-intercepts, let y be zero and solve for x.

$$x^3 - 4x = 0 \quad \text{Let } y \text{ be zero.}$$
$$x(x^2 - 4) = 0 \quad \text{Factor out common monomial factor.}$$
$$x(x + 2)(x - 2) = 0 \quad \text{Factor.}$$
$$x = 0, -2, \text{ or } 2 \quad \text{Solve for } x.$$

Because this equation has three solutions, you can conclude that the graph has three x-intercepts:

$(0, 0)$, $(-2, 0)$, and $(2, 0)$. *x-intercepts*

To find the y-intercepts, let x be zero and solve for y. Doing this produces

$$y = x^3 - 4x = 0^3 - 4(0) = 0.$$

This equation has only one solution, so the graph has one y-intercept:

$(0, 0)$. *y-intercept*

(See Figure 1.18.)

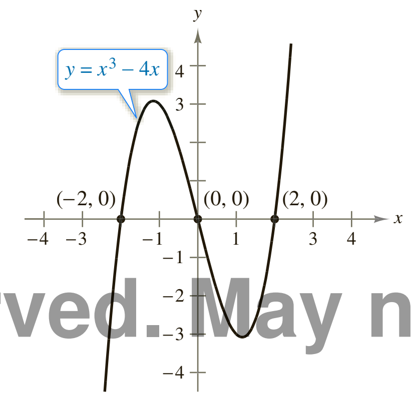


FIGURE 1.18

✓ **Checkpoint 3** *Worked-out solution available at LarsonAppliedCalculus.com*

Find the x- and y-intercepts of the graph of $y = x^2 - 2x - 3$. ■

Circles

Throughout this course, you will learn to recognize several types of graphs from their equations. For instance, you should recognize that the graph of a second-degree equation of the form

$$y = ax^2 + bx + c, \quad a \neq 0$$

is a parabola (see Example 2). Another easily recognized graph is that of a **circle.**

Consider the circle shown in Figure 1.19. A point (x, y) is on the circle if and only if its distance from the center (h, k) is r. By the Distance Formula,

$$\sqrt{(x-h)^2 + (y-k)^2} = r.$$

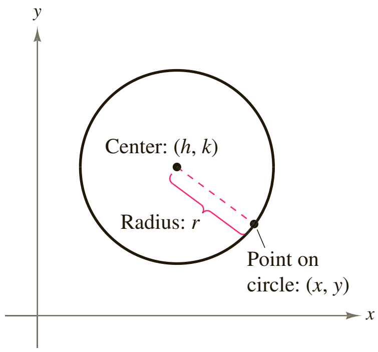


FIGURE 1.19

By squaring each side of this equation, you obtain the **standard form of the equation of a circle.**

Standard Form of the Equation of a Circle

The **standard form of the equation of a circle** is

$$(x-h)^2 + (y-k)^2 = r^2. \qquad \text{Center at } (h, k)$$

The point (h, k) is the **center** of the circle, and the positive number r is the **radius** of the circle. The standard form of the equation of a circle whose center is the origin, $(h, k) = (0, 0)$, is

$$x^2 + y^2 = r^2. \qquad \text{Center at } (0, 0)$$

EXAMPLE 4 **Finding the Equation of a Circle**

The point $(3, 4)$ lies on a circle whose center is at $(-1, 2)$. Find the standard form of the equation of this circle and sketch its graph.

SOLUTION The radius of the circle is the distance between $(-1, 2)$ and $(3, 4)$.

$$
\begin{aligned}
r &= \sqrt{[3-(-1)]^2 + (4-2)^2} && \text{Distance Formula} \\
&= \sqrt{(4)^2 + (2)^2} && \text{Simplify.} \\
&= \sqrt{16 + 4} && \text{Simplify.} \\
&= \sqrt{20} && \text{Radius}
\end{aligned}
$$

Using $(h, k) = (-1, 2)$ and $r = \sqrt{20}$, the standard form of the equation of the circle is

$$
\begin{aligned}
(x-h)^2 + (y-k)^2 &= r^2 && \text{Equation of a circle} \\
[x-(-1)]^2 + (y-2)^2 &= (\sqrt{20})^2 && \text{Substitute for } h, k, \text{ and } r. \\
(x+1)^2 + (y-2)^2 &= 20. && \text{Write in standard form.}
\end{aligned}
$$

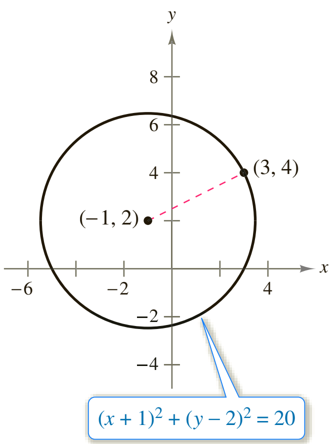


FIGURE 1.20

The graph of the equation of the circle is shown in Figure 1.20.

✓ **Checkpoint 4** *Worked-out solution available at LarsonAppliedCalculus.com*

The point $(1, 5)$ lies on a circle whose center is at $(-2, 1)$. Find the standard form of the equation of this circle and sketch its graph. ■

Edyta Pawlowska/Shutterstock.com

STUDY TIP

You can check the points of intersection in Figure 1.21 by verifying that the points are solutions of *both* of the original equations or by using the *intersect* feature of a graphing utility.

Points of Intersection

An ordered pair that is a solution of two different equations is called a **point of intersection** of the graphs of the two equations. For instance, Figure 1.21 shows that the graphs of

$$y = x^2 - 3 \quad \text{and} \quad y = x - 1$$

have two points of intersection: $(2, 1)$ and $(-1, -2)$. To find the points analytically, set the two y-values equal to each other and solve the equation

$$x^2 - 3 = x - 1$$

for x.

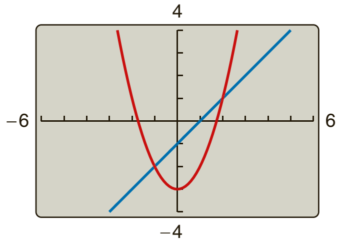


FIGURE 1.21

A common business application that involves a point of intersection is **break-even analysis.** The marketing of a new product typically requires an initial investment. When sufficient units have been sold so that the total revenue has offset the total cost, the sale of the product has reached the **break-even point.** The **total cost** of producing x units of a product is denoted by C, and the **total revenue** from the sale of x units of the product is denoted by R. So, you can find the break-even point by setting the cost C equal to the revenue R and solving for x. In other words, the break-even point corresponds to the point of intersection of the cost and revenue graphs.

EXAMPLE 5 Finding a Break-Even Point

A company manufactures a product at a cost of \$0.65 per unit and sells the product for \$1.20 per unit. The company's initial investment to produce the product was \$10,000. Will the company break even when it sells 18,000 units? How many units must the company sell to break even?

SOLUTION The total cost of producing x units of the product is given by

$C = 0.65x + 10{,}000.$ Cost equation

The total revenue from the sale of x units is given by

$R = 1.2x.$ Revenue equation

To find the break-even point, set the cost equal to the revenue and solve for x.

$R = C$ Set revenue equal to cost.

$1.2x = 0.65x + 10{,}000$ Substitute for R and C.

$0.55x = 10{,}000$ Subtract $0.65x$ from each side.

$x = \dfrac{10{,}000}{0.55}$ Divide each side by 0.55.

$x \approx 18{,}182$ Use a calculator.

So, the company will not break even when it sells 18,000 units. The company must sell 18,182 units before it breaks even. This result is shown graphically in Figure 1.22. Note in Figure 1.22 that sales less than 18,182 units correspond to a loss for the company ($R < C$), whereas sales greater than 18,182 units correspond to a profit for the company ($R > C$).

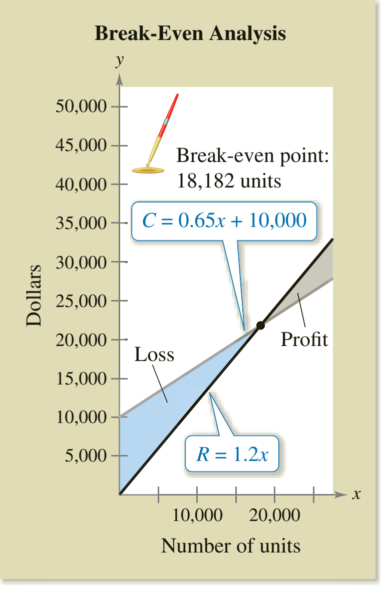
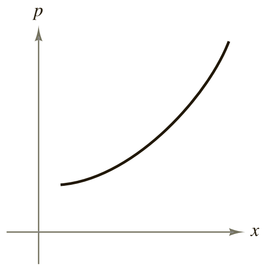


FIGURE 1.22

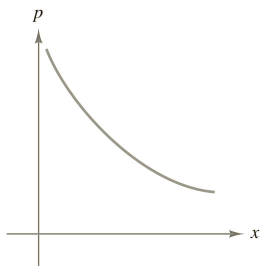
Checkpoint 5 Worked-out solution available at LarsonAppliedCalculus.com

How many units must the company in Example 5 sell to break even when the selling price is \$1.45 per unit?



Supply Curve

FIGURE 1.23



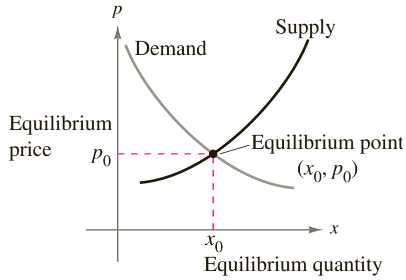
Demand Curve

FIGURE 1.24

Two types of equations that economists use to analyze a market are supply and demand equations. A **supply equation** shows the relationship between the unit price p of a product and the quantity supplied x. The graph of a supply equation is called a **supply curve.** (See Figure 1.23.) A typical supply curve rises because producers of a product want to sell more units when the unit price is higher.

