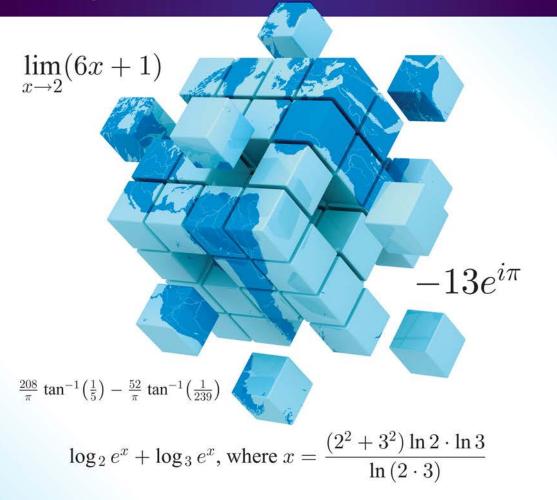
# **PRECALCULUS** Functions & Graphs | 13e

Earl W. Swokowski Jeffery A. Cole



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#### **ALGEBRA**

#### **QUADRATIC FORMULA**

If  $a \neq 0$ , the roots of  $ax^2 + bx + c = 0$  are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(x + y)(x - y) = x^{2} - y^{2}$$
  

$$(x + y)^{2} = x^{2} + 2xy + y^{2}$$
  

$$(x - y)^{2} = x^{2} - 2xy + y^{2}$$
  

$$(x + y)^{3} = x^{3} + 3x^{2}y + 3xy^{2} + y^{3}$$
  

$$(x - y)^{3} = x^{3} - 3x^{2}y + 3xy^{2} - y^{3}$$

#### SPECIAL FACTORING FORMULAS

$$x^{2} - y^{2} = (x + y)(x - y)$$

$$x^{2} + 2xy + y^{2} = (x + y)^{2}$$

$$x^{2} - 2xy + y^{2} = (x - y)^{2}$$

$$x^{3} - y^{3} = (x - y)(x^{2} + xy + y^{2})$$

$$x^{3} + y^{3} = (x + y)(x^{2} - xy + y^{2})$$

#### **EXPONENTS AND RADICALS**

$a^m a^n = a^{m+n}$	$a^{1/n} = \sqrt[n]{a}$
$(a^m)^n = a^{mn}$	$a^{m/n} = \sqrt[n]{a^m}$
$(ab)^n = a^n b^n$	$a^{m/n} = (\sqrt[n]{a})^m$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$
$\frac{a^m}{a^n}=a^{m-n}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
$a^{-n} = \frac{1}{a^n}$	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

# **BINOMIAL THEOREM**

SEQUENCES

sequence

$$(x + y)^{n} = x^{n} + {\binom{n}{1}}x^{n-1}y + {\binom{n}{2}}x^{n-2}y^{2} + \cdots + {\binom{n}{k}}x^{n-k}y^{k} + \cdots + y^{n},$$
  
where  
$${\binom{n}{k}} = \frac{n!}{k!(n-k)!}$$

#### **INEQUALITIES**

If a > b and b > c, then a > cIf a > b, then a + c > b + cIf a > b and c > 0, then ac > bcIf a > b and c < 0, then ac < bc

