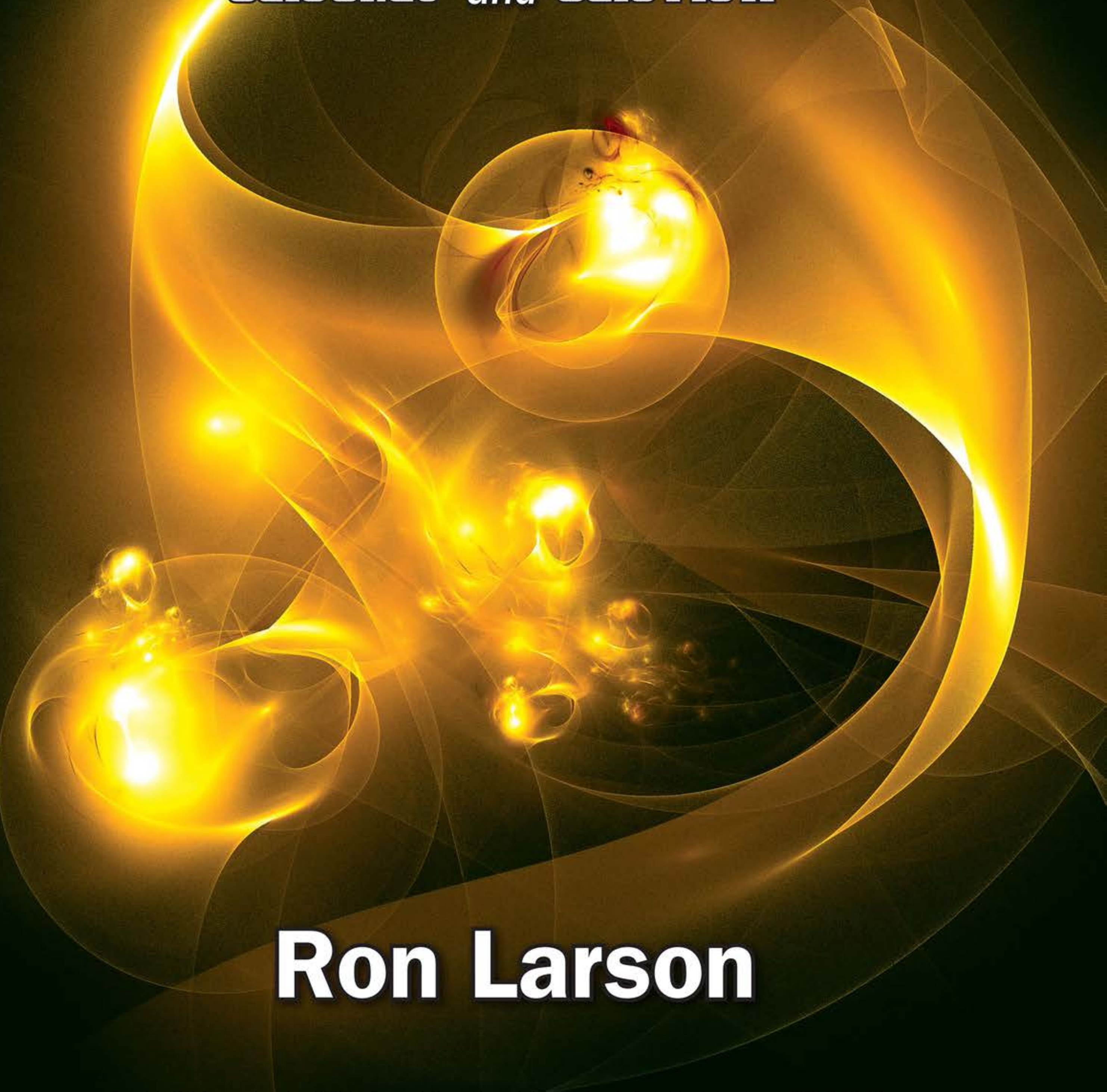
COLLEGE ALGEBRA

10E

with

CalcChat® and CalcView®

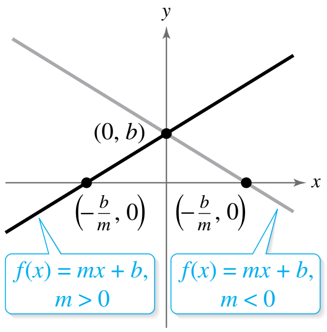


Ron Larson

GRAPHS OF PARENT FUNCTIONS

Linear Function

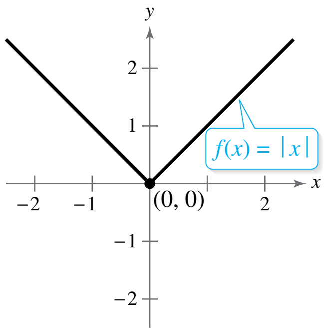
$f(x) = mx + b$



Domain: $(-\infty, \infty)$
Range ($m \neq 0$): $(-\infty, \infty)$
x-intercept: $(-b/m, 0)$
y-intercept: $(0, b)$
Increasing when $m > 0$
Decreasing when $m < 0$

Absolute Value Function

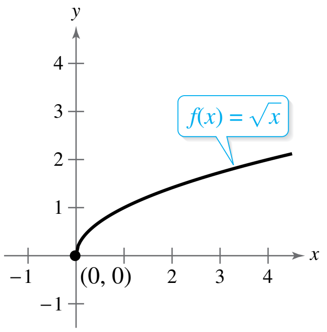
$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function
y-axis symmetry

Square Root Function

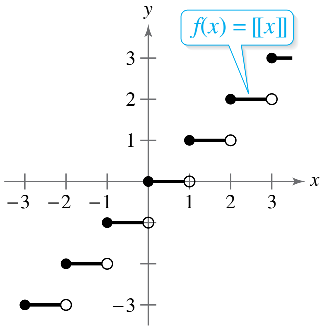
$f(x) = \sqrt{x}$



Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$

Greatest Integer Function

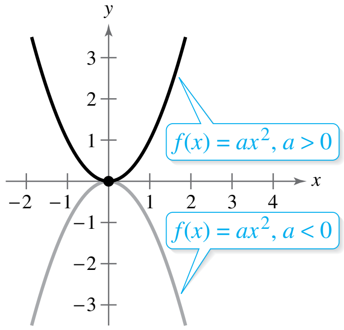
$f(x) = [\![x]\!]$



Domain: $(-\infty, \infty)$
Range: the set of integers
x-intercepts: in the interval $[0, 1)$
y-intercept: $(0, 0)$
Constant between each pair of consecutive integers
Jumps vertically one unit at each integer value

Quadratic (Squaring) Function

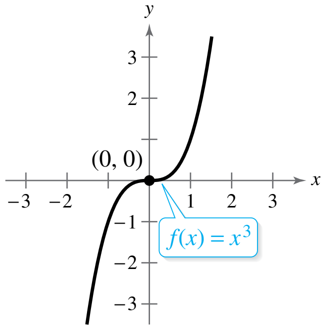
$f(x) = ax^2$



Domain: $(-\infty, \infty)$
Range ($a > 0$): $[0, \infty)$
Range ($a < 0$): $(-\infty, 0]$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$ for $a > 0$
Increasing on $(0, \infty)$ for $a > 0$
Increasing on $(-\infty, 0)$ for $a < 0$
Decreasing on $(0, \infty)$ for $a < 0$
Even function
y-axis symmetry
Relative minimum ($a > 0$), relative maximum ($a < 0$), or vertex: $(0, 0)$

Cubic Function

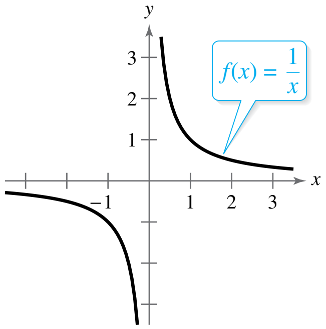
$f(x) = x^3$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing on $(-\infty, \infty)$
Odd function
Origin symmetry

Rational (Reciprocal) Function

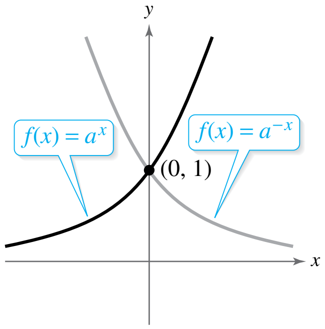
$f(x) = \frac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
No intercepts
Decreasing on $(-\infty, 0)$ and $(0, \infty)$
Odd function
Origin symmetry
Vertical asymptote: y-axis
Horizontal asymptote: x-axis

Exponential Function

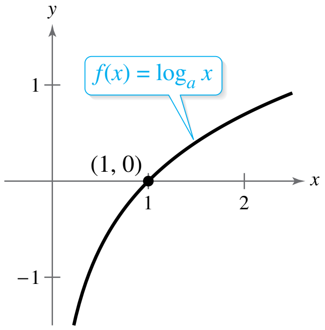
$f(x) = a^x, \ a > 1$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Intercept: $(0, 1)$
Increasing on $(-\infty, \infty)$
for $f(x) = a^x$
Decreasing on $(-\infty, \infty)$
for $f(x) = a^{-x}$
Horizontal asymptote: x-axis
Continuous

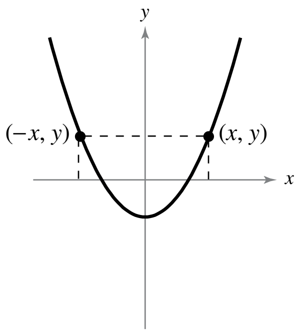
Logarithmic Function

$f(x) = \log_a x, \ a > 1$

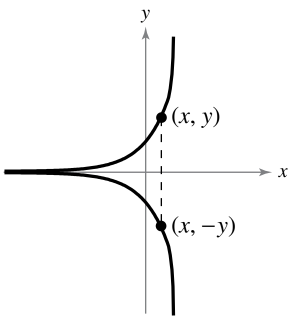


Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(1, 0)$
Increasing on $(0, \infty)$
Vertical asymptote: y-axis
Continuous
Reflection of graph of $f(x) = a^x$
in the line $y = x$

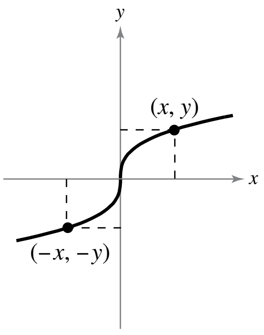
SYMMETRY



***y*-Axis Symmetry**



***x*-Axis Symmetry**



Origin Symmetry

COLLEGE ALGEBRA

10E

with

CalcChat® and CalcView®

Ron Larson
The Pennsylvania State University
The Behrend College

With the assistance of David C. Falvo
The Pennsylvania State University
The Behrend College

CENGAGE Learning®

Australia • Brazil • Mexico • Singapore • United Kingdom • United States

CENGAGE Learning

College Algebra
with CalcChat and CalcView
Tenth Edition

Ron Larson

Product Director: Terry Boyle
Product Manager: Gary Whalen
Senior Content Developer: Stacy Green
Associate Content Developer: Samantha Lugtu
Product Assistant: Katharine Werring
Media Developer: Lynh Pham
Marketing Manager: Ryan Ahern
Content Project Manager: Jennifer Risden
Manufacturing Planner: Doug Bertke
Production Service: Larson Texts, Inc.
Photo Researcher: Lumina Datamatics
Text Researcher: Lumina Datamatics
Illustrator: Larson Texts, Inc.
Text Designer: Larson Texts, Inc.
Cover Designer: Larson Texts, Inc.
Front Cover Image: betibup33/Shutterstock.com
Back Cover Image: SidorArt/Shutterstock.com
Compositor: Larson Texts, Inc.

For product information and technology assistance, contact us at **Cengage Learning Customer & Sales Support, 1-800-354-9706.**

For permission to use material from this text or product, submit all requests online at **www.cengage.com/permissions.** Further permissions questions can be emailed to **permissionrequest@cengage.com.**

Library of Congress Control Number: 2016944979

Student Edition:
ISBN: 978-1-337-28229-1

Loose-leaf Edition:
ISBN: 978-1-337-29152-1

Cengage Learning
20 Channel Center Street
Boston, MA 02210
USA

Cengage Learning is a leading provider of customized learning solutions with employees residing in nearly 40 different countries and sales in more than 125 countries around the world. Find your local representative at **www.cengage.com.**

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

To learn more about Cengage Learning Solutions, visit **www.cengage.com.**

Purchase any of our products at your local college store or at our preferred online store **www.cengagebrain.com.**

QR Code is a registered trademark of Denso Wave Incorporated

Printed in the United States of America
Print Number: 01 Print Year: 2016

Contents

P ▷ Prerequisites 1

1 ▷ Equations, Inequalities, and Mathematical Modeling 69

2 ▷ Functions and Their Graphs 159

7 ▷ Matrices and Determinants 493

8 ▷ Sequences, Series, and Probability 561

▷ Appendices

*Available at the text-specific website *www.cengagebrain.com*

Preface

Welcome to *College Algebra,* Tenth Edition. We are excited to offer you a new edition with even more resources that will help you understand and master algebra. This textbook includes features and resources that continue to make *College Algebra* a valuable learning tool for students and a trustworthy teaching tool for instructors.

College Algebra provides the clear instruction, precise mathematics, and thorough coverage that you expect for your course. Additionally, this new edition provides you with **free** access to three companion websites:

- **CalcView.com**—video solutions to selected exercises
- **CalcChat.com**—worked-out solutions to odd-numbered exercises and access to online tutors
- **LarsonPrecalculus.com**—companion website with resources to supplement your learning

These websites will help enhance and reinforce your understanding of the material presented in this text and prepare you for future mathematics courses. CalcView® and CalcChat® are also available as free mobile apps.

Features

NEW CalcView®

The website *CalcView.com* contains video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. The CalcView mobile app is available for free at the Apple® App Store® or Google Play™ store. The app features an embedded QR Code® reader that can be used to scan the on-page codes and go directly to the videos. You can also access the videos at *CalcView.com.*

UPDATED CalcChat®

In each exercise set, be sure to notice the reference to *CalcChat.com.* This website provides free step-by-step solutions to all odd-numbered exercises in many of our textbooks. Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. For over 14 years, hundreds of thousands of students have visited this site for help. The CalcChat mobile app is also available as a free download at the Apple® App Store® or Google Play™ store and features an embedded QR Code® reader.

App Store is a service mark of Apple Inc. Google Play is a trademark of Google Inc.
QR Code is a registered trademark of Denso Wave Incorporated.

REVISED LarsonPrecalculus.com

All companion website features have been updated based on this revision, plus we have added a new Collaborative Project feature. Access to these features is free. You can view and listen to worked-out solutions of Checkpoint problems in English or Spanish, explore examples, download data sets, watch lesson videos, and much more.

NEW Collaborative Project

You can find these extended group projects at *LarsonPrecalculus.com.* Check your understanding of the chapter concepts by solving in-depth, real-life problems. These collaborative projects provide an interesting and engaging way for you and other students to work together and investigate ideas.

REVISED Exercise Sets

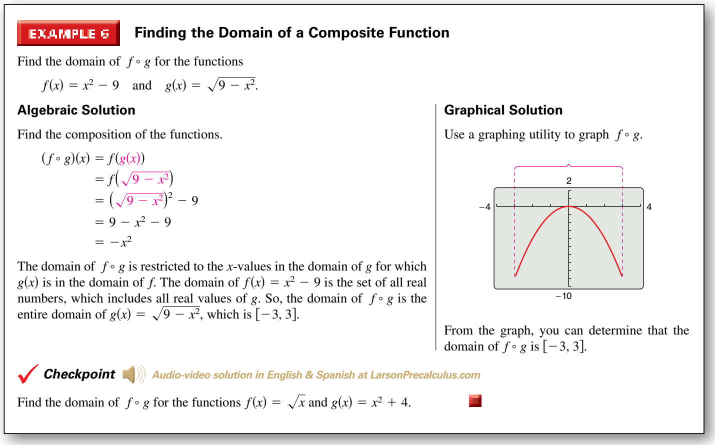
The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant, and include topics our users have suggested. The exercises have been reorganized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations. Error Analysis exercises have been added throughout the text to help you identify common mistakes.

Chapter Opener

Each Chapter Opener highlights real-life applications used in the examples and exercises.

Section Objectives

A bulleted list of learning objectives provides you the opportunity to preview what will be presented in the upcoming section.



EXAMPLE 6 **Finding the Domain of a Composite Function**

Find the domain of $f \circ g$ for the functions

$$f(x) = x^2 - 9 \quad \text{and} \quad g(x) = \sqrt{9 - x^2}.$$

Algebraic Solution

Find the composition of the functions.

$$\begin{aligned}(f \circ g)(x) &= f(g(x))\\ &= f\left(\sqrt{9 - x^2}\right)\\ &= \left(\sqrt{9 - x^2}\right)^2 - 9\\ &= 9 - x^2 - 9\\ &= -x^2\end{aligned}$$

The domain of $f \circ g$ is restricted to the x-values in the domain of g for which $g(x)$ is in the domain of f. The domain of $f(x) = x^2 - 9$ is the set of all real numbers, which includes all real values of g. So, the domain of $f \circ g$ is the entire domain of $g(x) = \sqrt{9 - x^2}$, which is $[-3, 3]$.

Graphical Solution

Use a graphing utility to graph $f \circ g$.

From the graph, you can determine that the domain of $f \circ g$ is $[-3, 3]$.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the domain of $f \circ g$ for the functions $f(x) = \sqrt{x}$ and $g(x) = x^2 + 4$.

Side-By-Side Examples

Throughout the text, we present solutions to many examples from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.

Remarks

These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show alternative or additional steps to a solution of an example.

Checkpoints

Accompanying every example, the Checkpoint problems encourage immediate practice and check your understanding of the concepts presented in the example. View and listen to worked-out solutions of the Checkpoint problems in English or Spanish at *LarsonPrecalculus.com.*

Technology

The technology feature gives suggestions for effectively using tools such as calculators, graphing utilities, and spreadsheet programs to help deepen your understanding of concepts, ease lengthy calculations, and provide alternate solution methods for verifying answers obtained by hand.

Historical Notes

These notes provide helpful information regarding famous mathematicians and their work.