A **demand equation** shows the relationship between the unit price p of a product and the quantity demanded x. The graph of a demand equation is called a **demand curve.** (See Figure 1.24.) A typical demand curve tends to show a decrease in the quantity demanded with each increase in price.

In an ideal situation, with no other factors present to influence the market, the production level should stabilize at the point of intersection of the graphs of the supply and demand equations. This point is called the **equilibrium point.** The x-coordinate of the equilibrium point is called the **equilibrium quantity** and the p-coordinate is called the **equilibrium price.** (See Figure 1.25.) You can find the equilibrium point by setting the demand equation equal to the supply equation and solving for x.



Equilibrium Point

FIGURE 1.25

EXAMPLE 6 Finding the Equilibrium Point

The demand and supply equations for an e-book reader are

$p = 195 - 5.8x$ Demand equation

$p = 150 + 3.2x$ Supply equation

where p is the price in dollars and x represents the number of units in millions. Find the equilibrium point for this market.

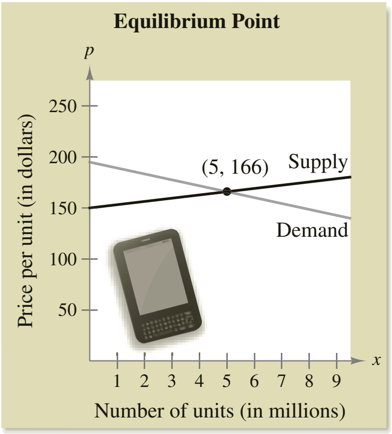


FIGURE 1.26

SOLUTION Begin by setting the demand equation equal to the supply equation.

$195 - 5.8x = 150 + 3.2x$ Set equations equal to each other.

$45 - 5.8x = 3.2x$ Subtract 150 from each side.

$45 = 9x$ Add $5.8x$ to each side.

$5 = x$ Divide each side by 9.

So, the equilibrium point occurs when the demand and supply are each five million units. (See Figure 1.26.) The price that corresponds to this x-value is obtained by substituting $x = 5$ into either of the original equations. For instance, substituting into the demand equation produces

$$p = 195 - 5.8(5) = 195 - 29 = \$166.$$

Note that when you substitute $x = 5$ into the supply equation, you obtain

$$p = 150 + 3.2(5) = 150 + 16 = \$166.$$

Checkpoint 6 *Worked-out solution available at LarsonAppliedCalculus.com*

The demand and supply equations for a streaming-media device are

$p = 113 - 3.5x$ Demand equation

$p = 89 + 2.5x$ Supply equation

where p is the price in dollars and x represents the number of units in millions. Find the equilibrium point for this market. ■

ALGEBRA TUTOR

For help with evaluating the expressions in Example 7, see the review of order of operations on page 70.

Mathematical Models

In this text, you will see many examples of the use of equations as **mathematical models** of real-life phenomena. In developing a mathematical model to represent actual data, you should strive for two (often conflicting) goals—accuracy and simplicity.

EXAMPLE 7 Using a Mathematical Model

The table shows the annual average crude oil production (in millions of barrels per day) in the United States from 2009 through 2013. *(Source: U.S. Energy Information Administration)*

Year	2009	2010	2011	2012	2013
Crude oil production	5.35	5.48	5.64	6.50	7.45

A mathematical model for these data is given by

$$y = 0.1671t^2 - 3.155t + 20.23$$

where y is the annual average crude oil production (in millions of barrels per day) and t is the year, with $t = 9$ corresponding to 2009. Use a graph and a table to compare the data with the model. Use the model to estimate the crude oil production in 2014.

SOLUTION Figure 1.27 and the table below compare the data with the model. The model appears to fit the data well.

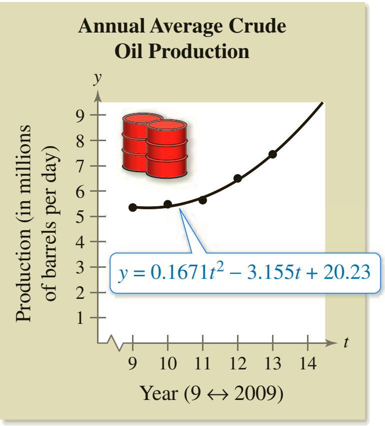


FIGURE 1.27

Year	2009	2010	2011	2012	2013
Crude oil production	5.35	5.48	5.64	6.50	7.45
Model	5.37	5.39	5.74	6.43	7.45

Using $t = 14$ to represent 2014, you can estimate the crude oil production in 2014 to be

$$y = 0.1671(14)^2 - 3.155(14) + 20.23 \approx 8.81 \text{ million barrels per day.}$$

✓ ***Checkpoint 7*** *Worked-out solution available at LarsonAppliedCalculus.com*

The table shows the annual sales (in billions of dollars) for Dollar Tree stores from 2006 through 2013. *(Source: Dollar Tree, Inc.)*

DATA

Year	2006	2007	2008	2009	2010	2011	2012	2013
Sales	3.97	4.24	4.64	5.23	5.88	6.63	7.39	7.84

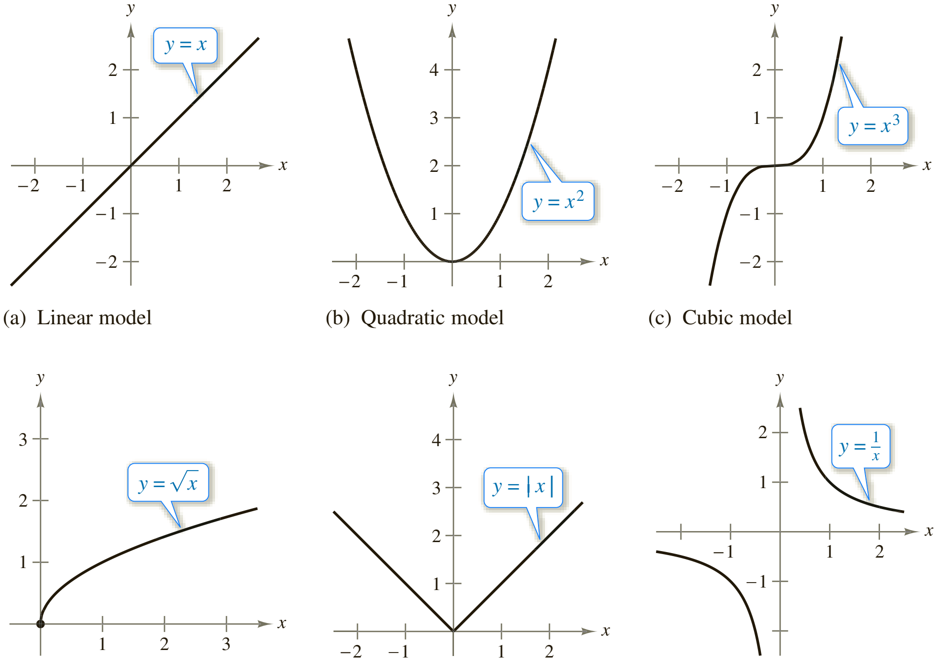
Spreadsheet at LarsonAppliedCalculus.com

A mathematical model for these data is given by

$$S = 0.0294t^2 + 0.030t + 2.63$$

where S is the annual sales (in billions of dollars) and t is the year, with $t = 6$ corresponding to 2006. Use a graph and a table to compare the data with the model. Use the model to estimate the sales in 2014. ■

Much of your study of calculus will center around the behavior of the graphs of mathematical models. Figure 1.28 shows the graphs of six basic algebraic equations. Familiarity with these graphs will help you in the creation and use of mathematical models.



(a) Linear model (b) Quadratic model (c) Cubic model

(d) Square root model (e) Absolute value model (f) Rational model

FIGURE 1.28

SUMMARIZE (Section 1.2)

1. Describe how to sketch the graph of an equation by hand *(page 11)*. For examples of sketching graphs by hand, see Examples 1 and 2.
2. Describe how to find the x- and y-intercepts of a graph *(page 13)*. For an example of finding the x- and y-intercepts of a graph, see Example 3.
3. State the standard form of the equation of a circle *(page 14)*. For an example of finding the standard form of the equation of a circle, see Example 4.
4. Describe how to find a point of intersection of the graphs of two equations *(page 15)*. For examples of finding points of intersection, see Examples 5 and 6.
5. Describe break-even analysis *(page 15)*. For an example of break-even analysis, see Example 5.
6. Describe supply equations, demand equations, and equilibrium points *(page 16)*. For an example of a supply equation, a demand equation, and an equilibrium point, see Example 6.
7. Describe a mathematical model *(page 17)*. For an example of a mathematical model, see Example 7.

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SKILLS WARM UP 1.2

The following warm-up exercises involve skills that were covered in a previous course. You will use these skills in the exercise set for this section. For additional help, review Appendices A.3 and A.4.

In Exercises 1–6, solve for y.

1. $5y - 12 = x$

2. $-y = 15 - x$

3. $x^3y + 2y = 1$

4. $x^2 + x - y^2 - 6 = 0$

5. $(x - 2)^2 + (y + 1)^2 = 9$

6. $(x + 6)^2 + (y - 5)^2 = 81$

In Exercises 7–10, evaluate the expression for the given value of x.

	Expression	*x-Value*		*Expression*	*x-Value*
7.	$y = 5x$	$x = -2$	**8.**	$y = 3x - 4$	$x = 3$
9.	$y = 4x^2 - 7$	$x = 0.5$	**10.**	$y = 9x^2 + 9x - 5$	$x = \frac{1}{3}$

In Exercises 11–14, factor the expression.

11. $x^2 - 3x + 2$

12. $x^2 + 5x + 6$

13. $y^2 - 3y + \frac{9}{4}$

14. $y^2 - 7y + \frac{49}{4}$

Exercises 1.2

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.

Matching In Exercises 1–6, match the equation with its graph. [The graphs are labeled (a)–(f).]

1. $y = x - 2$

2. $y = -\frac{1}{2}x + 2$

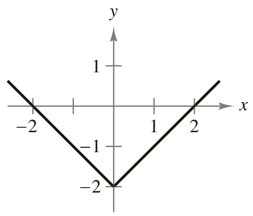
3. $y = x^2 + 2x$

4. $y = \sqrt{9 - x^2}$

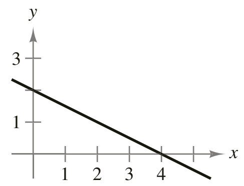
5. $y = |x| - 2$

6. $y = x^3 - x$

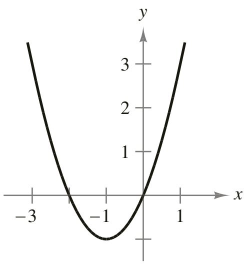
(a)



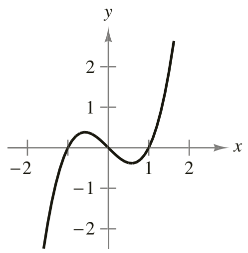
(b)



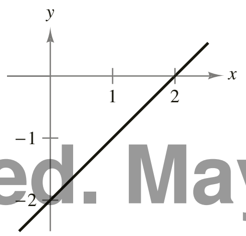
(c)



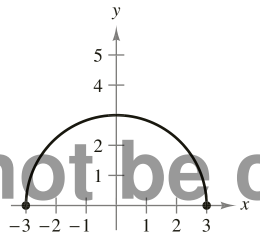
(d)



(e)



(f)



Sketching the Graph of an Equation In Exercises 7–22, sketch the graph of the equation. Use a graphing utility to verify your results. *See Examples 1 and 2.*

7. $y = 2x + 3$

8. $y = 1 - 4x$

9. $y = x^2 - 3$

10. $y = x^2 + 6$

11. $y = (x - 1)^2$

12. $y = (x + 5)^2$

13. $y = x^3 + 2$

14. $y = 1 - x^3$

15. $y = -\sqrt{x - 1}$

16. $y = \sqrt{x + 4}$

17. $y = |x + 1|$

18. $y = -|x - 2|$

19. $y = \dfrac{1}{x - 3}$

20. $y = \dfrac{1}{x + 2}$

21. $x = y^2 - 4$

22. $x = 4 - y^2$

Finding *x*- and *y*-Intercepts In Exercises 23–32, find the x- and y-intercepts of the graph of the equation. *See Example 3.*

23. $2x - y - 3 = 0$

24. $4x - 3y - 6 = 0$

25. $y = x^2 + x - 2$

26. $y = x^2 - 4x + 3$

27. $y = x^3 + 7x^2$

28. $y = x^3 - 9x$

29. $y = \dfrac{x^2 - 4}{x - 2}$

30. $y = \dfrac{x^2 + 3x}{2x}$

31. $x^2y - x^2 + 4y = 0$

32. $2x^2y + 8y - x^2 = 1$

Finding the Equation of a Circle In Exercises 33–40, find the standard form of the equation of the circle with the given characteristics and sketch its graph. *See Example 4.*

33. Center: $(0, 0)$; radius: 4

34. Center: $(0, 0)$; radius: 5

35. Center: $(2, -1)$; radius: 3

36. Center: $(-4, 3)$; radius: 2

37. Center: $(-1, 1)$; solution point: $(-1, 5)$

38. Center: $(2, -3)$; solution point: $(5, -7)$

39. Endpoints of a diameter: $(-6, -8), (6, 8)$

40. Endpoints of a diameter: $(0, -4), (6, 4)$

Finding Points of Intersection In Exercises 41–48, find the points of intersection (if any) of the graphs of the equations. Use a graphing utility to check your results.

41. $y = -x + 2,\ y = 2x - 1$

42. $y = -x + 7,\ y = \frac{3}{2}x - 8$

43. $y = -x^2 + 15,\ y = 3x + 11$

44. $y = x^2 - 5,\ y = x + 1$

45. $y = x^3,\ y = 2x$

46. $y = \sqrt{x},\ y = x$

47. $y = x^4 - 2x^2 + 1,\ y = 1 - x^2$

48. $y = x^3 - 2x^2 + x - 1,\ y = -x^2 + 3x - 1$

Finding a Break-Even Point In Exercises 49–54, C represents the total cost (in dollars) of producing x units of a product and R represents the total revenue (in dollars) from the sale of x units. How many units must the company sell to break even? *See Example 5.*

49. $C = 0.85x + 35{,}000,\ R = 1.55x$

50. $C = 6x + 500{,}000,\ R = 35x$

51. $C = 8650x + 250{,}000,\ R = 9950x$

52. $C = 2.5x + 10{,}000,\ R = 4.9x$

53. $C = 6x + 5000,\ R = 10x$

54. $C = 130x + 12{,}600,\ R = 200x$

55. Break-Even Analysis You are setting up a part-time business with an initial investment of \$21,000. The unit cost of the product is \$11.50, and the selling price is \$19.90.

(a) Find equations for the total cost C (in dollars) and total revenue R (in dollars) for x units.

(b) Find the break-even point by finding the point of intersection of the cost and revenue equations.

(c) How many units would yield a profit of \$1000?

56. Break-Even Analysis A 2015 Toyota Camry costs \$33,500 with a gasoline engine. A 2015 Toyota Avalon costs \$36,775 with a hybrid engine. The Camry gets 31 miles per gallon of gasoline and the Avalon gets 39 miles per gallon of gasoline. Assume that the price of gasoline is \$2.759. *(Source: Toyota Motor Sales, U.S.A., Inc. and U.S. Energy Information Administration)*

(a) Show that the cost C_g (in dollars) of driving the Toyota Camry x miles is

$$C_g = 33{,}500 + \frac{2.759x}{31}$$

and the cost C_h (in dollars) of driving the Toyota Avalon x miles is

$$C_h = 36{,}775 + \frac{2.759x}{39}.$$

(b) Find the break-even point. That is, find the mileage at which the hybrid-powered Toyota Avalon becomes more economical than the gasoline-powered Toyota Camry.

57. Supply and Demand The demand and supply equations for a fitness tracking band are given by

$p = 205 - 4x$ Demand equation

$p = 135 + 3x$ Supply equation

where p is the price (in dollars) and x represents the number of units (in thousands). Find the equilibrium point for this market.

58. Supply and Demand The demand and supply equations for an MP3 player are given by

$p = 190 - 15x$ Demand equation

$p = 75 + 8x$ Supply equation

where p is the price (in dollars) and x represents the number of units (in hundreds of thousands). Find the equilibrium point for this market.

59. E-Book Revenue The table shows the annual revenues (in billions of dollars) of e-books in the United States from 2009 through 2013. *(Source: Statista)*

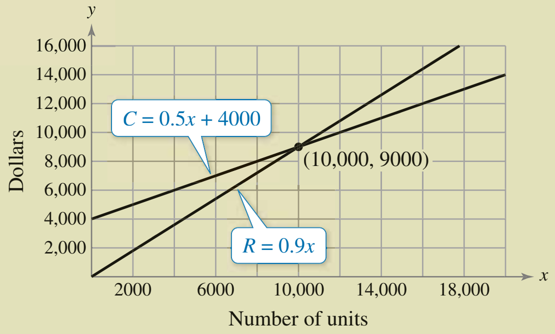
Year	2009	2010	2011	2012	2013
Revenue	0.82	1.52	2.31	3.35	4.52

A mathematical model for the data is given by $y = 0.00333t^3 - 0.0250t^2 + 0.252t - 1.85$, where y is the annual revenue (in billions of dollars) and t is the year, with $t = 9$ corresponding to 2009.

(a) Use a graph and a table to compare the data with the model.

(b) Use the model to predict the revenue in 2018.

60. HOW DO YOU SEE IT? The graph shows the cost and revenue equations for a product.



(a) For what number of units sold does the company break even?

(b) For what numbers of units sold is there a loss for the company?

(c) For what numbers of units sold is there a profit for the company?

61. Associate's Degrees A mathematical model for the numbers of associate's degrees conferred y (in thousands) from 2008 through 2012 is given by the equation $y = 7.79t^2 - 86.6t + 941$, where t represents the year, with $t = 8$ corresponding to 2008. *(Source: National Center for Education Statistics)*

(a) Use the model to complete the table.

Year	2008	2009	2010	2011	2012	2016
Degrees						

(b) This model was created using actual data from 2008 through 2012. How accurate do you think the model is in predicting the number of associate's degrees conferred in 2016? Explain your reasoning.

(c) Using this model, what is the prediction for the number of associate's degrees conferred in 2020? Do you think this prediction is valid?

62. Heart Transplants A mathematical model for the numbers of heart transplants y performed in the United States in the years 2009 through 2013 is given by $y = 19.000t^3 - 617.71t^2 + 6696.7t - 21{,}873$, where t represents the year, with $t = 9$ corresponding to 2009. *(Source: Organ Procurement and Transplantation Network)*

(a) Use a graphing utility or a spreadsheet to complete the table.

Year	2009	2010	2011	2012	2013
Transplants					

(b) Use your school's library, the Internet, or some other reference source to find the actual numbers of heart transplants in the years 2009 through 2013. Compare the actual numbers with those given by the model. How well does the model fit the data? Explain your reasoning.

(c) Using this model, what is the prediction for the number of heart transplants in 2019? Do you think this prediction is valid? What factors could affect this model's accuracy?