Algebra of Calculus

Throughout the text, special emphasis is given to the algebraic techniques used in calculus. Algebra of Calculus examples and exercises are integrated throughout the text and are identified by the symbol ƒ.

Summarize

The Summarize feature at the end of each section helps you organize the lesson's key concepts into a concise summary, providing you with a valuable study tool.

Vocabulary Exercises

The vocabulary exercises appear at the beginning of the exercise set for each section. These problems help you review previously learned vocabulary terms that you will use in solving the section exercises.

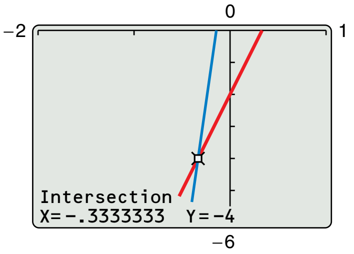
▷ **TECHNOLOGY** You can use a graphing utility to check that a solution is reasonable. One way is to graph the left side of the equation, then graph the right side of the equation, and determine the point of intersection. For instance, in Example 2, if you graph the equations

$y_1 = 6(x - 1) + 4$ The left side

and

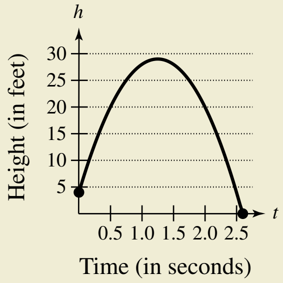
$y_2 = 3(7x + 1)$ The right side

in the same viewing window, they intersect at $x = -\frac{1}{3}$, as shown in the graph below.



```
Intersection
X=-.3333333  Y=-4
```

92. **HOW DO YOU SEE IT?** The graph represents the height h of a projectile after t seconds.



(a) Explain why h is a function of t.

(b) Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

(c) Approximate the domain of h.

(d) Is t a function of h? Explain.

How Do You See It?

The How Do You See It? feature in each section presents a real-life exercise that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

Project

The projects at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These projects are offered online at *LarsonPrecalculus.com.*

Chapter Summary

The Chapter Summary includes explanations and examples of the objectives taught in each chapter.

Instructor Resources

Annotated Instructor's Edition / ISBN-13: 978-1-337-28230-7
This is the complete student text plus point-of-use annotations for the instructor, including extra projects, classroom activities, teaching strategies, and additional examples. Answers to even-numbered text exercises, Vocabulary Checks, and Explorations are also provided.

Complete Solutions Manual (on instructor companion site)
This manual contains solutions to all exercises from the text, including Chapter Review Exercises and Chapter Tests, and Practice Tests with solutions.

Cengage Learning Testing Powered by Cognero (login.cengage.com)
CLT is a flexible online system that allows you to author, edit, and manage test bank content; create multiple test versions in an instant; and deliver tests from your LMS, your classroom, or wherever you want. This is available online via *www.cengage.com/login.*

Instructor Companion Site
Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online via *www.cengage.com/login.* Access and download PowerPoint® presentations, images, the instructor's manual, and more.

The Test Bank (on instructor companion site)
This contains text-specific multiple-choice and free response test forms.

Lesson Plans (on instructor companion site)
This manual provides suggestions for activities and lessons with notes on time allotment in order to ensure timeliness and efficiency during class.

MindTap for Mathematics
MindTap® is the digital learning solution that helps instructors engage and transform today's students into critical thinkers. Through paths of dynamic assignments and applications that you can personalize, real-time course analytics and an accessible reader, MindTap helps you turn cookie cutter into cutting edge, apathy into engagement, and memorizers into higher-level thinkers.

Enhanced WebAssign® ENHANCED WebAssign
Exclusively from Cengage Learning, Enhanced WebAssign combines the exceptional mathematics content that you know and love with the most powerful online homework solution, WebAssign. Enhanced WebAssign engages students with immediate feedback, rich tutorial content, and interactive, fully customizable e-books (YouBook), helping students to develop a deeper conceptual understanding of their subject matter. Quick Prep and Just In Time exercises provide opportunities for students to review prerequisite skills and content, both at the start of the course and at the beginning of each section. Flexible assignment options give instructors the ability to release assignments conditionally on the basis of students' prerequisite assignment scores. Visit us at **www.cengage.com/ewa** to learn more.

Student Resources

Student Study and Solutions Manual / ISBN-13: 978-1-337-29150-7
This guide offers step-by-step solutions for all odd-numbered text exercises, Chapter Tests, and Cumulative Tests. It also contains Practice Tests.

Note-Taking Guide / ISBN-13: 978-1-337-29151-4
This is an innovative study aid, in the form of a notebook organizer, that helps students develop a section-by-section summary of key concepts.

CengageBrain.com
To access additional course materials, please visit *www.cengagebrain.com.* At the *CengageBrain.com* home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where these resources can be found.

MindTap for Mathematics
MindTap® provides you with the tools you need to better manage your limited time—you can complete assignments whenever and wherever you are ready to learn with course material specially customized for you by your instructor and streamlined in one proven, easy-to-use interface. With an array of tools and apps—from note taking to flashcards—you'll get a true understanding of course concepts, helping you to achieve better grades and setting the groundwork for your future courses. This access code entitles you to one term of usage.

Enhanced WebAssign®
Enhanced WebAssign (assigned by the instructor) provides you with instant feedback on homework assignments. This online homework system is easy to use and includes helpful links to textbook sections, video examples, and problem-specific tutorials.

Acknowledgments

I would like to thank the many people who have helped me prepare the text and the supplements package. Their encouragement, criticisms, and suggestions have been invaluable.

Thank you to all of the instructors who took the time to review the changes in this edition and to provide suggestions for improving it. Without your help, this book would not be possible.

Reviewers of the Tenth Edition

Gurdial Arora, *Xavier University of Louisiana*
Russell C. Chappell, *Twinsburg High School, Ohio*
Darlene Martin, *Lawson State Community College*
John Fellers, *North Allegheny School District*
Professor Steven Sikes, *Collin College*
Ann Slate, *Surry Community College*
John Elias, *Glenda Dawson High School*
Kathy Wood, *Lansing Catholic High School*
Darin Bauguess, *Surry Community College*
Brianna Kurtz, *Daytona State College*

Reviewers of the Previous Editions

Timothy Andrew Brown, *South Georgia College;* Blair E. Caboot, *Keystone College;* Shannon Cornell, *Amarillo College;* Gayla Dance, *Millsaps College;* Paul Finster, *El Paso Community College;* Paul A. Flasch, *Pima Community College West Campus;* Vadas Gintautas, *Chatham University;* Lorraine A. Hughes, *Mississippi State University;* Shu-Jen Huang, *University of Florida;* Renyetta Johnson, *East Mississippi Community College;* George Keihany, *Fort Valley State University;* Mulatu Lemma, *Savannah State University;* William Mays Jr., *Salem Community College;* Marcella Melby, *University of Minnesota;* Jonathan Prewett, *University of Wyoming;* Denise Reid, *Valdosta State University;* David L. Sonnier, *Lyon College;* David H. Tseng, *Miami Dade College—Kendall Campus;* Kimberly Walters, *Mississippi State University;* Richard Weil, *Brown College;* Solomon Willis, *Cleveland Community College;* Bradley R. Young, *Darton College*

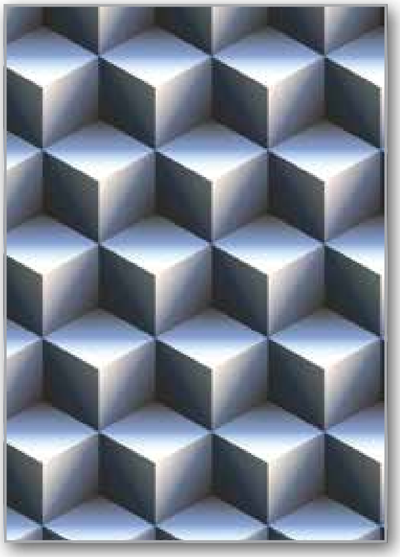
My thanks to Robert Hostetler, The Behrend College, The Pennsylvania State University, and David Heyd, The Behrend College, The Pennsylvania State University, for their significant contributions to previous editions of this text.

I would also like to thank the staff at Larson Texts, Inc. who assisted with proofreading the manuscript, preparing and proofreading the art package, and checking and typesetting the supplements.

On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O’Neil. If you have suggestions for improving this text, please feel free to write to me. Over the past two decades, I have received many useful comments from both instructors and students, and I value these comments very highly.

Ron Larson, Ph.D.
Professor of Mathematics
Penn State University
www.RonLarson.com

P Prerequisites



Computer Graphics *(page 56)*

Autocatalytic Chemical Reaction *(Exercise 84, page 40)*

Steel Beam Loading *(Exercise 81, page 33)*

Gallons of Water on Earth *(page 17)*

Change in Temperature *(page 7)*

Clockwise from top left, Suwit Ngaokaew/Shutterstock.com; Matt Antonino/Shutterstock.com; EpicStockMedia/Shutterstock.com; VladisChern/Shutterstock.com; iStockphoto.com/Photosbyjim

P.1 Review of Real Numbers and Their Properties

Real numbers can represent many real-life quantities. For example, in Exercises 49–52 on page 13, you will use real numbers to represent the federal surplus or deficit.

- **Represent and classify real numbers.**
- **Order real numbers and use inequalities.**
- **Find the absolute values of real numbers and find the distance between two real numbers.**
- **Evaluate algebraic expressions.**
- **Use the basic rules and properties of algebra.**

Real Numbers

Real numbers can describe quantities in everyday life such as age, miles per gallon, and population. Symbols such as

$$-5, 9, 0, \tfrac{4}{3}, 0.666\ .\ .\ .\ , 28.21, \sqrt{2}, \pi, \text{ and } \sqrt[3]{-32}$$

represent real numbers. Here are some important **subsets** (each member of a subset B is also a member of a set A) of the real numbers. The three dots, or *ellipsis points,* tell you that the pattern continues indefinitely.

$\{1, 2, 3, 4, .\ .\ .\}$ Set of natural numbers

$\{0, 1, 2, 3, 4, .\ .\ .\}$ Set of whole numbers

$\{.\ .\ .\ , -3, -2, -1, 0, 1, 2, 3, .\ .\ .\}$ Set of integers

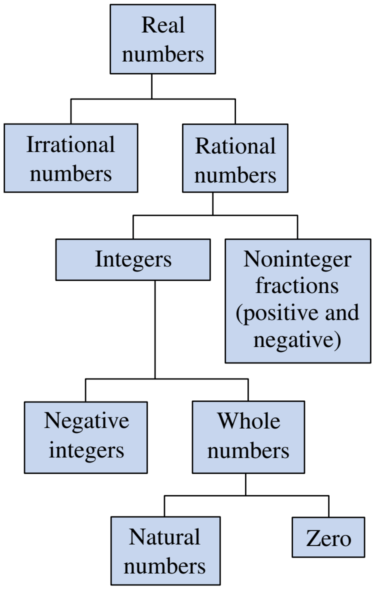
A real number is **rational** when it can be written as the ratio p/q of two integers, where $q \neq 0$. For example, the numbers

$$\tfrac{1}{3} = 0.3333\ .\ .\ . = 0.\overline{3}, \tfrac{1}{8} = 0.125, \text{ and } \tfrac{125}{111} = 1.126126\ .\ .\ . = 1.\overline{126}$$

are rational. The decimal representation of a rational number either repeats $\left(\text{as in } \frac{173}{55} = 3.1\overline{45}\right)$ or terminates $\left(\text{as in } \frac{1}{2} = 0.5\right)$. A real number that cannot be written as the ratio of two integers is **irrational.** The decimal representation of an irrational number neither terminates nor repeats. For example, the numbers

$$\sqrt{2} = 1.4142135\ .\ .\ . \approx 1.41 \quad \text{and} \quad \pi = 3.1415926\ .\ .\ . \approx 3.14$$

are irrational. (The symbol $\approx$ means "is approximately equal to.") Figure P.1 shows subsets of the real numbers and their relationships to each other.



Subsets of the real numbers
Figure P.1

EXAMPLE 1 Classifying Real Numbers

Determine which numbers in the set $\left\{-13, -\sqrt{5}, -1, -\frac{1}{3}, 0, \frac{5}{8}, \sqrt{2}, \pi, 7\right\}$ are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.

Solution

a. Natural numbers: $\{7\}$

b. Whole numbers: $\{0, 7\}$

c. Integers: $\{-13, -1, 0, 7\}$

d. Rational numbers: $\left\{-13, -1, -\frac{1}{3}, 0, \frac{5}{8}, 7\right\}$

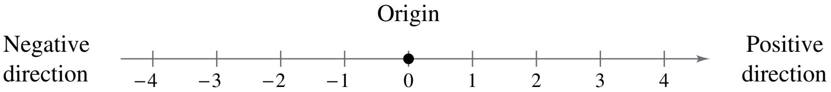
e. Irrational numbers: $\left\{-\sqrt{5}, \sqrt{2}, \pi\right\}$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Repeat Example 1 for the set $\left\{-\pi, -\frac{1}{4}, \frac{6}{3}, \frac{1}{2}\sqrt{2}, -7.5, -1, 8, -22\right\}$.

Michael G Smith/Shutterstock.com

Real numbers are represented graphically on the **real number line.** When you draw a point on the real number line that corresponds to a real number, you are **plotting** the real number. The point representing 0 on the real number line is the **origin.** Numbers to the right of 0 are positive, and numbers to the left of 0 are negative, as shown in the figure below. The term **nonnegative** describes a number that is either positive or zero.



As the next two number lines illustrate, there is a *one-to-one correspondence* between real numbers and points on the real number line.

Every real number corresponds to exactly one point on the real number line.

Every point on the real number line corresponds to exactly one real number.

EXAMPLE 2 Plotting Points on the Real Number Line

Plot the real numbers on the real number line.

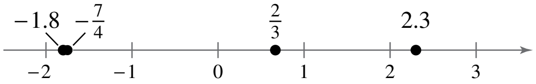
a. $-\dfrac{7}{4}$

b. 2.3

c. $\dfrac{2}{3}$

d. -1.8

Solution The figure below shows all four points.



a. The point representing the real number $-\frac{7}{4} = -1.75$ lies between -2 and -1, but closer to -2, on the real number line.

b. The point representing the real number 2.3 lies between 2 and 3, but closer to 2, on the real number line.

c. The point representing the real number $\frac{2}{3} = 0.666\ldots$ lies between 0 and 1, but closer to 1, on the real number line.

d. The point representing the real number -1.8 lies between -2 and -1, but closer to -2, on the real number line. Note that the point representing -1.8 lies slightly to the left of the point representing $-\frac{7}{4}$.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Plot the real numbers on the real number line.

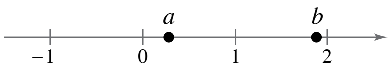
a. $\dfrac{5}{2}$ **b.** -1.6

c. $-\dfrac{3}{4}$ **d.** 0.7

Ordering Real Numbers

One important property of real numbers is that they are *ordered.*

> **Definition of Order on the Real Number Line**
>
> If a and b are real numbers, then a is *less than* b when $b - a$ is positive. The **inequality** $a < b$ denotes the **order** of a and b. This relationship can also be described by saying that b is *greater than* a and writing $b > a$. The inequality $a \leq b$ means that a is *less than or equal to* b, and the inequality $b \geq a$ means that b is *greater than or equal to* a. The symbols $<$, $>$, $\leq$, and $\geq$ are *inequality symbols.*



$a < b$ if and only if a lies to the left of b.
Figure P.2

Geometrically, this definition implies that $a < b$ if and only if a lies to the *left* of b on the real number line, as shown in Figure P.2.

EXAMPLE 3 Ordering Real Numbers

Place the appropriate inequality symbol ($<$ or $>$) between the pair of real numbers.

a. $-3, 0$ **b.** $-2, -4$ **c.** $\frac{1}{4}, \frac{1}{3}$

Solution

a. On the real number line, -3 lies to the left of 0, as shown in Figure P.3. So, you can say that -3 is *less than* 0, and write $-3 < 0$.

b. On the real number line, -2 lies to the right of -4, as shown in Figure P.4. So, you can say that -2 is *greater than* -4, and write $-2 > -4$.

c. On the real number line, $\frac{1}{4}$ lies to the left of $\frac{1}{3}$, as shown in Figure P.5. So, you can say that $\frac{1}{4}$ is *less than* $\frac{1}{3}$, and write $\frac{1}{4} < \frac{1}{3}$.

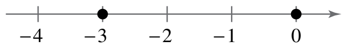


Figure P.3

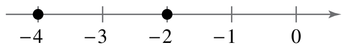


Figure P.4

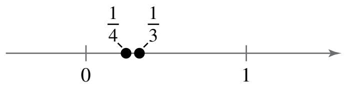


Figure P.5

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Place the appropriate inequality symbol ($<$ or $>$) between the pair of real numbers.

a. $1, -5$ **b.** $\frac{3}{2}, 7$ **c.** $-\frac{2}{3}, -\frac{3}{4}$

EXAMPLE 4 Interpreting Inequalities

See LarsonPrecalculus.com for an interactive version of this type of example.

Describe the subset of real numbers that the inequality represents.

a. $x \leq 2$ **b.** $-2 \leq x < 3$

Solution

a. The inequality $x \leq 2$ denotes all real numbers less than or equal to 2, as shown in Figure P.6.

b. The inequality $-2 \leq x < 3$ means that $x \geq -2$ *and* $x < 3$. This "double inequality" denotes all real numbers between -2 and 3, including -2 but not including 3, as shown in Figure P.7.

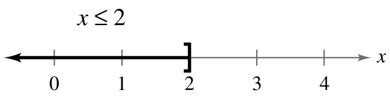


Figure P.6

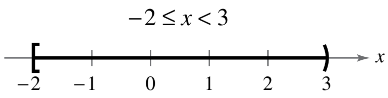


Figure P.7

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

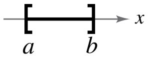
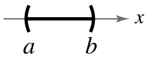
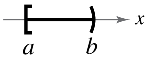
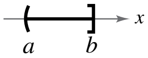
Describe the subset of real numbers that the inequality represents.

a. $x > -3$ **b.** $0 < x \leq 4$

Inequalities can describe subsets of real numbers called **intervals.** In the bounded intervals below, the real numbers a and b are the **endpoints** of each interval. The endpoints of a closed interval are included in the interval, whereas the endpoints of an open interval are not included in the interval.