63. Making a Conjecture Use a graphing utility to graph the equation $y = cx + 1$ for $c = 1, 2, 3, 4,$ and 5. Then make a conjecture about the x-coefficient and the graph of the equation.

64. Break-Even Point Define the break-even point for a business marketing a new product. Give examples of a linear cost equation and a linear revenue equation for which the break-even point is 10,000 units.

Finding Intercepts In Exercises 65–70, use a graphing utility to graph the equation and approximate the x- and y-intercepts of the graph.

65. $y = 0.24x^2 + 1.32x + 1.815$

66. $y = -0.56x^2 - 5.34x + 6.25$

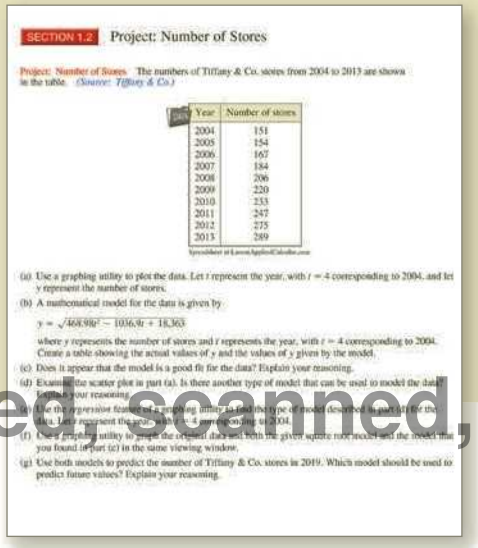
67. $y = \sqrt{0.3x^2 - 4.3x + 5.7}$

68. $y = \sqrt{-1.21x^2 + 2.34x + 5.6}$

69. $y = \dfrac{0.2x^2 + 1}{0.1x + 2.4}$

70. $y = \dfrac{0.4x - 5.3}{0.4x^2 + 5.3}$

71. Project: Number of Stores For a project analyzing the numbers of Tiffany & Co. stores from 2004 through 2013, visit this text's website at *LarsonAppliedCalculus.com*. *(Source: Tiffany & Co.)*



SECTION 1.2 Project: Number of Stores

Project: Number of Stores The numbers of Tiffany & Co. stores from 2004 to 2013 are shown in the table. *(Source: Tiffany & Co.)*

Year	Number of stores
2004	151
2005	154
2006	167
2007	184
2008	206
2009	220
2010	233
2011	247
2012	275
2013	289

Spreadsheet at LarsonAppliedCalculus.com

(a) Use a graphing utility to plot the data. Let t represent the year, with $t = 4$ corresponding to 2004, and let y represent the number of stores.

(b) A mathematical model for the data is given by

$$y = \sqrt{468.98t^2 - 1036.9t + 18{,}363}$$

where y represents the number of stores and t represents the year, with $t = 4$ corresponding to 2004. Create a table showing the actual values of y and the values of y given by the model.

(c) Does it appear that the model is a good fit for the data? Explain your reasoning.

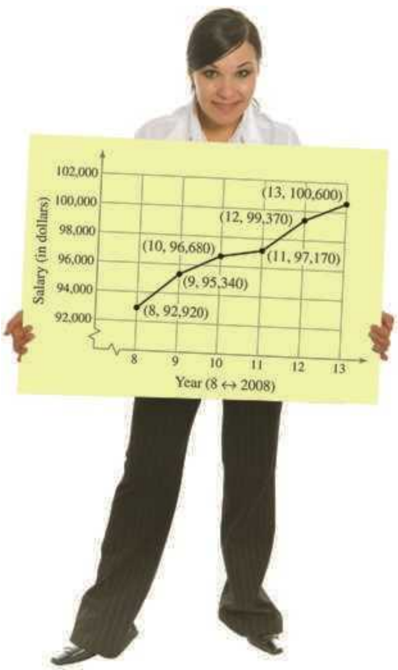
(d) Examine the scatter plot in part (a). Is there another type of model that can be used to model the data? Explain your reasoning.

(e) Use the *regression* feature of a graphing utility to find the type of model described in part (d) for the data. Let t represent the year, with $t = 4$ corresponding to 2004.

(f) Use a graphing utility to graph the original data and both the given square root model and the model that you found in part (e) in the same viewing window.

(g) Use both models to predict the number of Tiffany & Co. stores in 2019. Which model should be used to predict future values? Explain your reasoning.

1.3 Lines in the Plane and Slope



In Exercise 83 on page 32, you will use slope to analyze the average salaries of postsecondary education administrators.

- Use the slope-intercept form of a linear equation to sketch graphs.
- Find slopes of lines passing through two points.
- Use the point-slope form to write equations of lines.
- Find equations of parallel and perpendicular lines.
- Use linear equations to model and solve real-life problems.

Using Slope

The simplest mathematical model for relating two variables is the **linear equation**

$y = mx + b.$ Linear equation

This equation is called *linear* because its graph is a line. (In this text, the term *line* is used to mean *straight line*.) By letting $x = 0$, you can see that the line crosses the y-axis at

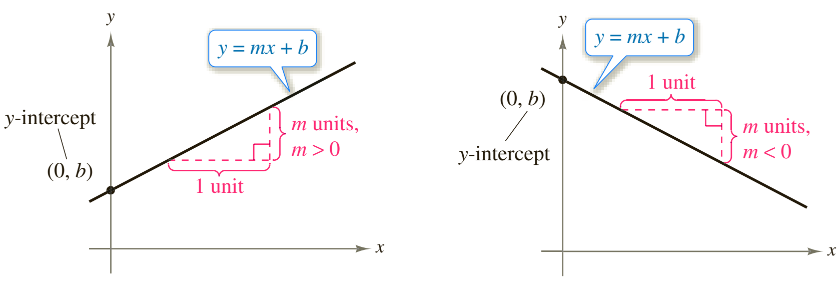
$y = b$

as shown in Figure 1.29. In other words, the y-intercept is $(0, b)$. The steepness or slope of the line is m.

$y = mx + b$

Slope ↑ ↑ y-intercept

The **slope** of a nonvertical line is the number of units the line rises (or falls) vertically for each unit of horizontal change from left to right, as shown in Figure 1.29.



Positive slope, line rises.

Negative slope, line falls.

FIGURE 1.29

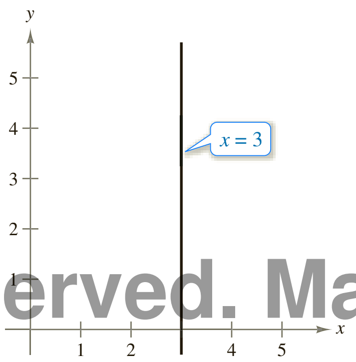
A linear equation that is written in the form $y = mx + b$ is said to be written in **slope-intercept form.**

The Slope-Intercept Form of the Equation of a Line

The graph of the equation

$y = mx + b$

is a line whose slope is m and whose y-intercept is $(0, b)$.



When a line is vertical, the slope is undefined.

FIGURE 1.30

A vertical line has an equation of the form

$x = a.$ Vertical line

Because the equation of a vertical line cannot be written in the form $y = mx + b$, it follows that the slope of a vertical line is undefined, as indicated in Figure 1.30.

Artpose Adam Borkowski/Shutterstock.com

Once you have determined the slope and the y-intercept of a line, it is a relatively simple matter to sketch its graph.

EXAMPLE 1 Graphing Linear Equations

Sketch the graph of each linear equation.

a. $y = 2x + 1$

b. $y = 2$

c. $x + y = 2$

SOLUTION

a. This equation is written in slope-intercept form.

$$y = 2x + 1 \qquad y = mx + b$$

Because $b = 1$, the y-intercept is $(0, 1)$. Moreover, because the slope is $m = 2$, the line *rises* two units for each unit the line moves to the right, as shown in Figure 1.31(a).

b. By writing this equation in slope-intercept form

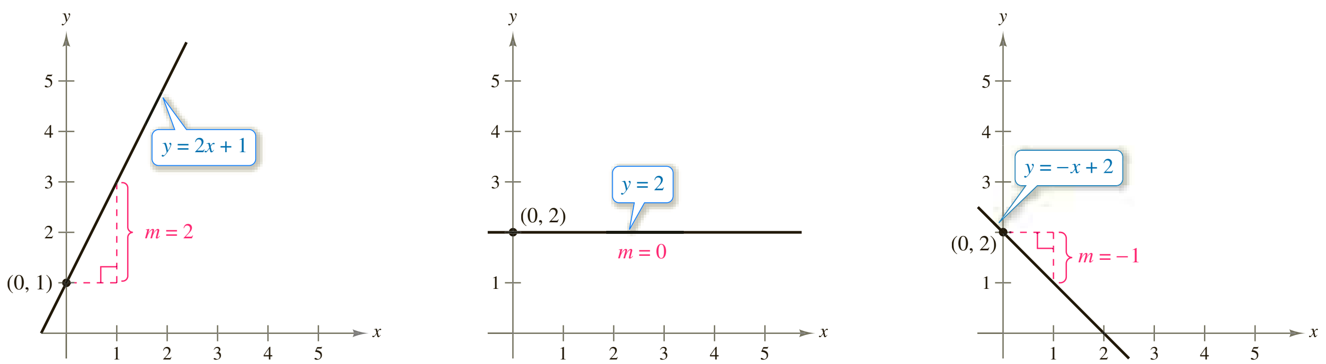
$$y = (0)x + 2 \qquad y = mx + b$$

you can see that the y-intercept is $(0, 2)$ and the slope is zero. A zero slope implies that the line is horizontal—that is, it does not rise *or* fall, as shown in Figure 1.31(b).

c. By writing this equation in slope-intercept form

$$x + y = 2 \qquad \text{Write original equation.}$$
$$y = -x + 2 \qquad \text{Subtract } x \text{ from each side.}$$
$$y = (-1)x + 2 \qquad \text{Write in slope-intercept form.}$$

you can see that the y-intercept is $(0, 2)$. Moreover, because the slope is $m = -1$, the line *falls* one unit for each unit the line moves to the right, as shown in Figure 1.31(c).



(a) When m is positive, the line rises from left to right.

(b) When m is zero, the line is horizontal.

(c) When m is negative, the line falls from left to right.

FIGURE 1.31

✓ ***Checkpoint 1*** *Worked-out solution available at LarsonAppliedCalculus.com*

Sketch the graph of each linear equation.

a. $y = 4x - 2$

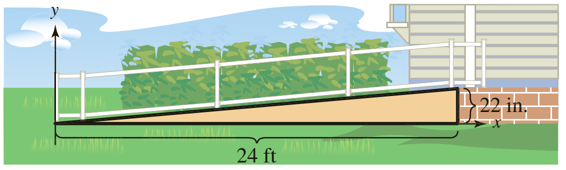
b. $x = 1$

c. $2x + y = 6$ ■

In real-life problems, the slope of a line can be interpreted as either a *ratio* or a *rate*. If the x- and y-axes have the same unit of measure, then the slope has no units and is a **ratio.** If the x- and y-axes have different units of measure, then the slope is a **rate** or **rate of change.**

EXAMPLE 2 Using Slope as a Ratio

The maximum recommended slope of a wheelchair ramp is $\frac{1}{12} \approx 0.083$. A business installs a wheelchair ramp that rises 22 inches over a horizontal length of 24 feet, as shown in the figure. Is the ramp steeper than recommended? *(Source: ADA Standards for Accessible Design)*



SOLUTION The horizontal length of the ramp is 24 feet or $12(24) = 288$ inches. So, the slope of the ramp is

$$\begin{aligned} \text{Slope} &= \frac{\text{vertical change}}{\text{horizontal change}} \\ &= \frac{22 \text{ in.}}{288 \text{ in.}} \\ &\approx 0.076. \end{aligned}$$

Because the slope of the ramp is less than 0.083, the ramp is not steeper than recommended. Note that the slope is a ratio and has no units.

Checkpoint 2 *Worked-out solution available at LarsonAppliedCalculus.com*

The business in Example 2 installs a second ramp that rises 27 inches over a horizontal length of 26 feet. Is the ramp steeper than recommended?

EXAMPLE 3 Using Slope as a Rate of Change

A manufacturing company determines that the total cost in dollars of producing x units of a product is

$$C = 25x + 3500.$$

Describe the practical significance of the y-intercept and slope of the line given by this equation.

SOLUTION The y-intercept $(0, 3500)$ tells you that the cost of producing zero units is \$3500. This is the **fixed cost** of production—it includes costs that must be paid regardless of the number of units produced. The slope, which is $m = 25$, tells you that the cost of producing each unit is \$25, as shown in Figure 1.32. Economists call the cost per unit the **marginal cost.** If the production increases by one unit, then the "margin" or extra amount of cost is \$25.

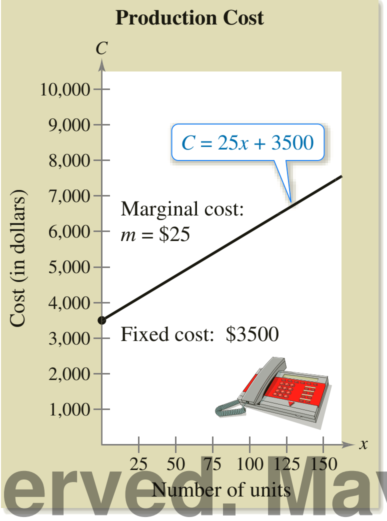


FIGURE 1.32

Checkpoint 3 *Worked-out solution available at LarsonAppliedCalculus.com*

A small business determines that the value of a digital copier t years after its purchase is $V = -300t + 1500$. Describe the practical significance of the y-intercept and slope of the line given by this equation. ■

Finding the Slope of a Line

Given an equation of a nonvertical line, you can find its slope by writing the equation in slope-intercept form. When you are not given an equation, you can still find the slope of a line. For instance, suppose you want to find the slope of the line passing through the points (x_1, y_1) and (x_2, y_2), as shown in Figure 1.33. As you move from left to right along this line, a change of $(y_2 - y_1)$ units in the vertical direction corresponds to a change of $(x_2 - x_1)$ units in the horizontal direction. These two changes are denoted by the symbols

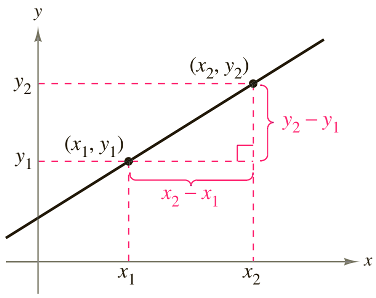


FIGURE 1.33

$$\Delta y = y_2 - y_1 = \text{the change in } y$$

and

$$\Delta x = x_2 - x_1 = \text{the change in } x.$$

(The symbol Δ is the Greek capital letter delta, and the symbols Δy and Δx are read as "delta y" and "delta x.") The ratio of Δy to Δx represents the slope of the line that passes through the points (x_1, y_1) and (x_2, y_2).

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Be sure you see that Δx represents a single number, not the product of two numbers (Δ and x). The same is true for Δy.

The Slope of a Line Passing Through Two Points

The **slope** m of the nonvertical line passing through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

where $x_1 \neq x_2$. Slope is not defined for vertical lines.

When this formula is used for slope, the *order of subtraction* is important. Given two points on a line, you may label either one of them as (x_1, y_1) and the other as (x_2, y_2). However, once you have done this, you must form the numerator and denominator using the same order of subtraction.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{y_1 - y_2}{x_1 - x_2} \qquad m = \frac{y_2 - y_1}{x_1 - x_2}$$

Correct   Correct   Incorrect

For instance, the slope of the line passing through the points $(3, 4)$ and $(5, 7)$ can be calculated as

$$m = \frac{7 - 4}{5 - 3} = \frac{3}{2}$$

or

$$m = \frac{4 - 7}{3 - 5} = \frac{-3}{-2} = \frac{3}{2}.$$

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EXAMPLE 4 Finding Slopes of Lines

Find the slope of the line passing through each pair of points.

a. $(-2, 0)$ and $(3, 1)$ **b.** $(-1, 2)$ and $(2, 2)$

c. $(0, 4)$ and $(1, -1)$ **d.** $(3, 4)$ and $(3, 1)$

SOLUTION

a. Letting $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (3, 1)$, you obtain a slope of

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{3 - (-2)} = \frac{1}{5}$$

Difference in y-values
Difference in x-values

as shown in Figure 1.34(a).

b. The slope of the line passing through $(-1, 2)$ and $(2, 2)$ is

$$m = \frac{2 - 2}{2 - (-1)} = \frac{0}{3} = 0.$$

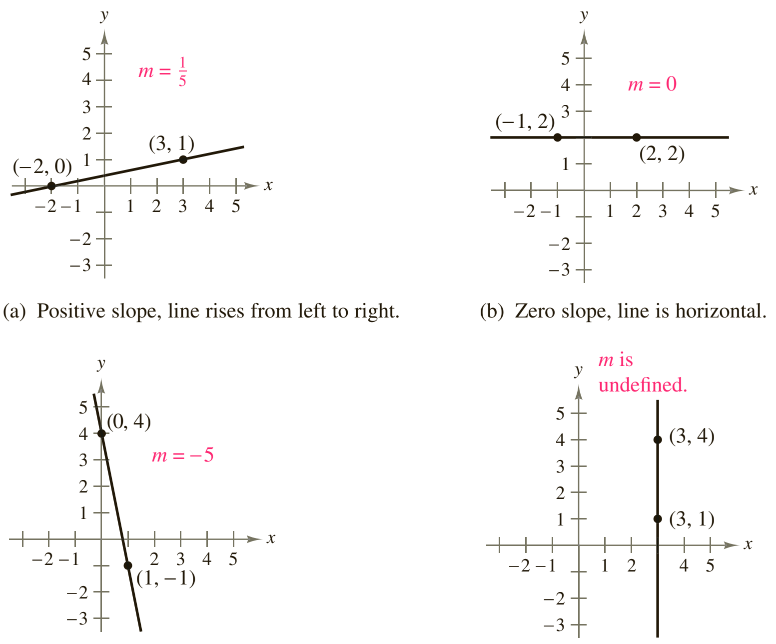
See Figure 1.34(b).

c. The slope of the line passing through $(0, 4)$ and $(1, -1)$ is

$$m = \frac{-1 - 4}{1 - 0} = \frac{-5}{1} = -5.$$

See Figure 1.34(c).

d. The slope of the vertical line passing through $(3, 4)$ and $(3, 1)$ is not defined because division by zero is undefined. [See Figure 1.34(d).]



(a) Positive slope, line rises from left to right.

(b) Zero slope, line is horizontal.

(c) Negative slope, line falls from left to right.

(d) Vertical line, undefined slope.

FIGURE 1.34

✓ ***Checkpoint 4*** *Worked-out solution available at LarsonAppliedCalculus.com*

Find the slope of the line passing through each pair of points.

a. $(-3, 2)$ and $(5, 18)$

b. $(-2, 1)$ and $(-4, 2)$

c. $(2, -4)$ and $(-2, -4)$ ■

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Writing Linear Equations

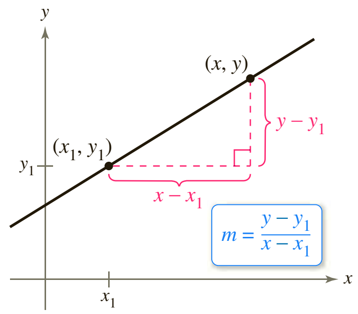
When you know the slope of a line and the coordinates of one point on the line, you can find an equation for the line. For instance, in Figure 1.35, let (x_1, y_1) be a point on the line whose slope is m. If (x, y) is any other point on the line, then

$$\frac{y - y_1}{x - x_1} = m.$$

This equation in the variables x and y can be rewritten in the form

$$y - y_1 = m(x - x_1)$$

which is the **point-slope form** of the equation of a line.