REMARK The reason that the four types of intervals at the right are called *bounded* is that each has a finite length. An interval that does not have a finite length is *unbounded* (see below).

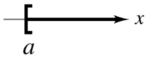
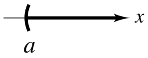
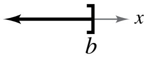
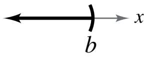
Bounded Intervals on the Real Number Line

Notation	**Interval Type**	**Inequality**	**Graph**
$[a, b]$	Closed	$a \leq x \leq b$	
(a, b)	Open	$a < x < b$	
$[a, b)$		$a \leq x < b$	
$(a, b]$		$a < x \leq b$	

The symbols ∞, **positive infinity,** and $-\infty$, **negative infinity,** do not represent real numbers. They are convenient symbols used to describe the unboundedness of an interval such as $(1, \infty)$ or $(-\infty, 3]$.

REMARK Whenever you write an interval containing ∞ or $-\infty$, always use a parenthesis and never a bracket next to these symbols. This is because ∞ and $-\infty$ are never included in the interval.

Unbounded Intervals on the Real Number Line

Notation	**Interval Type**	**Inequality**	**Graph**
$[a, \infty)$		$x \geq a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$		$x \leq b$	
$(-\infty, b)$	Open	$x < b$	
$(-\infty, \infty)$	Entire real line	$-\infty < x < \infty$	

EXAMPLE 5 Interpreting Intervals

a. The interval $(-1, 0)$ consists of all real numbers greater than -1 and less than 0.

b. The interval $[2, \infty)$ consists of all real numbers greater than or equal to 2.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Give a verbal description of the interval $[-2, 5)$.

EXAMPLE 6 Using Inequalities to Represent Intervals

a. The inequality $c \leq 2$ can represent the statement "c is at most 2."

b. The inequality $-3 < x \leq 5$ can represent "all x in the interval $(-3, 5]$."

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use inequality notation to represent the statement "x is less than 4 and at least -2."

Absolute Value and Distance

The **absolute value** of a real number is its *magnitude,* or the distance between the origin and the point representing the real number on the real number line.

Definition of Absolute Value

If a is a real number, then the **absolute value** of a is

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}.$$

Notice in this definition that the absolute value of a real number is never negative. For example, if $a = -5$, then $|-5| = -(-5) = 5$. The absolute value of a real number is either positive or zero. Moreover, 0 is the only real number whose absolute value is 0. So, $|0| = 0$.

Properties of Absolute Values

1. $|a| \geq 0$

2. $|-a| = |a|$

3. $|ab| = |a||b|$

4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \quad b \neq 0$

EXAMPLE 7 Finding Absolute Values

a. $|-15| = 15$

b. $\left|\frac{2}{3}\right| = \frac{2}{3}$

c. $|-4.3| = 4.3$

d. $-|-6| = -(6) = -6$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Evaluate each expression.

a. $|1|$

b. $-\left|\frac{3}{4}\right|$

c. $\frac{2}{|-3|}$

d. $-|0.7|$

EXAMPLE 8 Evaluating an Absolute Value Expression

Evaluate $\frac{|x|}{x}$ for (a) $x > 0$ and (b) $x < 0$.

Solution

a. If $x > 0$, then x is positive and $|x| = x$. So, $\frac{|x|}{x} = \frac{x}{x} = 1$.

b. If $x < 0$, then x is negative and $|x| = -x$. So, $\frac{|x|}{x} = \frac{-x}{x} = -1$.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Evaluate $\frac{|x + 3|}{x + 3}$ for (a) $x > -3$ and (b) $x < -3$.

The **Law of Trichotomy** states that for any two real numbers a and b, *precisely* one of three relationships is possible:

$$a = b, \quad a < b, \quad \text{or} \quad a > b.$$

Law of Trichotomy

EXAMPLE 9 Comparing Real Numbers

Place the appropriate symbol ($<$, $>$, or $=$) between the pair of real numbers.

a. $|-4| \; \square \; |3|$ **b.** $|-10| \; \square \; |10|$ **c.** $-|-7| \; \square \; |-7|$

Solution

a. $|-4| > |3|$ because $|-4| = 4$ and $|3| = 3$, and 4 is greater than 3.

b. $|-10| = |10|$ because $|-10| = 10$ and $|10| = 10$.

c. $-|-7| < |-7|$ because $-|-7| = -7$ and $|-7| = 7$, and -7 is less than 7.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Place the appropriate symbol ($<$, $>$, or $=$) between the pair of real numbers.

a. $|-3| \; \square \; |4|$

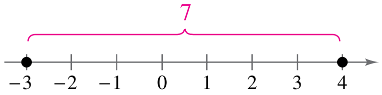
b. $-|-4| \; \square \; -|4|$

c. $|-3| \; \square \; -|-3|$

Absolute value can be used to find the distance between two points on the real number line. For example, the distance between -3 and 4 is

$$\begin{aligned} |-3 - 4| &= |-7| \\ &= 7 \end{aligned}$$

as shown in Figure P.8.



The distance between -3 and 4 is 7.

Figure P.8

Distance Between Two Points on the Real Number Line

Let a and b be real numbers. The **distance between a and b** is

$$d(a, b) = |b - a| = |a - b|.$$

One application of finding the distance between two points on the real number line is finding a change in temperature.

EXAMPLE 10 Finding a Distance

Find the distance between -25 and 13.

Solution

The distance between -25 and 13 is

$$|-25 - 13| = |-38| = 38.$$

Distance between -25 and 13

The distance can also be found as follows.

$$|13 - (-25)| = |38| = 38$$

Distance between -25 and 13

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

a. Find the distance between 35 and -23.

b. Find the distance between -35 and -23.

c. Find the distance between 35 and 23.

VladisChern/Shutterstock.com

Algebraic Expressions

One characteristic of algebra is the use of letters to represent numbers. The letters are **variables,** and combinations of letters and numbers are **algebraic expressions.** Here are a few examples of algebraic expressions.

$$5x, \qquad 2x - 3, \qquad \frac{4}{x^2 + 2}, \qquad 7x + y$$

Definition of an Algebraic Expression

An **algebraic expression** is a collection of letters (**variables**) and real numbers (**constants**) combined using the operations of addition, subtraction, multiplication, division, and exponentiation.

The **terms** of an algebraic expression are those parts that are separated by *addition.* For example, $x^2 - 5x + 8 = x^2 + (-5x) + 8$ has three terms: x^2 and $-5x$ are the **variable terms** and 8 is the **constant term.** For terms such as x^2, $-5x$, and 8, the numerical factor is the **coefficient.** Here, the coefficients are 1, -5, and 8.

EXAMPLE 11 Identifying Terms and Coefficients

	Algebraic Expression	Terms	Coefficients
a.	$5x - \frac{1}{7}$	$5x, -\frac{1}{7}$	$5, -\frac{1}{7}$
b.	$2x^2 - 6x + 9$	$2x^2, -6x, 9$	$2, -6, 9$
c.	$\frac{3}{x} + \frac{1}{2}x^4 - y$	$\frac{3}{x}, \frac{1}{2}x^4, -y$	$3, \frac{1}{2}, -1$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Identify the terms and coefficients of $-2x + 4$.

The **Substitution Principle** states, "If $a = b$, then b can replace a in any expression involving a." Use the Substitution Principle to **evaluate** an algebraic expression by substituting numerical values for each of the variables in the expression. The next example illustrates this.

EXAMPLE 12 Evaluating Algebraic Expressions

	Expression	Value of Variable	Substitute.	Value of Expression
a.	$-3x + 5$	$x = 3$	$-3(3) + 5$	$-9 + 5 = -4$
b.	$3x^2 + 2x - 1$	$x = -1$	$3(-1)^2 + 2(-1) - 1$	$3 - 2 - 1 = 0$
c.	$\frac{2x}{x + 1}$	$x = -3$	$\frac{2(-3)}{-3 + 1}$	$\frac{-6}{-2} = 3$

Note that you must substitute the value for *each* occurrence of the variable.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Evaluate $4x - 5$ when $x = 0$.

Basic Rules of Algebra

There are four arithmetic operations with real numbers: *addition, multiplication, subtraction,* and *division,* denoted by the symbols $+$, $\times$ or $\cdot$, $-$, and $\div$ or $/$, respectively. Of these, addition and multiplication are the two primary operations. Subtraction and division are the inverse operations of addition and multiplication, respectively.

Definitions of Subtraction and Division

Subtraction: Add the opposite.

$$a - b = a + (-b)$$

Division: Multiply by the reciprocal.

If $b \neq 0$, then $a/b = a\left(\frac{1}{b}\right) = \frac{a}{b}$.

In these definitions, $-b$ is the **additive inverse** (or opposite) of b, and $1/b$ is the **multiplicative inverse** (or reciprocal) of b. In the fractional form a/b, a is the **numerator** of the fraction and b is the **denominator.**

The properties of real numbers below are true for variables and algebraic expressions as well as for real numbers, so they are often called the **Basic Rules of Algebra.** Formulate a verbal description of each of these properties. For example, the first property states that *the order in which two real numbers are added does not affect their sum.*

Basic Rules of Algebra

Let a, b, and c be real numbers, variables, or algebraic expressions.

Property		**Example**
Commutative Property of Addition:	$a + b = b + a$	$4x + x^2 = x^2 + 4x$
Commutative Property of Multiplication:	$ab = ba$	$(4 - x)x^2 = x^2(4 - x)$
Associative Property of Addition:	$(a + b) + c = a + (b + c)$	$(x + 5) + x^2 = x + (5 + x^2)$
Associative Property of Multiplication:	$(ab)c = a(bc)$	$(2x \cdot 3y)(8) = (2x)(3y \cdot 8)$
Distributive Properties:	$a(b + c) = ab + ac$	$3x(5 + 2x) = 3x \cdot 5 + 3x \cdot 2x$
	$(a + b)c = ac + bc$	$(y + 8)y = y \cdot y + 8 \cdot y$
Additive Identity Property:	$a + 0 = a$	$5y^2 + 0 = 5y^2$
Multiplicative Identity Property:	$a \cdot 1 = a$	$(4x^2)(1) = 4x^2$
Additive Inverse Property:	$a + (-a) = 0$	$5x^3 + (-5x^3) = 0$
Multiplicative Inverse Property:	$a \cdot \frac{1}{a} = 1, \quad a \neq 0$	$(x^2 + 4)\left(\frac{1}{x^2 + 4}\right) = 1$

Subtraction is defined as "adding the opposite," so the Distributive Properties are also true for subtraction. For example, the "subtraction form" of $a(b + c) = ab + ac$ is $a(b - c) = ab - ac$. Note that the operations of subtraction and division are neither commutative nor associative. The examples

$$7 - 3 \neq 3 - 7 \quad \text{and} \quad 20 \div 4 \neq 4 \div 20$$

show that subtraction and division are not commutative. Similarly

$$5 - (3 - 2) \neq (5 - 3) - 2 \quad \text{and} \quad 16 \div (4 \div 2) \neq (16 \div 4) \div 2$$

demonstrate that subtraction and division are not associative.

EXAMPLE 13 Identifying Rules of Algebra

Identify the rule of algebra illustrated by the statement.

a. $(5x^3)2 = 2(5x^3)$ **b.** $(4x + 3) - (4x + 3) = 0$

c. $7x \cdot \frac{1}{7x} = 1, \quad x \neq 0$ **d.** $(2 + 5x^2) + x^2 = 2 + (5x^2 + x^2)$

Solution

a. This statement illustrates the Commutative Property of Multiplication. In other words, you obtain the same result whether you multiply $5x^3$ by 2, or 2 by $5x^3$.

b. This statement illustrates the Additive Inverse Property. In terms of subtraction, this property states that when any expression is subtracted from itself, the result is 0.

c. This statement illustrates the Multiplicative Inverse Property. Note that x must be a nonzero number. The reciprocal of x is undefined when x is 0.

d. This statement illustrates the Associative Property of Addition. In other words, to form the sum $2 + 5x^2 + x^2$, it does not matter whether 2 and $5x^2$, or $5x^2$ and x^2 are added first.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Identify the rule of algebra illustrated by the statement.

a. $x + 9 = 9 + x$ **b.** $5(x^3 \cdot 2) = (5x^3)2$ **c.** $(2 + 5x^2)y^2 = 2 \cdot y^2 + 5x^2 \cdot y^2$

REMARK Notice the difference between the *opposite of a number* and a *negative number.* If a is already negative, then its opposite, $-a$, is positive. For example, if $a = -5$, then

$$-a = -(-5) = 5.$$

Properties of Negation and Equality

Let a, b, and c be real numbers, variables, or algebraic expressions.

Property	Example
1. $(-1)a = -a$	$(-1)7 = -7$
2. $-(-a) = a$	$-(-6) = 6$
3. $(-a)b = -(ab) = a(-b)$	$(-5)3 = -(5 \cdot 3) = 5(-3)$
4. $(-a)(-b) = ab$	$(-2)(-x) = 2x$
5. $-(a + b) = (-a) + (-b)$	$-(x + 8) = (-x) + (-8)$ $= -x - 8$
6. If $a = b$, then $a \pm c = b \pm c$.	$\frac{1}{2} + 3 = 0.5 + 3$
7. If $a = b$, then $ac = bc$.	$4^2 \cdot 2 = 16 \cdot 2$
8. If $a \pm c = b \pm c$, then $a = b$.	$1.4 - 1 = \frac{7}{5} - 1$ ⇨ $1.4 = \frac{7}{5}$
9. If $ac = bc$ and $c \neq 0$, then $a = b$.	$3x = 3 \cdot 4$ ⇨ $x = 4$

REMARK The "or" in the Zero-Factor Property includes the possibility that either or both factors may be zero. This is an *inclusive or,* and it is generally the way the word "or" is used in mathematics.

Properties of Zero

Let a and b be real numbers, variables, or algebraic expressions.

1. $a + 0 = a$ and $a - 0 = a$

2. $a \cdot 0 = 0$

3. $\frac{0}{a} = 0, \quad a \neq 0$

4. $\frac{a}{0}$ is undefined.

5. **Zero-Factor Property:** If $ab = 0$, then $a = 0$ or $b = 0$.

Properties and Operations of Fractions

Let a, b, c, and d be real numbers, variables, or algebraic expressions such that $b \neq 0$ and $d \neq 0$.

1. **Equivalent Fractions:** $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.
2. **Rules of Signs:** $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ and $\frac{-a}{-b} = \frac{a}{b}$
3. **Generate Equivalent Fractions:** $\frac{a}{b} = \frac{ac}{bc}, \quad c \neq 0$
4. **Add or Subtract with Like Denominators:** $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$
5. **Add or Subtract with Unlike Denominators:** $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$
6. **Multiply Fractions:** $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
7. **Divide Fractions:** $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}, \quad c \neq 0$

REMARK In Property 1, the phrase "if and only if" implies two statements. One statement is: If $a/b = c/d$, then $ad = bc$. The other statement is: If $ad = bc$, where $b \neq 0$ and $d \neq 0$, then $a/b = c/d$.

EXAMPLE 14 Properties and Operations of Fractions

a. $\frac{x}{5} = \frac{3 \cdot x}{3 \cdot 5} = \frac{3x}{15}$ **b.** $\frac{7}{x} \div \frac{3}{2} = \frac{7}{x} \cdot \frac{2}{3} = \frac{14}{3x}$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

a. Multiply fractions: $\frac{3}{5} \cdot \frac{x}{6}$ **b.** Add fractions: $\frac{x}{10} + \frac{2x}{5}$

If a, b, and c are integers such that $ab = c$, then a and b are **factors** or **divisors** of c. A **prime number** is an integer that has exactly two positive factors—itself and 1—such as 2, 3, 5, 7, and 11. The numbers 4, 6, 8, 9, and 10 are **composite** because each can be written as the product of two or more prime numbers. The **Fundamental Theorem of Arithmetic** states that every positive integer greater than 1 is a prime number or can be written as the product of prime numbers in precisely one way (disregarding order). For example, the *prime factorization* of 24 is $24 = 2 \cdot 2 \cdot 2 \cdot 3$.

REMARK The number 1 is neither prime nor composite.

Summarize (Section P.1)

1. Explain how to represent and classify real numbers *(pages 2 and 3)*. For examples of representing and classifying real numbers, see Examples 1 and 2.
2. Explain how to order real numbers and use inequalities *(pages 4 and 5)*. For examples of ordering real numbers and using inequalities, see Examples 3–6.
3. State the definition of the absolute value of a real number *(page 6)*. For examples of using absolute value, see Examples 7–10.
4. Explain how to evaluate an algebraic expression *(page 8)*. For examples involving algebraic expressions, see Examples 11 and 12.
5. State the basic rules and properties of algebra *(pages 9–11)*. For examples involving the basic rules and properties of algebra, see Examples 13 and 14.

P.1 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. The decimal representation of an ________ number neither terminates nor repeats.

2. The point representing 0 on the real number line is the ________.

3. The distance between the origin and a point representing a real number on the real number line is the ________ ________ of the real number.

4. A number that can be written as the product of two or more prime numbers is a ________ number.

5. The ________ of an algebraic expression are those parts that are separated by addition.

6. The ________ ________ states that if $ab = 0$, then $a = 0$ or $b = 0$.

Skills and Applications

Classifying Real Numbers **In Exercises 7–10, determine which numbers in the set are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.**

7. $\left\{-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{2}, 0, 1, -4, 2, -11\right\}$

8. $\left\{\sqrt{5}, -7, -\frac{7}{3}, 0, 3.14, \frac{5}{4}, -3, 12, 5\right\}$

9. $\{2.01, 0.\overline{6}, -13, 0.010110111\ldots, 1, -6\}$

10. $\left\{25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, \frac{1}{2}\pi, 7, -11.1, 13\right\}$

Plotting Points on the Real Number Line **In Exercises 11 and 12, plot the real numbers on the real number line.**

11. (a) 3 (b) $\frac{7}{2}$ (c) $-\frac{5}{2}$ (d) -5.2

12. (a) 8.5 (b) $\frac{4}{3}$ (c) -4.75 (d) $-\frac{8}{3}$

Plotting and Ordering Real Numbers **In Exercises 13–16, plot the two real numbers on the real number line. Then place the appropriate inequality symbol (< or >) between them.**

13. $-4, -8$ **14.** $1, \frac{16}{3}$

15. $\frac{5}{6}, \frac{2}{3}$ **16.** $-\frac{8}{7}, -\frac{3}{7}$

Interpreting an Inequality or an Interval **In Exercises 17–24, (a) give a verbal description of the subset of real numbers represented by the inequality or the interval, (b) sketch the subset on the real number line, and (c) state whether the subset is bounded or unbounded.**

17. $x \leq 5$ **18.** $x < 0$

19. $-2 < x < 2$ **20.** $0 < x \leq 6$

21. $[4, \infty)$ **22.** $(-\infty, 2)$

23. $[-5, 2)$ **24.** $(-1, 2]$

Using Inequality and Interval Notation **In Exercises 25–28, use inequality notation and interval notation to describe the set.**

25. y is nonnegative. **26.** y is no more than 25.