Any two points on a nonvertical line can be used to determine the slope of the line.

FIGURE 1.35

Point-Slope Form of the Equation of a Line

The equation of the line with slope m passing through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

The point-slope form is most useful for *finding* the equation of a nonvertical line. You should remember this formula—it is used throughout the text.

EXAMPLE 5 Using the Point-Slope Form

Find the slope-intercept form of the equation of the line that has a slope of 3 and passes through the point $(1, -2)$.

SOLUTION Use the point-slope form with $m = 3$ and $(x_1, y_1) = (1, -2)$.

$$y - y_1 = m(x - x_1)$$ Point-slope form

$$y - (-2) = 3(x - 1)$$ Substitute for m, x_1, and y_1.

$$y + 2 = 3x - 3$$ Simplify.

$$y = 3x - 5$$ Write in slope-intercept form. See Figure 1.36.

So, the slope-intercept form of the equation of the line is $y = 3x - 5$.

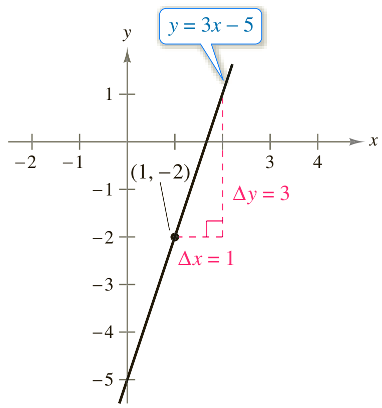


FIGURE 1.36

✓ ***Checkpoint 5*** *Worked-out solution available at LarsonAppliedCalculus.com*

Find the slope-intercept form of the equation of the line that has a slope of 2 and passes through the point $(-1, 2)$. ■

The point-slope form can be used to find an equation of the line passing through two points (x_1, y_1) and (x_2, y_2). To do this, first find the slope of the line

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

and then use the point-slope form to obtain the equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$ Two-point form

This is sometimes called the **two-point form** of the equation of a line.

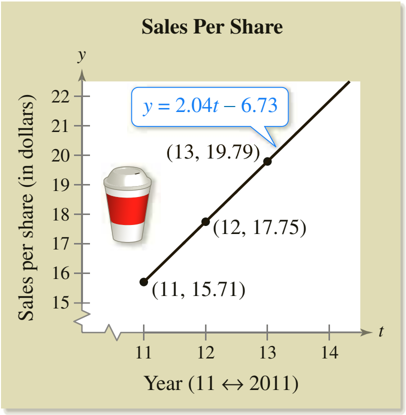


FIGURE 1.37

EXAMPLE 6 Estimating Sales Per Share

The sales per share for Starbucks Corporation was \$15.71 in 2011 and \$17.75 in 2012. Using only this information, write a linear equation that gives the sales per share in terms of the year. Then estimate the sales per share in 2013. *(Source: Starbucks Corp.)*

SOLUTION Let $t = 11$ represent 2011. Then the two given values are represented by the data points

$$(11, 15.71) \quad \text{and} \quad (12, 17.75).$$

The slope of the line through these points is

$$m = \frac{17.75 - 15.71}{12 - 11} = 2.04.$$

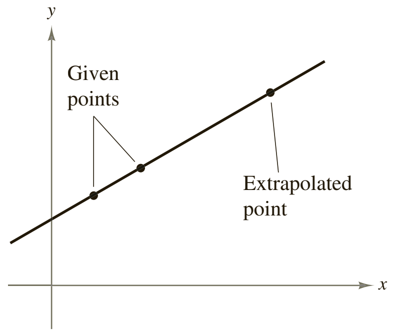
Using the point-slope form, you can find the equation that relates the sales per share y and the year t to be

$$y = 2.04t - 6.73.$$

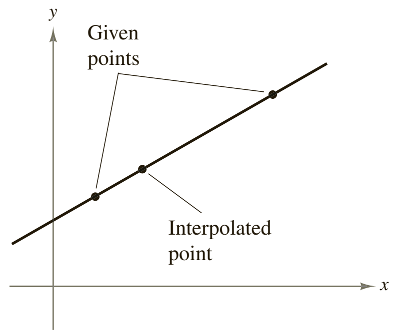
Using $t = 13$ to represent 2013, you can estimate that the 2013 sales per share was

$$y = 2.04(13) - 6.73 = 26.52 - 6.73 = 19.79.$$

According to this equation, the sales per share in 2013 was \$19.79, as shown in Figure 1.37. (In this case, the estimate is fairly good—the actual sales per share in 2013 was \$19.77.)



(a) Linear extrapolation



(b) Linear interpolation

FIGURE 1.38

✓ Checkpoint 6 *Worked-out solution available at LarsonAppliedCalculus.com*

The sales per share for Amazon.com was \$105.65 in 2011 and \$134.40 in 2012. Using only this information, write a linear equation that gives the sales per share in terms of the year. Then estimate the sales per share in 2013. *(Source: Amazon.com)* ■

The estimation method illustrated in Example 6 is called **linear extrapolation.** Note in Figure 1.38(a) that an extrapolated point does not lie between the given points. When the estimated point lies between two given points, as shown in Figure 1.38(b), the procedure is called **linear interpolation.**

Because the slope of a vertical line is not defined, its equation cannot be written in slope-intercept form. However, every line has an equation that can be written in the **general form**

$$Ax + By + C = 0 \qquad \text{General form}$$

where A and B are not both zero. For instance, the vertical line $x = a$ can be represented by the general form

$$x - a = 0. \qquad \text{General form of vertical line}$$

The five most common forms of equations of lines are summarized below.

Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$

Parallel and Perpendicular Lines

Slope can be used to decide whether two nonvertical lines in a plane are parallel, perpendicular, or neither.

Parallel and Perpendicular Lines

1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is, $m_1 = m_2$.
2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is, $m_1 = -1/m_2$.

EXAMPLE 7 Finding Parallel and Perpendicular Lines

Find the slope-intercept form of the equations of the lines that pass through the point $(2, -1)$ and are (a) parallel to and (b) perpendicular to the line $2x - 3y = 5$.

SOLUTION By writing the given equation in slope-intercept form

$$2x - 3y = 5 \qquad \text{Write original equation.}$$
$$-3y = -2x + 5 \qquad \text{Subtract } 2x \text{ from each side.}$$
$$y = \frac{2}{3}x - \frac{5}{3} \qquad \text{Write in slope-intercept form.}$$

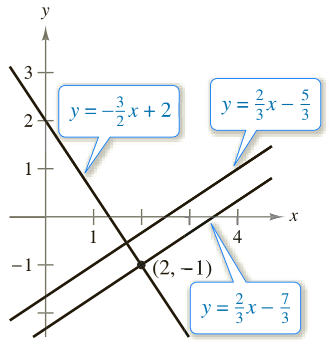


FIGURE 1.39

you can see that it has a slope of $m = \frac{2}{3}$, as shown in Figure 1.39.

a. Any line parallel to the given line must also have a slope of $\frac{2}{3}$. So, the line through $(2, -1)$ that is parallel to the given line has the following equation.

$$y - (-1) = \frac{2}{3}(x - 2) \qquad \text{Write in point-slope form.}$$
$$y + 1 = \frac{2}{3}x - \frac{4}{3} \qquad \text{Simplify.}$$
$$y = \frac{2}{3}x - \frac{4}{3} - 1 \qquad \text{Solve for } y.$$
$$y = \frac{2}{3}x - \frac{7}{3} \qquad \text{Write in slope-intercept form.}$$

b. Any line perpendicular to the given line must have a slope of $-\frac{3}{2}$ (because $-\frac{3}{2}$ is the negative reciprocal of $\frac{2}{3}$). So, the line through $(2, -1)$ that is perpendicular to the given line has the following equation.

$$y - (-1) = -\frac{3}{2}(x - 2) \qquad \text{Write in point-slope form.}$$
$$y + 1 = -\frac{3}{2}x + 3 \qquad \text{Simplify.}$$
$$y = -\frac{3}{2}x + 3 - 1 \qquad \text{Solve for } y.$$
$$y = -\frac{3}{2}x + 2 \qquad \text{Write in slope-intercept form.}$$

TECH TUTOR

On a graphing utility, lines will not appear to have the correct slopes unless you use a viewing window that has a *square setting*. For instance, try graphing the lines in Example 7 using the standard setting $-10 \le x \le 10$ and $-10 \le y \le 10$. Then reset the viewing window with the square setting $-9 \le x \le 9$ and $-6 \le y \le 6$. On which setting do the lines $y = \frac{2}{3}x - \frac{5}{3}$ and $y = -\frac{3}{2}x + 2$ appear to be perpendicular?

✓ **Checkpoint 7** *Worked-out solution available at LarsonAppliedCalculus.com*

Find the slope-intercept form of the equations of the lines that pass through the point $(2, 1)$ and are (a) parallel to and (b) perpendicular to the line $2x - 4y = 5$. ■

Extended Application: Linear Depreciation

Most business expenses can be deducted the same year they occur. One exception to this is the cost of property that has a useful life of more than 1 year, such as buildings, cars, or equipment. Such costs must be **depreciated** over the useful life of the property. When the *same amount* is depreciated each year, the procedure is called **linear depreciation** or **straight-line depreciation.** The *book value* is the difference between the original value and the total amount of depreciation accumulated to date.

EXAMPLE 8 Depreciating Equipment

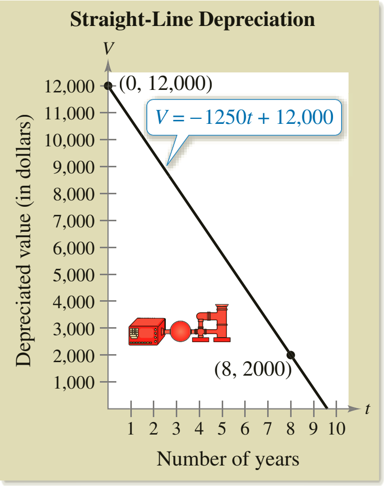


FIGURE 1.40

Your company has purchased a \$12,000 machine that has a useful life of 8 years. The salvage value at the end of 8 years is \$2000. Write a linear equation that describes the book value of the machine each year.

SOLUTION Let V represent the value of the machine at the end of year t. You can represent the initial value of the machine by the ordered pair (0, 12,000) and the salvage value of the machine by the ordered pair (8, 2000). The slope of the line is

$$m = \frac{2000 - 12,000}{8 - 0} = \frac{-\$1250}{1 \text{ year}} \qquad m = \frac{V_2 - V_1}{t_2 - t_1}$$

which represents the annual depreciation in *dollars per year*. Using the point-slope form, you can write the equation of the line as shown.

$$V - 12,000 = -1250(t - 0) \qquad \text{Write in point-slope form.}$$

$$V = -1250t + 12,000 \qquad \text{Write in slope-intercept form.}$$

The graph of this equation is shown in Figure 1.40.

✓ Checkpoint 8 *Worked-out solution available at LarsonAppliedCalculus.com*

Write a linear equation for the machine in Example 8 when the salvage value at the end of 8 years is \$1000. ■

SUMMARIZE (Section 1.3)

1. State the slope-intercept form of the equation of a line *(page 22)*. For an example of an equation in slope-intercept form, see Example 1.
2. Explain how to decide whether the slope of a line is a ratio or a rate of change *(page 24)*. For an example of a slope that is a ratio, see Example 2. For an example of a slope that is a rate of change, see Example 3.
3. Explain how to find the slope of a line passing through two points *(page 25)*. For an example of finding the slope of a line passing through two points, see Example 4.
4. State the point-slope form of the equation of a line *(page 27)*. For examples of using the point-slope form, see Examples 5 and 6.
5. Explain how to decide whether two lines are parallel, perpendicular, or neither *(page 29)*. For an example of finding equations of parallel and perpendicular lines, see Example 7.
6. Describe a real-life example of how a linear equation can be used to analyze the depreciation of property *(page 30, Example 8)*.

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SKILLS WARM UP 1.3

The following warm-up exercises involve skills that were covered in a previous course. You will use these skills in the exercise set for this section. For additional help, review Appendix A.3.

In Exercises 1 and 2, simplify the expression.

1. $\dfrac{5-(-2)}{-3-4}$

2. $\dfrac{-4-(-10)}{7-5}$

3. Evaluate $-\dfrac{1}{m}$ when $m = -3$.

4. Evaluate $-\dfrac{1}{m}$ when $m = \dfrac{6}{7}$.

In Exercises 5–10, solve for y in terms of x.

5. $-4x + y = 7$

6. $3x - y = 7$

7. $y - 2 = 3(x - 4)$

8. $y - (-5) = -1[x - (-2)]$

9. $y - (-3) = \dfrac{4-(-2)}{11-3}(x - 12)$

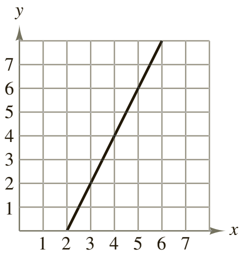
10. $y - 1 = \dfrac{-3-1}{-7-(-1)}[x - (-1)]$

Exercises 1.3

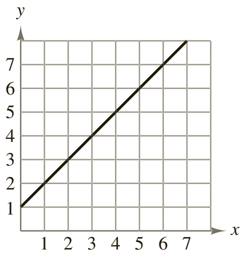
See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.

Estimating Slope In Exercises 1–4, estimate the slope of the line.

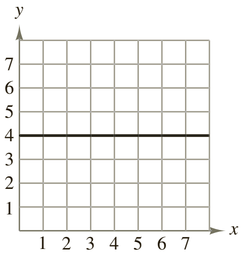
1.



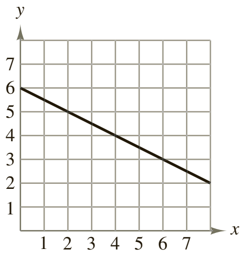
2.



3.



4.



Finding the Slope and *y*-Intercept In Exercises 5–16, find the slope and y-intercept (if possible) of the equation of the line.

5. $y = x + 7$

6. $y = 4x + 3$

7. $5x + y = 20$

8. $2x + y = 40$

9. $7x + 6y = 30$

10. $8x + 3y = 12$

11. $3x - y = 15$

12. $2x - 3y = 24$

13. $x = 4$

14. $x + 5 = 0$

15. $y - 9 = 0$

16. $y + 1 = 0$

Graphing Linear Equations In Exercises 17–26, sketch the graph of the linear equation. Use a graphing utility to verify your result. *See Example 1.*

17. $y = -2$

18. $y = -4$

19. $y = -2x + 1$

20. $y = 3x - 2$

21. $3x + 2y = 4$

22. $4x + 5y = 20$

23. $2x - y - 3 = 0$

24. $x + 2y + 10 = 0$

25. $3x + 5y + 30 = 0$

26. $-5x + 2y - 20 = 0$

Finding Slopes of Lines In Exercises 27–40, find the slope of the line passing through the pair of points. *See Example 4.*

27. $(0, -2), (8, 0)$

28. $(-1, 0), (1, 5)$

29. $(3, -4), (5, 2)$

30. $(1, 2), (-2, 2)$

31. $(4, -1), (2, 7)$

32. $\left(\frac{11}{3}, -2\right), \left(\frac{11}{3}, -10\right)$

33. $(-8, -3), (-8, -5)$

34. $(2, -1), (-2, -5)$

35. $(-2, 6), (1, 6)$

36. $(3, -13), (-2, -3)$

37. $\left(\frac{1}{4}, -2\right), \left(-\frac{3}{8}, 1\right)$

38. $\left(-\frac{3}{2}, -5\right), \left(\frac{5}{6}, 4\right)$

39. $\left(\frac{2}{3}, \frac{5}{2}\right), \left(\frac{1}{4}, -\frac{5}{6}\right)$

40. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

Finding Points on a Line In Exercises 41–48, use the point on the line and the slope of the line to find three additional points through which the line passes. (There are many correct answers.)

	Point	*Slope*		*Point*	*Slope*
41.	$(2, 1)$	$m = 0$	**42.**	$(-5, -3)$	$m = 0$
43.	$(1, 7)$	$m = -3$	**44.**	$(7, -2)$	$m = 2$
45.	$(6, -4)$	$m = \frac{2}{3}$	**46.**	$(-1, -6)$	$m = -\frac{1}{2}$
47.	$(-8, 1)$	m is undefined.	**48.**	$(-3, 4)$	m is undefined.

Using Slope In Exercises 49–52, use the concept of slope to determine whether the three points are collinear.

49. $(-2, 1), (-1, 0), (2, -2)$

50. $(0, 4), (7, -6), (-5, 11)$

51. $(-2, -1), (0, 3), (2, 7)$

52. $(4, 1), (-2, -2), (8, 3)$

Using the Point-Slope Form In Exercises 53–60, find an equation of the line that passes through the given point and has the given slope. Then sketch the line. *See Example 5.*

	Point	*Slope*		*Point*	*Slope*
53.	$(0, 3)$	$m = \frac{3}{4}$	**54.**	$(0, 0)$	$m = \frac{2}{3}$
55.	$(-2, 7)$	$m = 0$	**56.**	$(-2, 4)$	$m = 0$
57.	$(-1, -2)$	$m = -4$	**58.**	$(-1, -4)$	$m = -2$
59.	$(\frac{8}{3}, 0)$	$m = \frac{1}{4}$	**60.**	$(\frac{3}{2}, 0)$	$m = -\frac{1}{6}$

Writing an Equation of a Line In Exercises 61–70, find an equation of the line that passes through the points. Then sketch the line.

61. $(4, 3), (0, -5)$ **62.** $(-2, -4), (1, 5)$

63. $(2, 3), (2, -2)$ **64.** $(6, 1), (10, 1)$

65. $(3, -1), (-2, -1)$ **66.** $(2, 5), (2, -10)$

67. $(-\frac{1}{2}, 4), (\frac{1}{2}, 8)$ **68.** $(-\frac{1}{4}, 1), (\frac{1}{4}, 5)$

69. $(-\frac{1}{3}, 1), (-\frac{2}{3}, \frac{5}{6})$ **70.** $(\frac{7}{8}, \frac{3}{4}), (\frac{5}{4}, -\frac{1}{4})$

Writing an Equation of a Line In Exercises 71–74, find an equation of the line with the given characteristics.