27. t is at least 10 and at most 22.

28. k is less than 5 but no less than -3.

Evaluating an Absolute Value Expression **In Exercises 29–38, evaluate the expression.**

29. $|-10|$ **30.** $|0|$

31. $|3 - 8|$ **32.** $|6 - 2|$

33. $|-1| - |-2|$ **34.** $-3 - |-3|$

35. $5|-5|$ **36.** $-4|-4|$

37. $\dfrac{|x + 2|}{x + 2}, \quad x < -2$ **38.** $\dfrac{|x - 1|}{x - 1}, \quad x > 1$

Comparing Real Numbers **In Exercises 39–42, place the appropriate symbol (<, >, or =) between the pair of real numbers.**

39. $|-4|$ ☐ $|4|$ **40.** -5 ☐ $-|5|$

41. $-|-6|$ ☐ $|-6|$ **42.** $-|-2|$ ☐ $-|2|$

Finding a Distance **In Exercises 43–46, find the distance between *a* and *b*.**

43. $a = 126, b = 75$ **44.** $a = -20, b = 30$

45. $a = -\frac{5}{2}, b = 0$ **46.** $a = -\frac{1}{4}, b = -\frac{11}{4}$

Using Absolute Value Notation **In Exercises 47 and 48, use absolute value notation to represent the situation.**

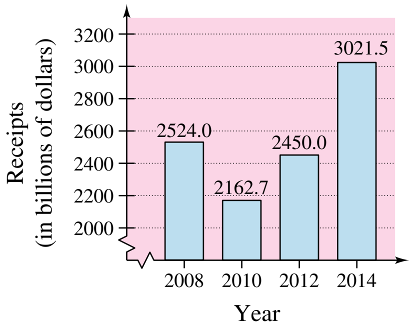
47. The distance between x and 5 is no more than 3.

48. The distance between x and -10 is at least 6.

The symbol ▦ and a red exercise number indicates that a video solution can be seen at *CalcView.com.*

Federal Deficit

In Exercises 49–52, use the bar graph, which shows the receipts of the federal government (in billions of dollars) for selected years from 2008 through 2014. In each exercise, you are given the expenditures of the federal government. Find the magnitude of the surplus or deficit for the year. *(Source: U.S. Office of Management and Budget)*



	Year	Receipts, R	Expenditures, E	$\lvert R - E \rvert$
49.	2008		\$2982.5 billion	
50.	2010		\$3457.1 billion	
51.	2012		\$3537.0 billion	
52.	2014		\$3506.1 billion	

Identifying Terms and Coefficients

In Exercises 53–58, identify the terms. Then identify the coefficients of the variable terms of the expression.

53. $7x + 4$
54. $2x - 3$
55. $6x^3 - 5x$
56. $4x^3 + 0.5x - 5$
57. $3\sqrt{3}x^2 + 1$
58. $2\sqrt{2}x^2 - 3$

Evaluating an Algebraic Expression

In Exercises 59–64, evaluate the expression for each value of x. (If not possible, state the reason.)

59. $4x - 6$ (a) $x = -1$ (b) $x = 0$
60. $9 - 7x$ (a) $x = -3$ (b) $x = 3$
61. $x^2 - 3x + 2$ (a) $x = 0$ (b) $x = -1$
62. $-x^2 + 5x - 4$ (a) $x = -1$ (b) $x = 1$
63. $\dfrac{x+1}{x-1}$ (a) $x = 1$ (b) $x = -1$
64. $\dfrac{x-2}{x+2}$ (a) $x = 2$ (b) $x = -2$

Michael G Smith/Shutterstock.com

Identifying Rules of Algebra

In Exercises 65–68, identify the rule(s) of algebra illustrated by the statement.

65. $\dfrac{1}{h+6}(h+6) = 1, \quad h \neq -6$
66. $(x+3) - (x+3) = 0$
67. $x(3y) = (x \cdot 3)y = (3x)y$
68. $\frac{1}{7}(7 \cdot 12) = \left(\frac{1}{7} \cdot 7\right)12 = 1 \cdot 12 = 12$

Operations with Fractions

In Exercises 69–72, perform the operation. (Write fractional answers in simplest form.)

69. $\dfrac{2x}{3} - \dfrac{x}{4}$
70. $\dfrac{3x}{4} + \dfrac{x}{5}$
71. $\dfrac{3x}{10} \cdot \dfrac{5}{6}$
72. $\dfrac{2x}{3} \div \dfrac{6}{7}$

Exploration

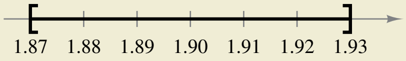
True or False?

In Exercises 73–75, determine whether the statement is true or false. Justify your answer.

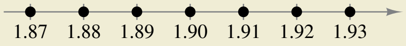
73. Every nonnegative number is positive.
74. If $a > 0$ and $b < 0$, then $ab > 0$.
75. If $a < 0$ and $b < 0$, then $ab > 0$.

76. HOW DO YOU SEE IT? Match each description with its graph. Which types of real numbers shown in Figure P.1 on page 2 may be included in a range of prices? a range of lengths? Explain.

(i)



(ii)



(a) The price of an item is within \$0.03 of \$1.90.

(b) The distance between the prongs of an electric plug may not differ from 1.9 centimeters by more than 0.03 centimeter.

77. Conjecture

(a) Use a calculator to complete the table.

n	0.0001	0.01	1	100	10,000
$\dfrac{5}{n}$					

(b) Use the result from part (a) to make a conjecture about the value of $5/n$ as n (i) approaches 0, and (ii) increases without bound.

P.2 Exponents and Radicals

Real numbers and algebraic expressions are often written with exponents and radicals. For example, in Exercise 69 on page 25, you will use an expression involving rational exponents to find the times required for a funnel to empty for different water heights.

- **Use properties of exponents.**
- **Use scientific notation to represent real numbers.**
- **Use properties of radicals.**
- **Simplify and combine radical expressions.**
- **Rationalize denominators and numerators.**
- **Use properties of rational exponents.**

Integer Exponents and Their Properties

Repeated *multiplication* can be written in **exponential form.**

Repeated Multiplication	Exponential Form
$a \cdot a \cdot a \cdot a \cdot a$	a^5
$(-4)(-4)(-4)$	$(-4)^3$
$(2x)(2x)(2x)(2x)$	$(2x)^4$

Exponential Notation

If a is a real number and n is a positive integer, then

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdot \cdot a}_{n \text{ factors}}$$

where n is the **exponent** and a is the **base.** You read a^n as "a to the nth **power.**"

An exponent can also be negative or zero. Properties 3 and 4 below show how to use negative and zero exponents.

Properties of Exponents

Let a and b be real numbers, variables, or algebraic expressions, and let m and n be integers. (All denominators and bases are nonzero.)

Property	Example
1. $a^m a^n = a^{m+n}$	$3^2 \cdot 3^4 = 3^{2+4} = 3^6 = 729$
2. $\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{x^7}{x^4} = x^{7-4} = x^3$
3. $a^{-n} = \dfrac{1}{a^n} = \left(\dfrac{1}{a}\right)^n$	$y^{-4} = \dfrac{1}{y^4} = \left(\dfrac{1}{y}\right)^4$
4. $a^0 = 1$	$(x^2 + 1)^0 = 1$
5. $(ab)^m = a^m b^m$	$(5x)^3 = 5^3 x^3 = 125x^3$
6. $(a^m)^n = a^{mn}$	$(y^3)^{-4} = y^{3(-4)} = y^{-12} = \dfrac{1}{y^{12}}$
7. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	$\left(\dfrac{2}{x}\right)^3 = \dfrac{2^3}{x^3} = \dfrac{8}{x^3}$
8. $\lvert a^2 \rvert = \lvert a \rvert^2 = a^2$	$\lvert(-2)^2\rvert = \lvert -2 \rvert^2 = 2^2 = 4 = (-2)^2$

iStockphoto.com/micropic

The properties of exponents listed on the preceding page apply to *all* integers m and n, not just to positive integers, as shown in Examples 1–4.

It is important to recognize the difference between expressions such as $(-2)^4$ and -2^4. In $(-2)^4$, the parentheses tell you that the exponent applies to the negative sign as well as to the 2, but in $-2^4 = -(2^4)$, the exponent applies only to the 2. So, $(-2)^4 = 16$ and $-2^4 = -16$.

EXAMPLE 1 Evaluating Exponential Expressions

a. $(-5)^2 = (-5)(-5) = 25$ Negative sign is part of the base.

b. $-5^2 = -(5)(5) = -25$ Negative sign is *not* part of the base.

c. $2 \cdot 2^4 = 2^{1+4} = 2^5 = 32$ Property 1

d. $\dfrac{4^4}{4^6} = 4^{4-6} = 4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$ Properties 2 and 3

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Evaluate each expression.

a. -3^4 **b.** $(-3)^4$

c. $3^2 \cdot 3$ **d.** $\dfrac{3^5}{3^8}$

TECHNOLOGY When using a calculator to evaluate exponential expressions, it is important to know when to use parentheses because the calculator follows the order of operations. For example, here is how you would evaluate $(-2)^4$ on a graphing utility.

$\boxed{(}\ \boxed{(-)}\ 2\ \boxed{)}\ \boxed{\wedge}\ 4\ \boxed{\text{ENTER}}$

The display will be 16. If you omit the parentheses, the display will be -16.

EXAMPLE 2 Evaluating Algebraic Expressions

Evaluate each algebraic expression when $x = 3$.

a. $5x^{-2}$ **b.** $\dfrac{1}{3}(-x)^3$

Solution

a. When $x = 3$, the expression $5x^{-2}$ has a value of

$$5x^{-2} = 5(3)^{-2} = \frac{5}{3^2} = \frac{5}{9}.$$

b. When $x = 3$, the expression $\frac{1}{3}(-x)^3$ has a value of

$$\frac{1}{3}(-x)^3 = \frac{1}{3}(-3)^3 = \frac{1}{3}(-27) = -9.$$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Evaluate each algebraic expression when $x = 4$.

a. $-x^{-2}$ **b.** $\dfrac{1}{4}(-x)^4$

EXAMPLE 3 Using Properties of Exponents

Use the properties of exponents to simplify each expression.

a. $(-3ab^4)(4ab^{-3})$ **b.** $(2xy^2)^3$ **c.** $3a(-4a^2)^0$ **d.** $\left(\dfrac{5x^3}{y}\right)^2$

Solution

a. $(-3ab^4)(4ab^{-3}) = (-3)(4)(a)(a)(b^4)(b^{-3}) = -12a^2b$

b. $(2xy^2)^3 = 2^3(x)^3(y^2)^3 = 8x^3y^6$

c. $3a(-4a^2)^0 = 3a(1) = 3a$

d. $\left(\dfrac{5x^3}{y}\right)^2 = \dfrac{5^2(x^3)^2}{y^2} = \dfrac{25x^6}{y^2}$

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use the properties of exponents to simplify each expression.

a. $(2x^{-2}y^3)(-x^4y)$ **b.** $(4a^2b^3)^0$ **c.** $(-5z)^3(z^2)$ **d.** $\left(\dfrac{3x^4}{x^2y^2}\right)^2$

REMARK Rarely in algebra is there only one way to solve a problem. Do not be concerned when the steps you use to solve a problem are not exactly the same as the steps presented in this text. It is important to use steps that you understand and, of course, steps that are justified by the rules of algebra. For example, the fractional form of Property 3 is

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m.$$

So, you might prefer the steps below for Example 4(d).

$$\left(\frac{3x^2}{y}\right)^{-2} = \left(\frac{y}{3x^2}\right)^2 = \frac{y^2}{9x^4}$$

EXAMPLE 4 Rewriting with Positive Exponents

a. $x^{-1} = \dfrac{1}{x}$ Property 3

b. $\dfrac{1}{3x^{-2}} = \dfrac{1(x^2)}{3}$ Property 3 (The exponent -2 does not apply to 3.)

$= \dfrac{x^2}{3}$ Simplify.

c. $\dfrac{12a^3b^{-4}}{4a^{-2}b} = \dfrac{12a^3 \cdot a^2}{4b \cdot b^4}$ Property 3

$= \dfrac{3a^5}{b^5}$ Property 1

d. $\left(\dfrac{3x^2}{y}\right)^{-2} = \dfrac{3^{-2}(x^2)^{-2}}{y^{-2}}$ Properties 5 and 7

$= \dfrac{3^{-2}x^{-4}}{y^{-2}}$ Property 6

$= \dfrac{y^2}{3^2x^4}$ Property 3

$= \dfrac{y^2}{9x^4}$ Simplify.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Rewrite each expression with positive exponents. Simplify, if possible.

a. $2a^{-2}$ **b.** $\dfrac{3a^{-3}b^4}{15ab^{-1}}$

c. $\left(\dfrac{x}{10}\right)^{-1}$ **d.** $(-2x^2)^3(4x^3)^{-1}$

There are about 366 billion billion gallons of water on Earth. It is convenient to write such a number in scientific notation.

Scientific Notation

Exponents provide an efficient way of writing and computing with very large (or very small) numbers. For example, there are about 366 billion billion gallons of water on Earth—that is, 366 followed by 18 zeros.

$$366{,}000{,}000{,}000{,}000{,}000{,}000$$

It is convenient to write such numbers in **scientific notation.** This notation has the form $\pm c \times 10^n$, where $1 \le c < 10$ and n is an integer. So, the number of gallons of water on Earth, written in scientific notation, is

$$3.66 \times 100{,}000{,}000{,}000{,}000{,}000{,}000 = 3.66 \times 10^{20}.$$

The *positive* exponent 20 tells you that the number is *large* (10 or more) and that the decimal point has been moved 20 places. A *negative* exponent tells you that the number is *small* (less than 1). For example, the mass (in grams) of one electron is approximately

$$9.1 \times 10^{-28} = 0.\underbrace{00000000000000000000000000091}_{\text{28 decimal places}}.$$

EXAMPLE 5 Scientific Notation

a. $0.0000782 = 7.82 \times 10^{-5}$

b. $836{,}100{,}000 = 8.361 \times 10^{8}$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Write 45,850 in scientific notation.

EXAMPLE 6 Decimal Notation

a. $-9.36 \times 10^{-6} = -0.00000936$

b. $1.345 \times 10^{2} = 134.5$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Write -2.718×10^{-3} in decimal notation.

TECHNOLOGY Most calculators automatically switch to scientific notation when showing large (or small) numbers that exceed the display range.

To enter numbers in scientific notation, your calculator should have an exponential entry key labeled

$\boxed{\text{EE}}$ or $\boxed{\text{EXP}}$.

Consult the user's guide for instructions on keystrokes and how your calculator displays numbers in scientific notation.

EXAMPLE 7 Using Scientific Notation

Evaluate $\dfrac{(2{,}400{,}000{,}000)(0.0000045)}{(0.00003)(1500)}$.

Solution Begin by rewriting each number in scientific notation. Then simplify.

$$\begin{aligned}\frac{(2{,}400{,}000{,}000)(0.0000045)}{(0.00003)(1500)} &= \frac{(2.4 \times 10^{9})(4.5 \times 10^{-6})}{(3.0 \times 10^{-5})(1.5 \times 10^{3})}\\ &= \frac{(2.4)(4.5)(10^{3})}{(4.5)(10^{-2})}\\ &= (2.4)(10^{5})\\ &= 240{,}000\end{aligned}$$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Evaluate (24,000,000,000)(0.00000012)(300,000).

EpicStockMedia/Shutterstock.com

Radicals and Their Properties

A **square root** of a number is one of its two equal factors. For example, 5 is a square root of 25 because 5 is one of the two equal factors of 25. In a similar way, a **cube root** of a number is one of its three equal factors, as in $125 = 5^3$.

Definition of *n*th Root of a Number

Let a and b be real numbers and let $n \geq 2$ be a positive integer. If

$$a = b^n$$

then b is an ***n*th root of *a*.** If $n = 2$, then the root is a **square root.** If $n = 3$, then the root is a **cube root.**

Some numbers have more than one nth root. For example, both 5 and -5 are square roots of 25. The *principal square* root of 25, written as $\sqrt{25}$, is the positive root, 5.

Principal *n*th Root of a Number

Let a be a real number that has at least one nth root. The **principal *n*th root of *a*** is the nth root that has the same sign as a. It is denoted by a **radical symbol**

$$\sqrt[n]{a}. \qquad \text{Principal } n\text{th root}$$

The positive integer $n \geq 2$ is the **index** of the radical, and the number a is the **radicand.** When $n = 2$, omit the index and write $\sqrt{a}$ rather than $\sqrt[2]{a}$. (The plural of index is *indices.*)

A common misunderstanding is that the square root sign implies both negative and positive roots. This is not correct. The square root sign implies only a positive root. When a negative root is needed, you must use the negative sign with the square root sign.

Incorrect: $\sqrt{4} = \pm 2$ ✗ *Correct:* $-\sqrt{4} = -2$ *and* $\sqrt{4} = 2$

EXAMPLE 8 Evaluating Radical Expressions

a. $\sqrt{36} = 6$ because $6^2 = 36$.

b. $-\sqrt{36} = -6$ because $-(\sqrt{36}) = -(\sqrt{6^2}) = -(6) = -6$.

c. $\sqrt[3]{\dfrac{125}{64}} = \dfrac{5}{4}$ because $\left(\dfrac{5}{4}\right)^3 = \dfrac{5^3}{4^3} = \dfrac{125}{64}$.

d. $\sqrt[5]{-32} = -2$ because $(-2)^5 = -32$.

e. $\sqrt[4]{-81}$ is not a real number because no real number raised to the fourth power produces -81.

✓ Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Evaluate each expression, if possible.

a. $-\sqrt{144}$ **b.** $\sqrt{-144}$

c. $\sqrt{\dfrac{25}{64}}$ **d.** $-\sqrt[3]{\dfrac{8}{27}}$

Here are some generalizations about the *n*th roots of real numbers.

Generalizations About *n*th Roots of Real Numbers			
Real Number a	Integer $n > 0$	Root(s) of a	Example
$a > 0$	n is even.	$\sqrt[n]{a}, \ -\sqrt[n]{a}$	$\sqrt[4]{81} = 3, \ -\sqrt[4]{81} = -3$
$a > 0$ or $a < 0$	n is odd.	$\sqrt[n]{a}$	$\sqrt[3]{-8} = -2$
$a < 0$	n is even.	No real roots	$\sqrt{-4}$ is not a real number.
$a = 0$	n is even or odd.	$\sqrt[n]{0} = 0$	$\sqrt[5]{0} = 0$

Integers such as 1, 4, 9, 16, 25, and 36 are **perfect squares** because they have integer square roots. Similarly, integers such as 1, 8, 27, 64, and 125 are **perfect cubes** because they have integer cube roots.

Properties of Radicals

Let a and b be real numbers, variables, or algebraic expressions such that the roots below are real numbers, and let m and n be positive integers.

Property	**Example**
1. $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$	$\sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$
2. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$	$\sqrt{5} \cdot \sqrt{7} = \sqrt{5 \cdot 7} = \sqrt{35}$
3. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}, \quad b \neq 0$	$\dfrac{\sqrt[4]{27}}{\sqrt[4]{9}} = \sqrt[4]{\dfrac{27}{9}} = \sqrt[4]{3}$
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt[3]{\sqrt{10}} = \sqrt[6]{10}$
5. $\left(\sqrt[n]{a}\right)^n = a$	$\left(\sqrt{3}\right)^2 = 3$
6. For n even, $\sqrt[n]{a^n} = \lvert a \rvert$.	$\sqrt{(-12)^2} = \lvert -12 \rvert = 12$
For n odd, $\sqrt[n]{a^n} = a$.	$\sqrt[3]{(-12)^3} = -12$

A common use of Property 6 is $\sqrt{a^2} = \lvert a \rvert$.

EXAMPLE 9 Using Properties of Radicals

Use the properties of radicals to simplify each expression.

a. $\sqrt{8} \cdot \sqrt{2}$ **b.** $\left(\sqrt[3]{5}\right)^3$

c. $\sqrt[3]{x^3}$ **d.** $\sqrt[6]{y^6}$

Solution

a. $\sqrt{8} \cdot \sqrt{2} = \sqrt{8 \cdot 2} = \sqrt{16} = 4$ **b.** $\left(\sqrt[3]{5}\right)^3 = 5$

c. $\sqrt[3]{x^3} = x$ **d.** $\sqrt[6]{y^6} = \lvert y \rvert$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use the properties of radicals to simplify each expression.

a. $\dfrac{\sqrt{125}}{\sqrt{5}}$ **b.** $\sqrt[3]{125^2}$ **c.** $\sqrt[3]{x^2} \cdot \sqrt[3]{x}$ **d.** $\sqrt{\sqrt{x}}$

Simplifying Radical Expressions

An expression involving radicals is in **simplest form** when the three conditions below are satisfied.

1. All possible factors are removed from the radical.
2. All fractions have radical-free denominators (a process called *rationalizing the denominator* accomplishes this).
3. The index of the radical is reduced.

To simplify a radical, factor the radicand into factors whose exponents are multiples of the index. Write the roots of these factors outside the radical. The "leftover" factors make up the new radicand.

REMARK When you simplify a radical, it is important that both the original and the simplified expressions are defined for the same values of the variable. For instance, in Example 10(c), $\sqrt{75x^3}$ and $5x\sqrt{3x}$ are both defined only for nonnegative values of x. Similarly, in Example 10(e), $\sqrt[4]{(5x)^4}$ and $5|x|$ are both defined for all real values of x.

EXAMPLE 10 Simplifying Radical Expressions

a. $\sqrt[3]{24} = \sqrt[3]{\overset{\substack{\text{Perfect cube}\\ \downarrow}}{8} \cdot \overset{\substack{\text{Leftover factor}\\ \downarrow}}{3}} = \sqrt[3]{2^3 \cdot 3} = 2\sqrt[3]{3}$

b. $\sqrt[4]{48} = \sqrt[4]{\overset{\substack{\text{Perfect}\\ \text{4th power}\\ \downarrow}}{16} \cdot \overset{\substack{\text{Leftover}\\ \text{factor}\\ \downarrow}}{3}} = \sqrt[4]{2^4 \cdot 3} = 2\sqrt[4]{3}$

c. $\sqrt{75x^3} = \sqrt{25x^2 \cdot 3x} = \sqrt{(5x)^2 \cdot 3x} = 5x\sqrt{3x}$

d. $\sqrt[3]{24a^4} = \sqrt[3]{8a^3 \cdot 3a} = \sqrt[3]{(2a)^3 \cdot 3a} = 2a\sqrt[3]{3a}$

e. $\sqrt[4]{(5x)^4} = |5x| = 5|x|$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Simplify each radical expression.

a. $\sqrt{32}$ **b.** $\sqrt[3]{250}$ **c.** $\sqrt{24a^5}$ **d.** $\sqrt[3]{-135x^3}$

Radical expressions can be combined (added or subtracted) when they are **like radicals**—that is, when they have the same index and radicand. For example, $\sqrt{2}$, $3\sqrt{2}$, and $\frac{1}{2}\sqrt{2}$ are like radicals, but $\sqrt{3}$ and $\sqrt{2}$ are unlike radicals. To determine whether two radicals can be combined, first simplify each radical.

EXAMPLE 11 Combining Radical Expressions

a.
$$\begin{aligned}
2\sqrt{48} - 3\sqrt{27} &= 2\sqrt{16 \cdot 3} - 3\sqrt{9 \cdot 3} && \text{Find square factors.}\\
&= 8\sqrt{3} - 9\sqrt{3} && \text{Find square roots and multiply by coefficients.}\\
&= (8 - 9)\sqrt{3} && \text{Combine like radicals.}\\
&= -\sqrt{3} && \text{Simplify.}
\end{aligned}$$

b.
$$\begin{aligned}
\sqrt[3]{16x} - \sqrt[3]{54x^4} &= \sqrt[3]{8 \cdot 2x} - \sqrt[3]{27x^3 \cdot 2x} && \text{Find cube factors.}\\
&= 2\sqrt[3]{2x} - 3x\sqrt[3]{2x} && \text{Find cube roots.}\\
&= (2 - 3x)\sqrt[3]{2x} && \text{Combine like radicals.}
\end{aligned}$$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Simplify each radical expression.

a. $3\sqrt{8} + \sqrt{18}$ **b.** $\sqrt[3]{81x^5} - \sqrt[3]{24x^2}$

Rationalizing Denominators and Numerators

To rationalize a denominator or numerator of the form $a - b\sqrt{m}$ or $a + b\sqrt{m}$, multiply both numerator and denominator by a **conjugate:** $a + b\sqrt{m}$ and $a - b\sqrt{m}$ are conjugates of each other. If $a = 0$, then the rationalizing factor for $\sqrt{m}$ is itself, $\sqrt{m}$. For cube roots, choose a rationalizing factor that produces a perfect cube radicand.

EXAMPLE 12 Rationalizing Single-Term Denominators

a. $$\frac{5}{2\sqrt{3}} = \frac{5}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$ $\sqrt{3}$ is rationalizing factor.

$$= \frac{5\sqrt{3}}{2(3)}$$ Multiply.

$$= \frac{5\sqrt{3}}{6}$$ Simplify.

b. $$\frac{2}{\sqrt[3]{5}} = \frac{2}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}$$ $\sqrt[3]{5^2}$ is rationalizing factor.

$$= \frac{2\sqrt[3]{5^2}}{\sqrt[3]{5^3}}$$ Multiply.

$$= \frac{2\sqrt[3]{25}}{5}$$ Simplify.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Rationalize the denominator of each expression.

a. $\dfrac{5}{3\sqrt{2}}$ **b.** $\dfrac{1}{\sqrt[3]{25}}$

EXAMPLE 13 Rationalizing a Denominator with Two Terms

$$\frac{2}{3 + \sqrt{7}} = \frac{2}{3 + \sqrt{7}} \cdot \frac{3 - \sqrt{7}}{3 - \sqrt{7}}$$ Multiply numerator and denominator by conjugate of denominator.

$$= \frac{2(3 - \sqrt{7})}{3(3 - \sqrt{7}) + \sqrt{7}(3 - \sqrt{7})}$$ Distributive Property

$$= \frac{2(3 - \sqrt{7})}{3(3) - 3(\sqrt{7}) + \sqrt{7}(3) - \sqrt{7}(\sqrt{7})}$$ Distributive Property

$$= \frac{2(3 - \sqrt{7})}{(3)^2 - (\sqrt{7})^2}$$ Simplify.

$$= \frac{2(3 - \sqrt{7})}{2}$$ Simplify.

$$= 3 - \sqrt{7}$$ Divide out common factor.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Rationalize the denominator: $\dfrac{8}{\sqrt{6} - \sqrt{2}}$.

Sometimes it is necessary to rationalize the numerator of an expression. For instance, in Section P.5 you will use the technique shown in Example 14 on the next page to rationalize the numerator of an expression from calculus.

EXAMPLE 14 Rationalizing a Numerator

$$\frac{\sqrt{5}-\sqrt{7}}{2} = \frac{\sqrt{5}-\sqrt{7}}{2} \cdot \frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}}$$ Multiply numerator and denominator by conjugate of numerator.

$$= \frac{(\sqrt{5})^2-(\sqrt{7})^2}{2(\sqrt{5}+\sqrt{7})}$$ Simplify.

$$= \frac{5-7}{2(\sqrt{5}+\sqrt{7})}$$ Property 5 of radicals

$$= \frac{-2}{2(\sqrt{5}+\sqrt{7})}$$ Simplify.

$$= \frac{-1}{\sqrt{5}+\sqrt{7}}$$ Divide out common factor.

REMARK Do not confuse the expression $\sqrt{5}+\sqrt{7}$ with the expression $\sqrt{5+7}$. In general, $\sqrt{x+y}$ does not equal $\sqrt{x}+\sqrt{y}$. Similarly, $\sqrt{x^2+y^2}$ does not equal $x+y$.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Rationalize the numerator: $\dfrac{2-\sqrt{2}}{3}$.

Rational Exponents and Their Properties

Definition of Rational Exponents

If a is a real number and n is a positive integer such that the principal nth root of a exists, then $a^{1/n}$ is defined as

$$a^{1/n} = \sqrt[n]{a}.$$

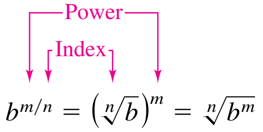
Moreover, if m is a positive integer, then

$$a^{m/n} = (a^{1/n})^m.$$

$1/n$ and m/n are called **rational exponents** of a.

REMARK If m and n have no common factors, then it is also true that $a^{m/n} = (a^m)^{1/n}$.

The numerator of a rational exponent denotes the *power* to which the base is raised, and the denominator denotes the *index* or the *root* to be taken.



$$b^{m/n} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$$

When you are working with rational exponents, the properties of integer exponents still apply. For example, $2^{1/2}2^{1/3} = 2^{(1/2)+(1/3)} = 2^{5/6}$.

EXAMPLE 15 Changing From Radical to Exponential Form

a. $\sqrt{3} = 3^{1/2}$

b. $\sqrt{(3xy)^5} = \sqrt[2]{(3xy)^5} = (3xy)^{5/2}$

c. $2x\sqrt[4]{x^3} = (2x)(x^{3/4}) = 2x^{1+(3/4)} = 2x^{7/4}$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Write (a) $\sqrt[3]{27}$, (b) $\sqrt{x^3y^5z}$, and (c) $3x\sqrt[3]{x^2}$ in exponential form.

TECHNOLOGY There are four methods of evaluating radicals on most graphing utilities. For square roots, you can use the *square root key* $\boxed{\sqrt{\ }}$. For cube roots, you can use the *cube root key* $\boxed{\sqrt[3]{\ }}$. For other roots, first convert the radical to exponential form and then use the *exponential key* $\boxed{\wedge}$, or use the *x*th root key $\boxed{\sqrt[x]{\ }}$ (or menu choice). Consult the user's guide for your graphing utility for specific keystrokes.

EXAMPLE 16 Changing From Exponential to Radical Form

See LarsonPrecalculus.com for an interactive version of this type of example.

a. $(x^2 + y^2)^{3/2} = \left(\sqrt{x^2 + y^2}\right)^3 = \sqrt{(x^2 + y^2)^3}$

b. $2y^{3/4}z^{1/4} = 2(y^3z)^{1/4} = 2\sqrt[4]{y^3z}$

c. $a^{-3/2} = \dfrac{1}{a^{3/2}} = \dfrac{1}{\sqrt{a^3}}$

d. $x^{0.2} = x^{1/5} = \sqrt[5]{x}$

✓ Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Write each expression in radical form.

a. $(x^2 - 7)^{-1/2}$ **b.** $-3b^{1/3}c^{2/3}$

c. $a^{0.75}$ **d.** $(x^2)^{2/5}$

Rational exponents are useful for evaluating roots of numbers on a calculator, for reducing the index of a radical, and for simplifying expressions in calculus.

EXAMPLE 17 Simplifying with Rational Exponents

a. $(-32)^{-4/5} = \left(\sqrt[5]{-32}\right)^{-4} = (-2)^{-4} = \dfrac{1}{(-2)^4} = \dfrac{1}{16}$

b. $(-5x^{5/3})(3x^{-3/4}) = -15x^{(5/3)-(3/4)} = -15x^{11/12}, \quad x \neq 0$

c. $\sqrt[9]{a^3} = a^{3/9} = a^{1/3} = \sqrt[3]{a}$ Reduce index.

d. $\sqrt[3]{\sqrt{125}} = \sqrt[6]{125} = \sqrt[6]{(5)^3} = 5^{3/6} = 5^{1/2} = \sqrt{5}$

e. $(2x - 1)^{4/3}(2x - 1)^{-1/3} = (2x - 1)^{(4/3)-(1/3)} = 2x - 1, \quad x \neq \frac{1}{2}$

REMARK The expression in Example 17(b) is not defined when $x = 0$ because $0^{-3/4}$ is not a real number. Similarly, the expression in Example 17(e) is not defined when $x = \frac{1}{2}$ because

$$\left(2 \cdot \tfrac{1}{2} - 1\right)^{-1/3} = (0)^{-1/3}$$

is not a real number.

✓ Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Simplify each expression.

a. $(-125)^{-2/3}$ **b.** $(4x^2y^{3/2})(-3x^{-1/3}y^{-3/5})$

c. $\sqrt[3]{\sqrt[4]{27}}$ **d.** $(3x + 2)^{5/2}(3x + 2)^{-1/2}$

Summarize (Section P.2)

1. Make a list of the properties of exponents *(page 14)*. For examples that use these properties, see Examples 1–4.
2. State the definition of scientific notation *(page 17)*. For examples involving scientific notation, see Examples 5–7.
3. Make a list of the properties of radicals *(page 19)*. For examples involving radicals, see Examples 8 and 9.
4. Explain how to simplify a radical expression *(page 20)*. For examples of simplifying radical expressions, see Examples 10 and 11.
5. Explain how to rationalize a denominator or a numerator *(page 21)*. For examples of rationalizing denominators and numerators, see Examples 12–14.
6. State the definition of a rational exponent *(page 22)*. For examples involving rational exponents, see Examples 15–17.

P.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. In the exponential form a^n, n is the ________ and a is the ________.

2. A convenient way of writing very large or very small numbers is ________ ________.

3. One of the two equal factors of a number is a __________ __________ of the number.

4. In the radical form $\sqrt[n]{a}$, the positive integer n is the ________ of the radical and the number a is the ________.

5. Radical expressions can be combined (added or subtracted) when they are ________ ________.

6. The expressions $a + b\sqrt{m}$ and $a - b\sqrt{m}$ are ________ of each other.

7. The process used to create a radical-free denominator is known as ________ the denominator.

8. In the expression $b^{m/n}$, m denotes the ________ to which the base is raised and n denotes the ________ or root to be taken.