71. A vertical line through $(3, 0)$

72. A horizontal line through $(0, -5)$

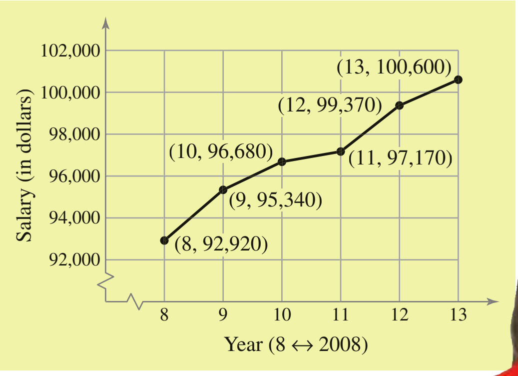
73. A line with a y-intercept at -10 and parallel to all horizontal lines

74. A line with an x-intercept at -5 and parallel to all vertical lines

Finding Parallel and Perpendicular Lines In Exercises 75–82, find equations of the lines that pass through the given point and are (a) parallel to the given line and (b) perpendicular to the given line. Then use a graphing utility to graph all three equations in the same viewing window using a square setting. *See Example 7.*

	Point	*Line*		*Point*	*Line*
75.	$(-3, 2)$	$x + y = 7$	**76.**	$(2, 1)$	$4x - 2y = 3$
77.	$(-\frac{2}{3}, \frac{7}{8})$	$3x + 4y = 7$	**78.**	$(\frac{7}{8}, \frac{3}{4})$	$5x + 3y = 0$
79.	$(-1, 0)$	$y + 3 = 0$	**80.**	$(2, 5)$	$y + 4 = 0$
81.	$(1, 1)$	$x - 2 = 0$	**82.**	$(12, -3)$	$x - 5 = 0$

83. Average Salary The graph shows the average salaries (in dollars) of postsecondary education administrators from 2008 through 2013. *(Source: U.S. Bureau of Labor Statistics)*

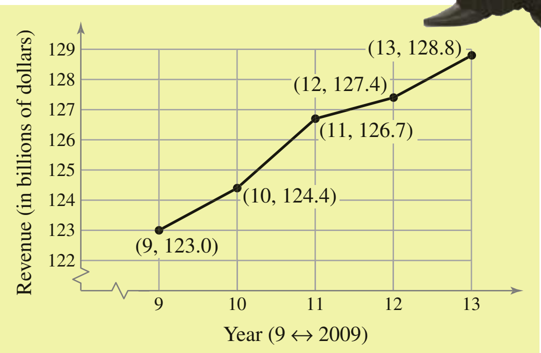


(a) Determine the time periods in which the average salary increased the greatest and the least.

(b) Find the slope of the line segment connecting the points for the years 2008 and 2013.

(c) Interpret the meaning of the slope in part (b) in the context of the problem.

84. Revenue The graph shows the revenue (in billions of dollars) for AT&T for the years 2009 through 2013. *(Source: AT&T Inc.)*



(a) Determine the years in which the revenue increased the greatest and the least.

(b) Find the slope of the line segment connecting the points for the years 2009 and 2013.

(c) Interpret the meaning of the slope in part (b) in the context of the problem.

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85. Road Grade You are driving on a road that has a 6% uphill grade. This means that the slope of the road is $\frac{6}{100}$. Approximate the amount of vertical change in your position when you drive 200 feet.

86. Temperature Conversion Use the fact that water freezes at 0°C (32°F) and boils at 100°C (212°F).

(a) Write a linear equation that expresses the relationship between the temperature in degrees Celsius C and the temperature in degrees Fahrenheit F.

(b) A person has a temperature of 102.2°F. What is this temperature on the Celsius scale?

(c) The temperature in a room is 76°F. What is this temperature on the Celsius scale?

87. Population The resident population of Wisconsin (in thousands) was 5655 in 2009 and 5743 in 2013. Assume that the relationship between the population y and the year t is linear. Let $t = 0$ represent 2000. *(Source: U.S. Census Bureau)*

(a) Write a linear model for the data. What is the slope and what does it tell you about the population?

(b) Use the model to estimate the population in 2011.

(c) Use your school's library, the Internet, or some other reference source to find the actual population in 2011. How close was your estimate?

(d) Do you think your model could be used to predict the population in 2018? Explain.

88. Personal Income Personal income (in billions of dollars) in the United States was 12,430 in 2008 and 14,167 in 2013. Assume that the relationship between the personal income y and the time t (in years) is linear. Let $t = 0$ represent 2000. *(Source: U.S. Bureau of Economic Analysis)*

(a) Write a linear model for the data.

(b) Estimate the personal incomes in 2011 and 2014.

(c) Use your school's library, the Internet, or some other reference source to find the actual personal incomes in 2011 and 2014. How close were your estimates?

89. Linear Depreciation A small business purchases a piece of equipment for $1025. After 5 years, the equipment will be outdated, having no value.

(a) Write a linear equation giving the value y (in dollars) of the equipment in terms of the time t (in years), $0 \leq t \leq 5$.

(b) Use a graphing utility to graph the equation.

(c) Move the cursor along the graph and estimate (to two-decimal-place accuracy) the value of the equipment after 3 years.

(d) Move the cursor along the graph and estimate (to two-decimal-place accuracy) the time when the value of the equipment will be $600.

90. Linear Depreciation A hospital purchases a $500,000 magnetic resonance imaging (MRI) machine that has a useful life of 9 years. The salvage value at the end of 9 years is $77,000.

(a) Write a linear equation that describes the value y (in dollars) of the MRI machine in terms of the time t (in years), $0 \leq t \leq 9$.

(b) Find the value of the machine after 5 years.

(c) Find the time when the value of the equipment will be $160,000.

91. Choosing a Job As a salesperson, you receive a monthly salary of $2000, plus a commission of 7% of sales. You are offered a new job at $2300 per month, plus a commission of 5% of sales.

(a) Write linear equations for your monthly wage W (in dollars) in terms of your monthly sales S (in dollars) for your current job and for your job offer.

(b) Use a graphing utility to graph each equation and find the point of intersection. What does the point of intersection signify?

(c) You think you can sell $20,000 worth of a product per month. Should you change jobs? Explain.

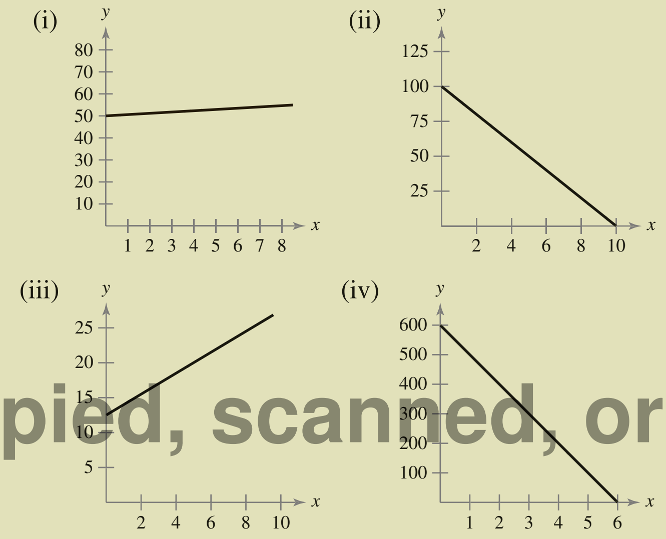
92. HOW DO YOU SEE IT? Match the description of the situation with its graph. Then write the equation of the line. [The graphs are labeled (i), (ii), (iii), and (iv).]

(a) You are paying $10 per week to repay a $100 loan.

(b) An employee is paid $12.50 per hour plus $1.50 for each unit produced per hour.

(c) A sales representative receives $50 per day for food plus $0.58 for each mile traveled.

(d) A computer that was purchased for $600 depreciates $100 per year.



QUIZ YOURSELF

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.

Take this quiz as you would take a quiz in class. When you are done, check your work against the answers given in the back of the book.

In Exercises 1–3, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

1. $(3, -2), (-3, 1)$ **2.** $\left(\frac{1}{4}, -\frac{3}{2}\right), \left(\frac{1}{2}, 2\right)$ **3.** $(-12, 4), (6, -2)$

4. Use the Distance Formula to show that the points $(4, 0)$, $(2, 1)$, and $(-1, -5)$ are vertices of a right triangle.

5. The resident population of Georgia (in thousands) was 9810 in 2011 and 9992 in 2013. Use the Midpoint Formula to estimate the population in 2012. *(Source: U.S. Census Bureau)*

In Exercises 6–8, sketch the graph of the equation and label the intercepts.

6. $y = 5x + 2$ **7.** $y = x^2 + x - 6$ **8.** $y = |x - 3|$

In Exercises 9–11, find the standard form of the equation of the circle with the given characteristics and sketch its graph.

9. Center: $(0, 0)$; radius: 9

10. Center: $(-1, 0)$; radius: 6

11. Center: $(2, -2)$; solution point: $(-1, 2)$

12. A business manufactures a product at a cost of \$4.55 per unit and sells the product for \$7.19 per unit. The company's initial investment to produce the product was \$12,500. How many units must the company sell to break even?

In Exercises 13–15, find an equation of the line that passes through the given point and has the given slope. Then sketch the line.

13. $(0, -3)$; $m = 0$ **14.** $(1, 1)$; $m = 2$ **15.** $(6, 5)$; $m = -\frac{1}{3}$

In Exercises 16–18, find an equation of the line that passes through the points. Then sketch the line.

16. $(1, -1), (-4, 5)$ **17.** $(-2, 3), (-2, 2)$ **18.** $\left(\frac{5}{2}, 2\right), (0, 2)$

19. Find equations of the lines that pass through the point $(3, -5)$ and are

(a) parallel to the line $x + 4y = -2$.

(b) perpendicular to the line $x + 4y = -2$.

20. A company had sales of \$1,330,000 in 2011 and \$1,800,000 in 2015. The company's sales can be modeled by a linear equation. Predict the sales in 2019 and 2022.

21. A company reimburses its sales representatives \$218 per day for lodging and meals, plus \$0.56 per mile driven. Write a linear equation giving the daily cost C (in dollars) in terms of x, the number of miles driven.

22. Your annual salary was \$35,700 in 2013 and \$39,100 in 2015. Assume your salary can be modeled by a linear equation.

(a) Write a linear equation giving your salary S (in dollars) in terms of the time t (in years). Let $t = 13$ represent 2013.

(b) Use the linear model to predict your salary in 2020.