Skills and Applications

Evaluating Exponential Expressions **In Exercises 9–14, evaluate each expression.**

9. (a) $5 \cdot 5^3$ (b) $\dfrac{5^2}{5^4}$

10. (a) $(3^3)^0$ (b) -3^2

11. (a) $(2^3 \cdot 3^2)^2$ (b) $\left(-\dfrac{3}{5}\right)^3\left(\dfrac{5}{3}\right)^2$

12. (a) $\dfrac{3}{3^{-4}}$ (b) $48(-4)^{-3}$

13. (a) $\dfrac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}}$ (b) $(-2)^0$

14. (a) $3^{-1} + 2^{-2}$ (b) $(3^{-2})^2$

Evaluating an Algebraic Expression **In Exercises 15–20, evaluate the expression for the given value of x.**

15. $-3x^3, \quad x = 2$

16. $7x^{-2}, \quad x = 4$

17. $6x^0, \quad x = 10$

18. $2x^3, \quad x = -3$

19. $-3x^4, \quad x = -2$

20. $12(-x)^3, \quad x = -\frac{1}{3}$

Using Properties of Exponents **In Exercises 21–26, simplify each expression.**

21. (a) $(5z)^3$ (b) $5x^4(x^2)$

22. (a) $(-2x)^2$ (b) $(4x^3)^0$

23. (a) $6y^2(2y^0)^2$ (b) $(-z)^3(3z^4)$

24. (a) $\dfrac{7x^2}{x^3}$ (b) $\dfrac{12(x + y)^3}{9(x + y)}$

25. (a) $\left(\dfrac{4}{y}\right)^3\left(\dfrac{3}{y}\right)^4$ (b) $\left(\dfrac{b^{-2}}{a^{-2}}\right)\left(\dfrac{b}{a}\right)^2$

26. (a) $[(x^2y^{-2})^{-1}]^{-1}$ (b) $(5x^2z^6)^3(5x^2z^6)^{-3}$

Rewriting with Positive Exponents **In Exercises 27–30, rewrite each expression with positive exponents. Simplify, if possible.**

27. (a) $(x + 5)^0$ (b) $(2x^2)^{-2}$

28. (a) $(4y^{-2})(8y^4)$ (b) $(z + 2)^{-3}(z + 2)^{-1}$

29. (a) $\left(\dfrac{x^{-3}y^4}{5}\right)^{-3}$ (b) $\left(\dfrac{a^{-2}}{b^{-2}}\right)\left(\dfrac{b}{a}\right)^3$

30. (a) $\dfrac{3^n \cdot 3^{2n}}{3^{3n} \cdot 3^2}$ (b) $\dfrac{x^2 \cdot x^n}{x^3 \cdot x^n}$

Scientific Notation **In Exercises 31 and 32, write the number in scientific notation.**

31. 10,250.4

32. -0.000125

Decimal Notation **In Exercises 33–36, write the number in decimal notation.**

33. 3.14×10^{-4}

34. -2.058×10^6

35. Light year: 9.46×10^{12} kilometers

36. Diameter of a human hair: 9.0×10^{-6} meter

Using Scientific Notation **In Exercises 37 and 38, evaluate each expression without using a calculator.**

37. (a) $(2.0 \times 10^9)(3.4 \times 10^{-4})$
(b) $(1.2 \times 10^7)(5.0 \times 10^{-3})$

38. (a) $\dfrac{6.0 \times 10^8}{3.0 \times 10^{-3}}$ (b) $\dfrac{2.5 \times 10^{-3}}{5.0 \times 10^2}$

Evaluating Radical Expressions **In Exercises 39 and 40, evaluate each expression without using a calculator.**

39. (a) $\sqrt{9}$ (b) $\sqrt[3]{\frac{27}{8}}$

40. (a) $\sqrt[3]{27}$ (b) $\left(\sqrt{36}\right)^3$

Using Properties of Radicals **In Exercises 41 and 42, use the properties of radicals to simplify each expression.**

41. (a) $(\sqrt[5]{2})^5$ (b) $\sqrt[5]{32x^5}$

42. (a) $\sqrt{12} \cdot \sqrt{3}$ (b) $\sqrt[4]{(3x^2)^4}$

Simplifying Radical Expressions **In Exercises 43–50, simplify each radical expression.**

43. (a) $\sqrt{20}$ (b) $\sqrt[3]{128}$

44. (a) $\sqrt[3]{\frac{16}{27}}$ (b) $\sqrt{\frac{75}{4}}$

45. (a) $\sqrt{72x^3}$ (b) $\sqrt{54xy^4}$

46. (a) $\sqrt{\dfrac{18^2}{z^3}}$ (b) $\sqrt{\dfrac{32a^4}{b^2}}$

47. (a) $\sqrt[3]{16x^5}$ (b) $\sqrt{75x^2y^{-4}}$

48. (a) $\sqrt[4]{3x^4y^2}$ (b) $\sqrt[5]{160x^8z^4}$

49. (a) $2\sqrt{20x^2} + 5\sqrt{125x^2}$
(b) $8\sqrt{147x} - 3\sqrt{48x}$

50. (a) $3\sqrt[3]{54x^3} + \sqrt[3]{16x^3}$
(b) $\sqrt[3]{64x} - \sqrt[3]{27x^4}$

Rationalizing a Denominator **In Exercises 51–54, rationalize the denominator of the expression. Then simplify your answer.**

51. $\dfrac{1}{\sqrt{3}}$ **52.** $\dfrac{8}{\sqrt[3]{2}}$

53. $\dfrac{5}{\sqrt{14} - 2}$ **54.** $\dfrac{3}{\sqrt{5} + \sqrt{6}}$

Rationalizing a Numerator **In Exercises 55 and 56, rationalize the numerator of the expression. Then simplify your answer.**

55. $\dfrac{\sqrt{5} + \sqrt{3}}{3}$ **56.** $\dfrac{\sqrt{7} - 3}{4}$

Writing Exponential and Radical Forms **In Exercises 57–60, fill in the missing form of the expression.**

	Radical Form	Rational Exponent Form
57.	$\sqrt[3]{64}$	
58.	$x^2\sqrt{x}$	
59.		$3x^{-2/3}$
60.		$a^{0.4}$

Simplifying Expressions **In Exercises 61–68, simplify each expression.**

61. (a) $32^{-3/5}$ (b) $\left(\frac{16}{81}\right)^{-3/4}$

62. (a) $100^{-3/2}$ (b) $\left(\frac{9}{4}\right)^{-1/2}$

63. (a) $\sqrt[4]{3^2}$ (b) $\sqrt[6]{(x + 1)^4}$

64. (a) $\sqrt[6]{x^3}$ (b) $\sqrt[4]{(3x^2)^4}$

65. (a) $\sqrt{\sqrt{32}}$ (b) $\sqrt{\sqrt[4]{2x}}$

66. (a) $\sqrt{\sqrt{243(x + 1)}}$
(b) $\sqrt{\sqrt[3]{10a^7b}}$

67. (a) $(x - 1)^{1/3}(x - 1)^{2/3}$
(b) $(x - 1)^{1/3}(x - 1)^{-4/3}$

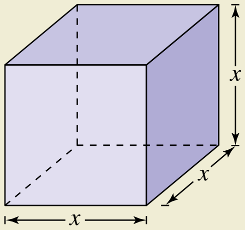
68. (a) $(4x + 3)^{5/2}(4x + 3)^{-5/3}$
(b) $(4x + 3)^{-5/2}(4x + 3)^{2/3}$

69. Mathematical Modeling A funnel is filled with water to a height of h centimeters. The formula

$$t = 0.03[12^{5/2} - (12 - h)^{5/2}], \quad 0 \le h \le 12$$

represents the amount of time t (in seconds) that it will take for the funnel to empty. Use the *table* feature of a graphing utility to find the times required for the funnel to empty for water heights of $h = 0, h = 1, h = 2, \ldots, h = 12$ centimeters.

70. HOW DO YOU SEE IT? Package A is a cube with a volume of 500 cubic inches. Package B is a cube with a volume of 250 cubic inches. Is the length x of a side of package A greater than, less than, or equal to twice the length of a side of package B? Explain.



Exploration

True or False? **In Exercises 71–74, determine whether the statement is true or false. Justify your answer.**

71. $\dfrac{x^{k+1}}{x} = x^k$ **72.** $(a^n)^k = a^{n^k}$

73. $(a + b)^2 = a^2 + b^2$

74. $\dfrac{a}{\sqrt{b}} = \dfrac{a^2}{(\sqrt{b})^2} = \dfrac{a^2}{b}$

iStockphoto.com/micropic

P.3 Polynomials and Special Products

Polynomials have many real-life applications. For example, in Exercise 81 on page 33, you will work with polynomials that model uniformly distributed safe loads for steel beams.

- **Write polynomials in standard form.**
- **Add, subtract, and multiply polynomials.**
- **Use special products to multiply polynomials.**
- **Use polynomials to solve real-life problems.**

Polynomials

One of the most common types of algebraic expressions is the **polynomial.** Some examples are $2x + 5$, $3x^4 - 7x^2 + 2x + 4$, and $5x^2y^2 - xy + 3$. The first two are *polynomials in x* and the third is a *polynomial in x and y*. The terms of a polynomial in x have the form ax^k, where a is the **coefficient** and k is the **degree** of the term. For example, the polynomial $2x^3 - 5x^2 + 1 = 2x^3 + (-5)x^2 + (0)x + 1$ has coefficients $2, -5, 0$, and 1.

Definition of a Polynomial in *x*

Let $a_0, a_1, a_2, \ldots, a_n$ be real numbers and let n be a nonnegative integer. A polynomial in x is an expression of the form

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

where $a_n \neq 0$. The polynomial is of **degree** n, a_n is the **leading coefficient,** and a_0 is the **constant term.**

Polynomials with one, two, and three terms are **monomials, binomials,** and **trinomials,** respectively. A polynomial written with descending powers of x is in **standard form.**

EXAMPLE 1 Writing Polynomials in Standard Form

Polynomial	Standard Form	Degree	Leading Coefficient
a. $4x^2 - 5x^7 - 2 + 3x$	$-5x^7 + 4x^2 + 3x - 2$	7	-5
b. $4 - 9x^2$	$-9x^2 + 4$	2	-9
c. 8	8 or $8x^0$	0	8

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Write the polynomial $6 - 7x^3 + 2x$ in standard form. Then identify the degree and leading coefficient of the polynomial.

A polynomial that has all zero coefficients is called the **zero polynomial,** denoted by 0. No degree is assigned to the zero polynomial. For polynomials in more than one variable, the degree of a *term* is the sum of the exponents of the variables in the term. The degree of the *polynomial* is the highest degree of its terms. For example, the degree of the polynomial $-2x^3y^6 + 4xy - x^7y^4$ is 11 because the sum of the exponents in the last term is the greatest. The leading coefficient of the polynomial is the coefficient of the highest-degree term. Expressions are not polynomials when a variable is underneath a radical or when a polynomial expression (with degree greater than 0) is in the denominator of a term. For example, the expressions $x^3 - \sqrt{3x} = x^3 - (3x)^{1/2}$ and $x^2 + (5/x) = x^2 + 5x^{-1}$ are not polynomials.

iStockphoto.com/Photosbyjim

Operations with Polynomials

You can add and subtract polynomials in much the same way you add and subtract real numbers. Add or subtract the *like terms* (terms having the same variables to the same powers) by adding or subtracting their coefficients. For example, $-3xy^2$ and $5xy^2$ are like terms and their sum is

$$-3xy^2 + 5xy^2 = (-3 + 5)xy^2 = 2xy^2.$$

EXAMPLE 2 Adding or Subtracting Polynomials

a. $(5x^3 - 7x^2 - 3) + (x^3 + 2x^2 - x + 8)$

$= (5x^3 + x^3) + (-7x^2 + 2x^2) + (-x) + (-3 + 8)$ Group like terms.

$= 6x^3 - 5x^2 - x + 5$ Combine like terms.

b. $(7x^4 - x^2 - 4x + 2) - (3x^4 - 4x^2 + 3x)$

$= 7x^4 - x^2 - 4x + 2 - 3x^4 + 4x^2 - 3x$ Distributive Property

$= (7x^4 - 3x^4) + (-x^2 + 4x^2) + (-4x - 3x) + 2$ Group like terms.

$= 4x^4 + 3x^2 - 7x + 2$ Combine like terms.

REMARK When a negative sign precedes an expression inside parentheses, remember to distribute the negative sign to each term inside the parentheses. In other words, multiply each term by -1.

$$-(3x^4 - 4x^2 + 3x) = -3x^4 + 4x^2 - 3x$$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the difference $(2x^3 - x + 3) - (x^2 - 2x - 3)$ and write the resulting polynomial in standard form.

To find the *product* of two polynomials, use the right and left Distributive Properties. For example, you can find the product of $3x - 2$ and $5x + 7$ by first treating $5x + 7$ as a single quantity.

$$\begin{aligned}(3x - 2)(5x + 7) &= 3x(5x + 7) - 2(5x + 7)\\ &= (3x)(5x) + (3x)(7) - (2)(5x) - (2)(7)\\ &= \underbrace{15x^2}_{\text{Product of First terms}} + \underbrace{21x}_{\text{Product of Outer terms}} - \underbrace{10x}_{\text{Product of Inner terms}} - \underbrace{14}_{\text{Product of Last terms}}\\ &= 15x^2 + 11x - 14\end{aligned}$$

Note that when using the **FOIL Method** above (which can be used only to multiply two binomials), some of the terms in the product may be like terms that can be combined into one term.

EXAMPLE 3 Finding a Product by the FOIL Method

Use the FOIL Method to find the product of $2x - 4$ and $x + 5$.

Solution

$$(2x - 4)(x + 5) = \overset{F}{2x^2} + \overset{O}{10x} - \overset{I}{4x} - \overset{L}{20} = 2x^2 + 6x - 20$$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use the FOIL Method to find the product of $3x - 1$ and $x - 5$.

When multiplying two polynomials, be sure to multiply *each* term of one polynomial by *each* term of the other. A vertical arrangement can be helpful.

EXAMPLE 4 A Vertical Arrangement for Multiplication

Multiply $x^2 - 2x + 2$ by $x^2 + 2x + 2$ using a vertical arrangement.

Solution

$$
\begin{array}{rl}
x^2 - 2x + 2 & \text{Write in standard form.}\\
\underline{\times\ x^2 + 2x + 2} & \text{Write in standard form.}\\
2x^2 - 4x + 4 \quad \Leftarrow & 2(x^2 - 2x + 2)\\
2x^3 - 4x^2 + 4x \qquad \Leftarrow & 2x(x^2 - 2x + 2)\\
\underline{x^4 - 2x^3 + 2x^2} \qquad\qquad \Leftarrow & x^2(x^2 - 2x + 2)\\
x^4 + 0x^3 + 0x^2 + 0x + 4 = x^4 + 4 & \text{Combine like terms.}
\end{array}
$$

So, $(x^2 - 2x + 2)(x^2 + 2x + 2) = x^4 + 4$.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Multiply $x^2 + 2x + 3$ by $x^2 - 2x + 3$ using a vertical arrangement.

Special Products

Some binomial products have special forms that occur frequently in algebra. You do not need to memorize these formulas because you can use the Distributive Property to multiply. However, becoming familiar with these formulas will enable you to manipulate the algebra more quickly.

Special Products

Let u and v be real numbers, variables, or algebraic expressions.

Special Product	**Example**
Sum and Difference of Same Terms	
$(u + v)(u - v) = u^2 - v^2$	$(x + 4)(x - 4) = x^2 - 4^2$ $= x^2 - 16$
Square of a Binomial	
$(u + v)^2 = u^2 + 2uv + v^2$	$(x + 3)^2 = x^2 + 2(x)(3) + 3^2$ $= x^2 + 6x + 9$
$(u - v)^2 = u^2 - 2uv + v^2$	$(3x - 2)^2 = (3x)^2 - 2(3x)(2) + 2^2$ $= 9x^2 - 12x + 4$
Cube of a Binomial	
$(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$	$(x + 2)^3 = x^3 + 3x^2(2) + 3x(2^2) + 2^3$ $= x^3 + 6x^2 + 12x + 8$
$(u - v)^3 = u^3 - 3u^2v + 3uv^2 - v^3$	$(x - 1)^3 = x^3 - 3x^2(1) + 3x(1^2) - 1^3$ $= x^3 - 3x^2 + 3x - 1$

EXAMPLE 5 Sum and Difference of Same Terms

Find the product of $5x + 9$ and $5x - 9$.

Solution

The product of a sum and a difference of the *same* two terms has no middle term and takes the form $(u + v)(u - v) = u^2 - v^2$.

$$(5x + 9)(5x - 9) = (5x)^2 - 9^2 = 25x^2 - 81$$

✓ ***Checkpoint*** ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Find the product of $3x - 2$ and $3x + 2$.

REMARK When squaring a binomial, note that the resulting middle term is always *twice* the product of the two terms of the binomial.

EXAMPLE 6 Square of a Binomial

Find $(6x - 5)^2$.

Solution

The square of the binomial $u - v$ is $(u - v)^2 = u^2 - 2uv + v^2$.

$$(6x - 5)^2 = (6x)^2 - 2(6x)(5) + 5^2 = 36x^2 - 60x + 25$$

✓ ***Checkpoint*** ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Find $(x + 10)^2$.

EXAMPLE 7 Cube of a Binomial

Find $(3x + 2)^3$.

Solution

The cube of the binomial $u + v$ is $(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$. Note the *decreasing* powers of u and the *increasing* powers of v. Letting $u = 3x$ and $v = 2$,

$$(3x + 2)^3 = (3x)^3 + 3(3x)^2(2) + 3(3x)(2^2) + 2^3 = 27x^3 + 54x^2 + 36x + 8.$$

✓ ***Checkpoint*** ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Find $(4x - 1)^3$.

EXAMPLE 8 Multiplying Two Trinomials

See LarsonPrecalculus.com for an interactive version of this type of example.

Find the product of $x + y - 2$ and $x + y + 2$.

Solution

One way to find this product is to group $x + y$ and form a special product.

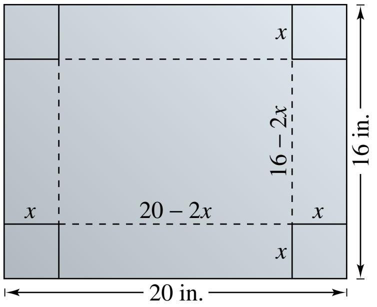
$$\begin{aligned}(x + y - 2)(x + y + 2) &= [(x + y) \overset{\substack{\text{Difference}\\ \downarrow}}{-} 2][(x + y) \overset{\substack{\text{Sum}\\ \downarrow}}{+} 2]\\ &= (x + y)^2 - 2^2 && \text{Sum and difference of same terms}\\ &= x^2 + 2xy + y^2 - 4 && \text{Square of a binomial}\end{aligned}$$

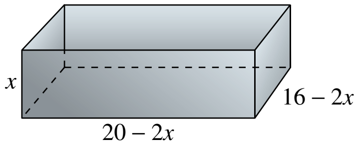
✓ ***Checkpoint*** ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Find the product of $x - 2 + 3y$ and $x - 2 - 3y$.

Application

EXAMPLE 9 Finding the Volume of a Box





An open box is made by cutting squares from the corners of a piece of metal that is 20 inches by 16 inches, as shown in the figure. The edge of each cut-out square is x inches. Find the volume of the box in terms of x. Then find the volume of the box when $x = 1$, $x = 2$, and $x = 3$.

Solution

The volume of a rectangular box is equal to the product of its length, width, and height. From the figure, the length is $20 - 2x$, the width is $16 - 2x$, and the height is x. So, the volume of the box is

$$\begin{aligned}\text{Volume} &= (20 - 2x)(16 - 2x)(x)\\ &= (320 - 72x + 4x^2)(x)\\ &= 320x - 72x^2 + 4x^3.\end{aligned}$$

When $x = 1$ inch, the volume of the box is

$$\begin{aligned}\text{Volume} &= 320(1) - 72(1)^2 + 4(1)^3\\ &= 252 \text{ cubic inches.}\end{aligned}$$

When $x = 2$ inches, the volume of the box is

$$\begin{aligned}\text{Volume} &= 320(2) - 72(2)^2 + 4(2)^3\\ &= 384 \text{ cubic inches.}\end{aligned}$$

When $x = 3$ inches, the volume of the box is

$$\begin{aligned}\text{Volume} &= 320(3) - 72(3)^2 + 4(3)^3\\ &= 420 \text{ cubic inches.}\end{aligned}$$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

In Example 9, find the volume of the box in terms of x when the piece of metal is 12 inches by 10 inches. Then find the volume when $x = 2$ and $x = 3$.

Summarize (Section P.3)

1. State the definition of a polynomial in x and explain what is meant by the standard form of a polynomial *(page 26)*. For an example of writing polynomials in standard form, see Example 1.
2. Explain how to add and subtract polynomials *(page 27)*. For an example of adding and subtracting polynomials, see Example 2.
3. Explain the FOIL Method *(page 27)*. For an example of finding a product using the FOIL Method, see Example 3.
4. Explain how to find binomial products that have special forms *(page 28)*. For examples of binomial products that have special forms, see Examples 5–8.

P.3 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. For the polynomial $a_nx^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_1x + a_0$, $a_n \neq 0$, the degree is ________, the leading coefficient is ________, and the constant term is ________.

2. A polynomial with one term is a ________, while a polynomial with two terms is a ________ and a polynomial with three terms is a ________.

3. To add or subtract polynomials, add or subtract the ________ ________ by adding or subtracting their coefficients.

4. The letters in "FOIL" stand for F ________, O ________, I ________, and L ________.

Skills and Applications

Writing Polynomials in Standard Form **In Exercises 5–10, (a) write the polynomial in standard form, (b) identify the degree and leading coefficient of the polynomial, and (c) state whether the polynomial is a monomial, a binomial, or a trinomial.**

5. $7x$

6. 3

7. $14x - \frac{1}{2}x^5$

8. $3 + 2x$

9. $1 + 6x^4 - 4x^5$

10. $-y + 25y^2 + 1$

Identifying Polynomials **In Exercises 11–16, determine whether the expression is a polynomial. If so, write the polynomial in standard form.**

11. $2x - 3x^3 + 8$

12. $5x^4 - 2x^2 + x^{-2}$

13. $\dfrac{3x + 4}{x}$

14. $\dfrac{x^2 + 2x - 3}{2}$

15. $y^2 - y^4 + y^3$

16. $y^4 - \sqrt{y}$

Adding or Subtracting Polynomials **In Exercises 17–24, add or subtract and write the result in standard form.**

17. $(6x + 5) - (8x + 15)$

18. $(2x^2 + 1) - (x^2 - 2x + 1)$

19. $(t^3 - 1) + (6t^3 - 5t)$

20. $(4y^2 - 3) + (-7y^2 + 9)$

21. $(15x^2 - 6) + (-8.3x^3 - 14.7x^2 - 17)$

22. $(15.6w^4 - 14w - 17.4) + (16.9w^4 - 9.2w + 13)$

23. $5z - [3z - (10z + 8)]$

24. $(y^3 + 1) - [(y^2 + 1) + (3y - 7)]$

Multiplying Polynomials **In Exercises 25–38, multiply the polynomials.**

25. $3x(x^2 - 2x + 1)$

26. $y^2(4y^2 + 2y - 3)$

27. $-5z(3z - 1)$

28. $-3x(5x + 2)$

29. $(1.5t^2 + 5)(-3t)$

30. $(2 - 3.5y)(2y^3)$

31. $-2x(0.1x + 17)$

32. $6y\left(5 - \frac{3}{8}y\right)$

33. $(x + 7)(x + 5)$

34. $(x - 8)(x + 4)$

35. $(3x - 5)(2x + 1)$

36. $(7x - 2)(4x - 3)$

37. $(x^2 - x + 2)(x^2 + x + 1)$

38. $(2x^2 - x + 4)(x^2 + 3x + 2)$

Finding Special Products **In Exercises 39–62, find the special product.**

39. $(x + 10)(x - 10)$

40. $(2x + 3)(2x - 3)$

41. $(x + 2y)(x - 2y)$

42. $(4a + 5b)(4a - 5b)$

43. $(2x + 3)^2$

44. $(5 - 8x)^2$

45. $(4x^3 - 3)^2$

46. $(8x + 3)^2$

47. $(x + 3)^3$

48. $(x - 2)^3$

49. $(2x - y)^3$

50. $(3x + 2y)^3$

51. $\left(\frac{1}{5}x - 3\right)\left(\frac{1}{5}x + 3\right)$

52. $(1.5x - 4)(1.5x + 4)$

53. $(-6x + 3y)(-6x - 3y)$

54. $(3a^3 - 4b^2)(3a^3 + 4b^2)$

55. $\left(\frac{1}{4}x - 5\right)^2$

56. $(2.4x + 3)^2$

57. $[(x - 3) + y]^2$

58. $[(x + 1) - y]^2$

59. $[(m - 3) + n][(m - 3) - n]$

60. $[(x - 3y) + z][(x - 3y) - z]$

61. $(u + 2)(u - 2)(u^2 + 4)$

62. $(x + y)(x - y)(x^2 + y^2)$

Operations with Polynomials **In Exercises 63–66, perform the operation.**

63. Subtract $4x^2 - 5$ from $-3x^3 + x^2 + 9$.

64. Subtract $-7t^4 + 5t^2 - 1$ from $2t^4 - 10t^3 - 4t$.

65. Multiply $y^2 + 3y - 5$ by $y^2 - 6y + 4$.

66. Multiply $x^2 + 4x - 1$ by $x^2 - x + 3$.

Finding a Product **In Exercises 67–70, find the product. (The expressions are not polynomials, but the formulas can still be used.)**

67. $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$ **68.** $(5 + \sqrt{x})(5 - \sqrt{x})$

69. $(x - \sqrt{5})^2$ **70.** $(x + \sqrt{3})^2$

71. **Cost, Revenue, and Profit** An electronics manufacturer can produce and sell x MP3 players per week. The total cost C (in dollars) of producing x MP3 players is $C = 93x + 35{,}000$, and the total revenue R (in dollars) is $R = 135x$.

(a) Find the profit P in terms of x.

(b) Find the profit obtained by selling 5000 MP3 players per week.

72. **Compound Interest** An investment of \$500 compounded annually for 2 years at an interest rate r (in decimal form) yields an amount of $500(1 + r)^2$.

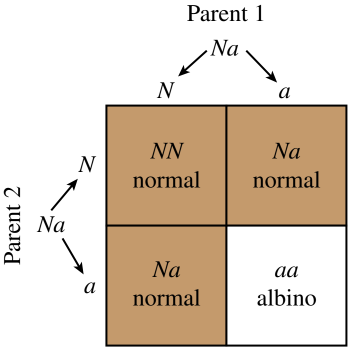
(a) Write this polynomial in standard form.

(b) Use a calculator to evaluate the polynomial for the values of r given in the table.

r	$2\frac{1}{2}\%$	3%	4%	$4\frac{1}{2}\%$	5%
$500(1 + r)^2$					

(c) What conclusion can you make from the table?

73. **Genetics** In deer, the gene N is for normal coloring and the gene a is for albino. Any gene combination with an N results in normal coloring. The Punnett square shows the possible gene combinations of an offspring and the resulting colors when both parents have the gene combination Na.



(a) What percent of the possible gene combinations result in albino coloring?

(b) Each parent's gene combination is represented by the polynomial $0.5N + 0.5a$. The product $(0.5N + 0.5a)^2$ represents the possible gene combinations of an offspring. Find this product.

(c) The coefficient of each term of the polynomial you wrote in part (b) is the probability (in decimal form) of the offspring having that gene combination. Use this polynomial to confirm your answer in part (a). Explain.

74. **Construction Management** A square-shaped foundation for a building with 100-foot sides is reduced by x feet on one side and extended by x feet on an adjacent side.

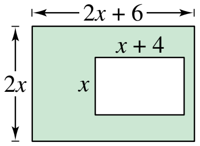
(a) The area of the new foundation is represented by $(100 - x)(100 + x)$. Find this product.

(b) Does the area of the foundation increase, decrease, or stay the same? Explain.

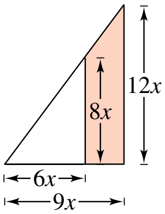
(c) Use the polynomial in part (a) to find the area of the new foundation when $x = 21$.

Geometry **In Exercises 75–78, find the area of the shaded region in terms of x. Write your result as a polynomial in standard form.**

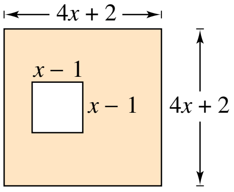
75.



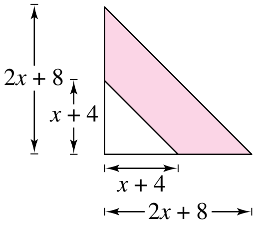
76.



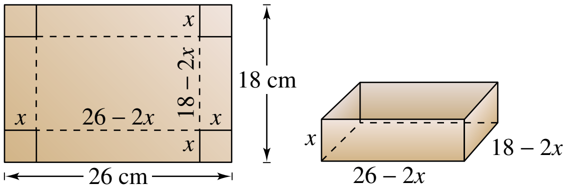
77.



78.



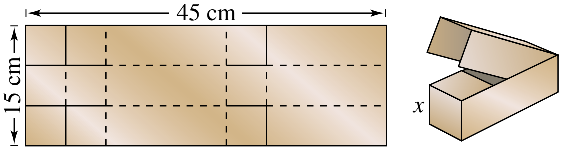
79. **Volume of a Box** A take-out fast-food restaurant is constructing an open box by cutting squares from the corners of the piece of cardboard shown in the figure. The edge of each cut-out square is x centimeters.



(a) Find the volume of the box in terms of x.

(b) Find the volume when $x = 1$, $x = 2$, and $x = 3$.

80. **Volume of a Box** An overnight shipping company designs a closed box by cutting along the solid lines and folding along the broken lines on the rectangular piece of corrugated cardboard shown in the figure.



(a) Find the volume of the shipping box in terms of x.

(b) Find the volume when $x = 3$, $x = 5$, and $x = 7$.

81. Engineering

A one-inch-wide steel beam has a uniformly distributed load. When the span of the beam is x feet and its depth is 6 inches, the safe load S (in pounds) is approximately $S_6 = (0.06x^2 - 2.42x + 38.71)^2$. When the depth is 8 inches, the safe load is approximately $S_8 = (0.08x^2 - 3.30x + 51.93)^2$.

(a) Approximate the difference of the safe loads for these two beams when the span is 12 feet.

(b) How does the difference of the safe loads change as the span increases?

82. Stopping Distance The stopping distance of an automobile is the distance traveled during the driver's reaction time plus the distance traveled after the driver applies the brakes. In an experiment, researchers measured these distances (in feet) when the automobile was traveling at a speed of x miles per hour on dry, level pavement, as shown in the bar graph. The distance traveled during the reaction time R was

$R = 1.1x$

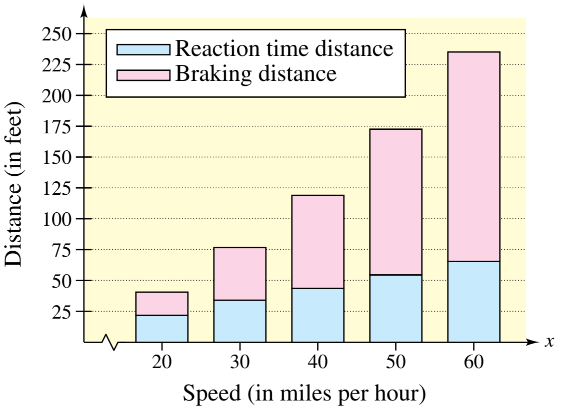
and the braking distance B was

$B = 0.0475x^2 - 0.001x + 0.23.$

(a) Determine the polynomial that represents the total stopping distance T.

(b) Use the result of part (a) to estimate the total stopping distance when $x = 30$, $x = 40$, and $x = 55$ miles per hour.

(c) Use the bar graph to make a statement about the total stopping distance required for increasing speeds.



iStockphoto.com/Photosbyjim

Exploration

True or False? **In Exercises 83–86, determine whether the statement is true or false. Justify your answer.**

83. The product of two binomials is always a second-degree polynomial.

84. The sum of two second-degree polynomials is always a second-degree polynomial.

85. The sum of two binomials is always a binomial.

86. The leading coefficient of the product of two polynomials is always the product of the leading coefficients of the two polynomials.

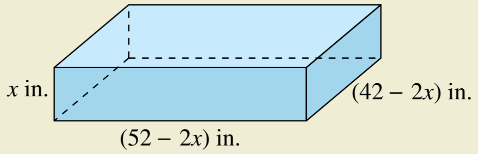
87. Degree of a Product Find the degree of the product of two polynomials of degrees m and n.

88. Degree of a Sum Find the degree of the sum of two polynomials of degrees m and n, where $m < n$.

89. Error Analysis Describe the error.

$(x - 3)^2 = x^2 + 9$ ✗

90. **HOW DO YOU SEE IT?** An open box has a length of $(52 - 2x)$ inches, a width of $(42 - 2x)$ inches, and a height of x inches, as shown.



(a) Describe a way that you could make the box from a rectangular piece of cardboard. Give the original dimensions of the cardboard.

(b) What degree is the polynomial that represents the volume of the box? Explain your reasoning.

(c) Describe a procedure for finding the value of x (to the nearest tenth of an inch) that yields the maximum possible volume of the box.

91. Think About It When the polynomial

$-x^3 + 3x^2 + 2x - 1$

is subtracted from an unknown polynomial, the difference is $5x^2 + 8$. Find the unknown polynomial.

92. Logical Reasoning Verify that $(x + y)^2$ is not equal to $x^2 + y^2$ by letting $x = 3$ and $y = 4$ and evaluating both expressions. Are there any values of x and y for which $(x + y)^2$ and $x^2 + y^2$ are equal? Explain.

P.4 Factoring Polynomials

Polynomial factoring has many real-life applications. For example, in Exercise 84 on page 40, you will use polynomial factoring to write an alternative form of an expression that models the rate of change of an autocatalytic chemical reaction.

- **Factor out common factors from polynomials.**
- **Factor special polynomial forms.**
- **Factor trinomials as the product of two binomials.**
- **Factor polynomials by grouping.**

Polynomials with Common Factors

The process of writing a polynomial as a product is called **factoring.** It is an important tool for solving equations and for simplifying rational expressions.

Unless noted otherwise, when you are asked to factor a polynomial, assume that you are looking for factors that have integer coefficients. If a polynomial does not factor using integer coefficients, then it is **prime** or **irreducible over the integers.** For example, the polynomial $x^2 - 3$ is irreducible over the integers. Over the *real numbers,* this polynomial factors as

$$x^2 - 3 = (x + \sqrt{3})(x - \sqrt{3}).$$

A polynomial is **completely factored** when each of its factors is prime. For example,

$$x^3 - x^2 + 4x - 4 = (x - 1)(x^2 + 4) \qquad \text{Completely factored}$$

is completely factored, but

$$x^3 - x^2 - 4x + 4 = (x - 1)(x^2 - 4) \qquad \text{Not completely factored}$$

is not completely factored. Its complete factorization is

$$x^3 - x^2 - 4x + 4 = (x - 1)(x + 2)(x - 2).$$

The simplest type of factoring involves a polynomial that can be written as the product of a monomial and another polynomial. The technique used here is the Distributive Property, $a(b + c) = ab + ac$, in the *reverse* direction.

$$ab + ac = a(b + c) \qquad a \text{ is a common factor.}$$

Factoring out any common factors is the first step in completely factoring a polynomial.

EXAMPLE 1 Factoring Out Common Factors

Factor each expression.

a. $6x^3 - 4x$ **b.** $-4x^2 + 12x - 16$ **c.** $(x - 2)(2x) + (x - 2)(3)$

Solution

a. $6x^3 - 4x = 2x(3x^2) - 2x(2)$ $\qquad$ $2x$ is a common factor.

$\qquad = 2x(3x^2 - 2)$

b. $-4x^2 + 12x - 16 = -4(x^2) + (-4)(-3x) + (-4)4$ $\qquad$ -4 is a common factor.

$\qquad = -4(x^2 - 3x + 4)$

c. $(x - 2)(2x) + (x - 2)(3) = (x - 2)(2x + 3)$ $\qquad$ $(x - 2)$ is a common factor.

✓ **Checkpoint** Audio-video solution in English & Spanish at LarsonPrecalculus.com

Factor each expression.

a. $5x^3 - 15x^2$ **b.** $-3 + 6x - 12x^3$ **c.** $(x + 1)(x^2) - (x + 1)(2)$

Suwit Ngaokaew/Shutterstock.com

Factoring Special Polynomial Forms

Some polynomials have special forms that arise from the special product forms on page 28. You should learn to recognize these forms.

Factoring Special Polynomial Forms

Factored Form	**Example**
Difference of Two Squares	
$u^2 - v^2 = (u + v)(u - v)$	$9x^2 - 4 = (3x)^2 - 2^2 = (3x + 2)(3x - 2)$
Perfect Square Trinomial	
$u^2 + 2uv + v^2 = (u + v)^2$	$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2 = (x + 3)^2$
$u^2 - 2uv + v^2 = (u - v)^2$	$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2 = (x - 3)^2$
Sum or Difference of Two Cubes	
$u^3 + v^3 = (u + v)(u^2 - uv + v^2)$	$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$
$u^3 - v^3 = (u - v)(u^2 + uv + v^2)$	$27x^3 - 1 = (3x)^3 - 1^3 = (3x - 1)(9x^2 + 3x + 1)$

The factored form of the difference of two squares is always a set of **conjugate pairs.**

$$u^2 \underset{\text{Difference}}{\underset{\uparrow}{-}} v^2 = \underbrace{(u \underset{\uparrow}{+} v)(u \underset{\uparrow}{-} v)}_{\text{Opposite signs}} \qquad \text{Conjugate pairs}$$

To recognize perfect square terms, look for coefficients that are squares of integers and variables raised to *even powers.*

EXAMPLE 2 Factoring Out a Common Factor First

$$\begin{aligned} 3 - 12x^2 &= 3(1 - 4x^2) && \text{3 is a common factor.} \\ &= 3[1^2 - (2x)^2] && \text{Rewrite } 1 - 4x^2 \text{ as the difference of two squares.} \\ &= 3(1 + 2x)(1 - 2x) && \text{Factor.} \end{aligned}$$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Factor $100 - 4y^2$.

REMARK In Example 2, note that the first step in factoring a polynomial is to check for any common factors. Once you have removed any common factors, it is often possible to recognize patterns that were not immediately obvious.

EXAMPLE 3 Factoring the Difference of Two Squares

a. $(x + 2)^2 - y^2 = [(x + 2) + y][(x + 2) - y]$

$$= (x + 2 + y)(x + 2 - y)$$

b.

$$\begin{aligned} 16x^4 - 81 &= (4x^2)^2 - 9^2 && \text{Rewrite as the difference of two squares.} \\ &= (4x^2 + 9)(4x^2 - 9) && \text{Factor.} \\ &= (4x^2 + 9)[(2x)^2 - 3^2] && \text{Rewrite } 4x^2 - 9 \text{ as the difference of two squares.} \\ &= (4x^2 + 9)(2x + 3)(2x - 3) && \text{Factor.} \end{aligned}$$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Factor $(x - 1)^2 - 9y^4$.

A **perfect square trinomial** is the square of a binomial, and it has the form

$$u^2 + 2uv + v^2 = (u + v)^2 \quad \text{or} \quad u^2 - 2uv + v^2 = (u - v)^2.$$

Like signs Like signs

Note that the first and last terms are squares and the middle term is twice the product of u and v.

EXAMPLE 4 Factoring Perfect Square Trinomials

Factor each trinomial.

a. $x^2 - 10x + 25$ **b.** $16x^2 + 24x + 9$

Solution

a. $x^2 - 10x + 25 = x^2 - 2(x)(5) + 5^2 = (x - 5)^2$

b. $16x^2 + 24x + 9 = (4x)^2 + 2(4x)(3) + 3^2 = (4x + 3)^2$

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Factor $9x^2 - 30x + 25$.

The next two formulas show the sum and difference of two cubes. Pay special attention to the signs of the terms.

Like signs Like signs

$$u^3 + v^3 = (u + v)(u^2 - uv + v^2) \qquad u^3 - v^3 = (u - v)(u^2 + uv + v^2)$$

Unlike signs Unlike signs

EXAMPLE 5 Factoring the Difference of Two Cubes

$x^3 - 27 = x^3 - 3^3$ Rewrite 27 as 3^3.

$= (x - 3)(x^2 + 3x + 9)$ Factor.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Factor $64x^3 - 1$.

EXAMPLE 6 Factoring the Sum of Two Cubes

a. $y^3 + 8 = y^3 + 2^3$ Rewrite 8 as 2^3.

$= (y + 2)(y^2 - 2y + 4)$ Factor.

b. $3x^3 + 192 = 3(x^3 + 64)$ 3 is a common factor.

$= 3(x^3 + 4^3)$ Rewrite 64 as 4^3.

$= 3(x + 4)(x^2 - 4x + 16)$ Factor.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Factor each expression.

a. $x^3 + 216$ **b.** $5y^3 + 135$

Trinomials with Binomial Factors

To factor a trinomial of the form $ax^2 + bx + c$, use the pattern below.

$$ax^2 + bx + c = (\overset{\substack{\text{Factors of } a\\ \downarrow}}{\square}x + \square)(\overset{\substack{\text{Factors of } a\\ \downarrow}}{\square}x + \underset{\substack{\uparrow\\ \text{Factors of } c}}{\square})$$

The goal is to find a combination of factors of a and c such that the sum of the outer and inner products is the middle term bx. For example, for the trinomial $6x^2 + 17x + 5$, you can write all possible factorizations and determine which one has outer and inner products whose sum is $17x$.

$$(6x + 5)(x + 1),\ (6x + 1)(x + 5),\ (2x + 1)(3x + 5),\ (2x + 5)(3x + 1)$$

The correct factorization is $(2x + 5)(3x + 1)$ because the sum of the outer (O) and inner (I) products is $17x$.

$$(2x + 5)(3x + 1) = \overset{\substack{\text{F}\\ \downarrow}}{6x^2} + \overset{\substack{\text{O}\\ \downarrow}}{2x} + \overset{\substack{\text{I}\\ \downarrow}}{15x} + \overset{\substack{\text{L}\\ \downarrow}}{5} = 6x^2 + \overset{\substack{\text{O + I}\\ \downarrow}}{17x} + 5$$

REMARK Factoring a trinomial can involve trial and error. However, it is relatively easy to check your answer by multiplying the factors. The product should be the original trinomial. For instance, in Example 7, verify that $(x - 3)(x - 4) = x^2 - 7x + 12$.

EXAMPLE 7 Factoring a Trinomial: Leading Coefficient Is 1

Factor $x^2 - 7x + 12$.

Solution For this trinomial, $a = 1$, $b = -7$, and $c = 12$. Because b is negative and c is positive, both factors of 12 must be negative. So, the possible factorizations of $x^2 - 7x + 12$ are

$$(x - 1)(x - 12), \quad (x - 2)(x - 6), \quad \text{and} \quad (x - 3)(x - 4).$$

Testing the middle term, you will find the correct factorization to be

$$x^2 - 7x + 12 = (x - 3)(x - 4). \qquad \text{O + I} = -4x - 3x = -7x$$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Factor $x^2 + x - 6$.

EXAMPLE 8 Factoring a Trinomial: Leading Coefficient Is Not 1

See LarsonPrecalculus.com for an interactive version of this type of example.

Factor $2x^2 + x - 15$.

Solution For this trinomial, $a = 2$, $b = 1$, and $c = -15$. Because c is negative, its factors must have unlike signs. The eight possible factorizations are below.

$$(2x - 1)(x + 15) \quad (2x + 1)(x - 15) \quad (2x - 3)(x + 5) \quad (2x + 3)(x - 5)$$
$$(2x - 5)(x + 3) \quad (2x + 5)(x - 3) \quad (2x - 15)(x + 1) \quad (2x + 15)(x - 1)$$

Testing the middle term, you will find the correct factorization to be

$$2x^2 + x - 15 = (2x - 5)(x + 3). \qquad \text{O + I} = 6x - 5x = x$$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Factor $2x^2 - 5x + 3$.

Factoring by Grouping

Sometimes, polynomials with more than three terms can be **factored by grouping.**

EXAMPLE 9 Factoring by Grouping

$$\begin{aligned} x^3 - 2x^2 - 3x + 6 &= (x^3 - 2x^2) - (3x - 6) && \text{Group terms.} \\ &= x^2(x - 2) - 3(x - 2) && \text{Factor each group.} \\ &= (x - 2)(x^2 - 3) && (x - 2) \text{ is a common factor.} \end{aligned}$$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Factor $x^3 + x^2 - 5x - 5$. ■

REMARK Sometimes, more than one grouping will work. For instance, another way to factor the polynomial in Example 9 is

$$\begin{aligned} &x^3 - 2x^2 - 3x + 6 \\ &\quad = (x^3 - 3x) - (2x^2 - 6) \\ &\quad = x(x^2 - 3) - 2(x^2 - 3) \\ &\quad = (x^2 - 3)(x - 2). \end{aligned}$$

Notice that this is the same result as in Example 9.

Factoring by grouping can eliminate some of the trial and error involved in factoring a trinomial. To factor a trinomial of the form $ax^2 + bx + c$ by grouping, choose factors of the product ac that sum to b and use these factors to rewrite the middle term. Example 10 illustrates this technique.

EXAMPLE 10 Factoring a Trinomial by Grouping

In the trinomial $2x^2 + 5x - 3$, $a = 2$ and $c = -3$, so the product ac is -6. Now, -6 factors as $(6)(-1)$ and $6 + (-1) = 5 = b$. So, rewrite the middle term as $5x = 6x - x$ and factor by grouping.

$$\begin{aligned} 2x^2 + 5x - 3 &= 2x^2 + 6x - x - 3 && \text{Rewrite middle term.} \\ &= (2x^2 + 6x) - (x + 3) && \text{Group terms.} \\ &= 2x(x + 3) - (x + 3) && \text{Factor } 2x^2 + 6x. \\ &= (x + 3)(2x - 1) && (x + 3) \text{ is a common factor.} \end{aligned}$$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use factoring by grouping to factor $2x^2 + 5x - 12$. ■

Guidelines for Factoring Polynomials

1. Factor out any common factors using the Distributive Property.
2. Factor according to one of the special polynomial forms.
3. Factor as $ax^2 + bx + c = (mx + r)(nx + s)$.
4. Factor by grouping.

Summarize (Section P.4)

1. Explain what it means to completely factor a polynomial *(page 34)*. For an example of factoring out common factors, see Example 1.
2. Make a list of the special polynomial forms of factoring *(page 35)*. For examples of factoring these special forms, see Examples 2–6.
3. Explain how to factor a trinomial of the form $ax^2 + bx + c$ *(page 37)*. For examples of factoring trinomials of this form, see Examples 7 and 8.
4. Explain how to factor a polynomial by grouping *(page 38)*. For examples of factoring by grouping, see Examples 9 and 10.

P.4 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. The process of writing a polynomial as a product is called ________.
2. A polynomial is ________ ________ when each of its factors is prime.
3. A ________ ________ ________ is the square of a binomial, and it has the form $u^2 + 2uv + v^2$ or $u^2 - 2uv + v^2$.
4. Sometimes, polynomials with more than three terms can be factored by ________.

Skills and Applications

Factoring Out a Common Factor

In Exercises 5–8, factor out the common factor.

5. $2x^3 - 6x$ **6.** $3z^3 - 6z^2 + 9z$

7. $3x(x - 5) + 8(x - 5)$ **8.** $(x + 3)^2 - 4(x + 3)$

Factoring the Difference of Two Squares

In Exercises 9–18, completely factor the difference of two squares.

9. $x^2 - 81$ **10.** $x^2 - 64$

11. $25y^2 - 4$ **12.** $4y^2 - 49$

13. $64 - 9z^2$ **14.** $81 - 36z^2$

15. $(x - 1)^2 - 4$ **16.** $25 - (z + 5)^2$

17. $81u^4 - 1$ **18.** $x^4 - 16y^4$

Factoring a Perfect Square Trinomial

In Exercises 19–24, factor the perfect square trinomial.

19. $x^2 - 4x + 4$ **20.** $4t^2 + 4t + 1$

21. $25z^2 - 30z + 9$ **22.** $36y^2 + 84y + 49$

23. $4y^2 - 12y + 9$ **24.** $9u^2 + 24uv + 16v^2$

Factoring the Sum or Difference of Two Cubes

In Exercises 25–32, factor the sum or difference of two cubes.

25. $x^3 - 8$ **26.** $x^3 + 125$

27. $8t^3 - 1$ **28.** $27z^3 + 1$

29. $27x^3 + 8$ **30.** $64y^3 - 125$

31. $u^3 + 27v^3$ **32.** $(x + 2)^3 - y^3$

Factoring a Trinomial

In Exercises 33–42, factor the trinomial.

33. $x^2 + x - 2$ **34.** $x^2 + 5x + 6$

35. $s^2 - 5s + 6$ **36.** $t^2 - t - 6$

37. $3x^2 + 10x - 8$ **38.** $2x^2 - 3x - 27$

39. $5x^2 + 31x + 6$ **40.** $8x^2 + 51x + 18$

41. $-5y^2 - 8y + 4$ **42.** $-6z^2 + 17z + 3$

Factoring by Grouping

In Exercises 43–48, factor by grouping.

43. $x^3 - x^2 + 2x - 2$ **44.** $x^3 + 5x^2 - 5x - 25$

45. $2x^3 - x^2 - 6x + 3$ **46.** $3x^3 + x^2 - 15x - 5$

47. $3x^5 + 6x^3 - 2x^2 - 4$

48. $8x^5 - 6x^2 + 12x^3 - 9$

Factoring a Trinomial by Grouping

In Exercises 49–52, factor the trinomial by grouping.

49. $2x^2 + 9x + 9$ **50.** $6x^2 + x - 2$

51. $6x^2 - x - 15$ **52.** $12x^2 - 13x + 1$

Factoring Completely

In Exercises 53–70, completely factor the expression.

53. $6x^2 - 54$ **54.** $12x^2 - 48$

55. $x^3 - x^2$ **56.** $x^3 - 16x$

57. $1 - 4x + 4x^2$ **58.** $-9x^2 + 6x - 1$

59. $2x^2 + 4x - 2x^3$ **60.** $9x^2 + 12x - 3x^3$

61. $(x^2 + 3)^2 - 16x^2$ **62.** $(x^2 + 8)^2 - 36x^2$

63. $2x^3 + x^2 - 8x - 4$

64. $3x^3 + x^2 - 27x - 9$

65. $2x(3x + 1) + (3x + 1)^2$

66. $4x(2x - 1) + (2x - 1)^2$

67. $2(x - 2)(x + 1)^2 - 3(x - 2)^2(x + 1)$

68. $2(x + 1)(x - 3)^2 - 3(x + 1)^2(x - 3)$

69. $5(2x + 1)^2(x + 1)^2 + (2x + 1)(x + 1)^3$

70. $7(3x + 2)^2(1 - x)^2 + (3x + 2)(1 - x^3)$

Fractional Coefficients

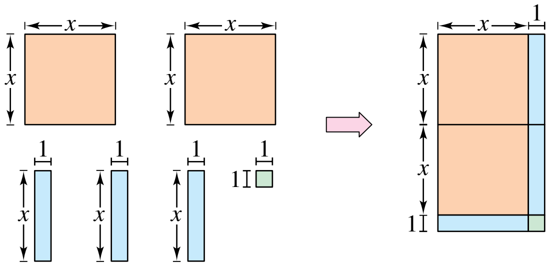
In Exercises 71–76, completely factor the expression. (*Hint:* The factors will contain fractional coefficients.)

71. $16x^2 - \frac{1}{9}$ **72.** $\frac{4}{25}y^2 - 64$

73. $z^2 + z + \frac{1}{4}$ **74.** $9y^2 - \frac{3}{2}y + \frac{1}{16}$

75. $y^3 + \frac{8}{27}$ **76.** $x^3 - \frac{27}{64}$

Geometric Modeling **In Exercises 77–80, draw a "geometric factoring model" to represent the factorization. For example, a factoring model for $2x^2 + 3x + 1 = (2x + 1)(x + 1)$ is shown below.**



77. $x^2 + 3x + 2 = (x + 2)(x + 1)$

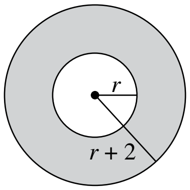
78. $x^2 + 4x + 3 = (x + 3)(x + 1)$

79. $2x^2 + 7x + 3 = (2x + 1)(x + 3)$

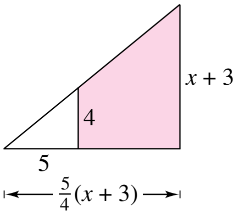
80. $3x^2 + 7x + 2 = (3x + 1)(x + 2)$

Geometry **In Exercises 81 and 82, write an expression in factored form for the area of the shaded portion of the figure.**

81.

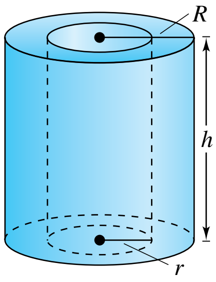


82.



83. Geometry The cylindrical shell shown in the figure has a volume of

$$V = \pi R^2 h - \pi r^2 h.$$



(a) Factor the expression for the volume.

(b) From the result of part (a), show that the volume is

$$2\pi(\text{average radius})(\text{thickness of the shell})h.$$

84. Chemistry The rate of change of an autocatalytic chemical reaction is

$$kQx - kx^2$$

where Q is the amount of the original substance, x is the amount of substance formed, and k is a constant of proportionality. Factor the expression.

Suwit Ngaokaew/Shutterstock.com

Factoring a Trinomial **In Exercises 85 and 86, find all values of b for which the trinomial is factorable.**

85. $x^2 + bx - 15$

86. $x^2 + bx + 24$

Factoring a Trinomial **In Exercises 87 and 88, find two integer values of c such that the trinomial is factorable. (There are many correct answers.)**

87. $2x^2 + 5x + c$

88. $3x^2 - x + c$

Exploration

True or False? **In Exercises 89 and 90, determine whether the statement is true or false. Justify your answer.**

89. The difference of two perfect squares can be factored as the product of conjugate pairs.

90. A perfect square trinomial can always be factored as the square of a binomial.

91. Error Analysis Describe the error.

$$9x^2 - 9x - 54 = (3x + 6)(3x - 9)$$
$$= 3(x + 2)(x - 3)$$

92. Think About It Is $(3x - 6)(x + 1)$ completely factored? Explain.

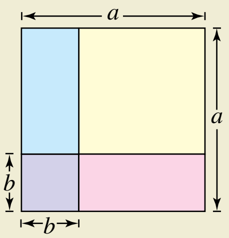
Factoring with Variables in the Exponents **In Exercises 93 and 94, factor the expression as completely as possible.**

93. $x^{2n} - y^{2n}$

94. $x^{3n} + y^{3n}$

95. Think About It Give an example of a polynomial that is prime.

96. HOW DO YOU SEE IT? The figure shows a large square with an area of a^2 that contains a smaller square with an area of b^2.



(a) Describe the regions that represent $a^2 - b^2$. How can you rearrange these regions to show that $a^2 - b^2 = (a - b)(a + b)$?

(b) How can you use the figure to show that $(a - b)^2 = a^2 - 2ab + b^2$?

(c) Draw another figure to show that $(a + b)^2 = a^2 + 2ab + b^2$. Explain how the figure shows this.

97. Difference of Two Sixth Powers Rewrite $u^6 - v^6$ as the difference of two squares. Then find a formula for completely factoring $u^6 - v^6$. Use your formula to completely factor $x^6 - 1$ and $x^6 - 64$.