### ABSOLUTE VALUE (d > 0)

|x| < d if and only if -d < x < d

MEANS

|x| > d if and only if either

$$x > d$$
 or  $x < -d$ 

Arithmetic mean A of n numbers

Geometric mean G of n numbers  $G = (a_1 a_2 \cdots a_n)^{1/n}, a_k > 0$ 

 $A = \frac{a_1 + a_2 + \dots + a_n}{n}$ 

$$> d$$
 or  $x < -d$ 

d or 
$$x < -d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

*n*th term of an arithmetic sequence with first

 $a_n = a_1 + (n-1)d$ 

Sum  $S_n$  of the first *n* terms of an arithmetic

term  $a_1$  and common difference d

or 
$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

*n*th term of a geometric sequence with first term  $a_1$  and common ratio r

$$a_n = a_1 r^{n-1}$$

Sum  $S_n$  of the first *n* terms of a geometric sequence

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

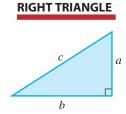
#### **EXPONENTIALS AND LOGARITHMS**

$$y = \log_{a} x \quad \text{means} \quad a^{y} = x$$
$$\log_{a} xy = \log_{a} x + \log_{a} y$$
$$\log_{a} \frac{x}{y} = \log_{a} x - \log_{a} y$$
$$\log_{a} x^{r} = r \log_{a} x$$
$$a^{\log_{a} x} = x$$
$$\log_{a} a^{x} = x$$
$$\log_{a} a = 1$$
$$\log x = \log_{10} x$$
$$\ln x = \log_{e} x$$
$$\log_{b} u = \frac{\log_{a} u}{\log_{a} b}$$

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# FORMULAS FROM GEOMETRY

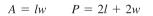
area A perimeter P circumference C volume V curved surface area S altitude h radius r

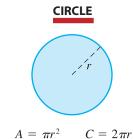






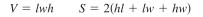




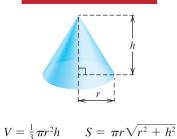


**RECTANGULAR BOX** 

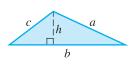




#### **RIGHT CIRCULAR CONE**

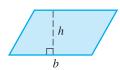


TRIANGLE



$$A = \frac{1}{2}bh \qquad P = a + b + c$$

PARALLELOGRAM



A = bh

#### **CIRCULAR SECTOR**



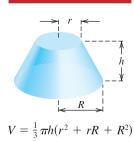
$$A = \frac{1}{2}r^2\theta \qquad s = r\theta$$

SPHERE

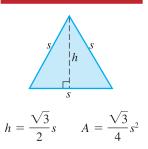


 $V = \frac{4}{3}\pi r^3 \qquad S = 4\pi r^2$ 

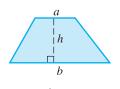
#### FRUSTUM OF A CONE



EQUILATERAL TRIANGLE

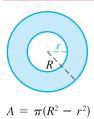




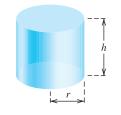




#### **CIRCULAR RING**

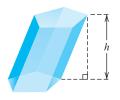


#### **RIGHT CIRCULAR CYLINDER**





PRISM



#### V = Bh with B the area of the base

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### **ANALYTIC GEOMETRY**

#### **DISTANCE FORMULA**

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$P_1(x_1, y_1)$$

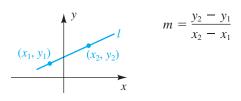
$$P_2(x_2, y_2)$$

$$x$$

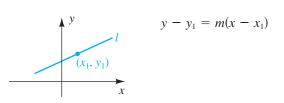
### EQUATION OF A CIRCLE

$$(x - h)^2 + (y - k)^2 = r^2$$

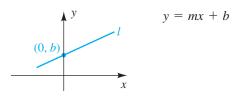
#### SLOPE *m* OF A LINE



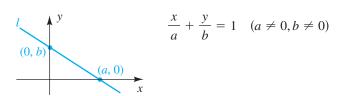
#### POINT-SLOPE FORM OF A LINE



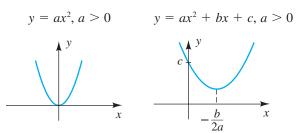
#### **SLOPE-INTERCEPT FORM OF A LINE**



#### **INTERCEPT FORM OF A LINE**



#### **GRAPH OF A QUADRATIC FUNCTION**



#### CONSTANTS

 $\pi \approx 3.14159$  $e \approx 2.71828$ 

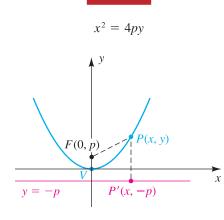
#### CONVERSIONS

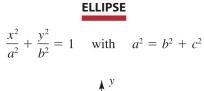
- 1 centimeter  $\approx 0.3937$  inch
- 1 meter  $\approx 3.2808$  feet
- 1 kilometer  $\approx 0.6214$  mile
- 1 gram  $\approx 0.0353$  ounce
- 1 kilogram  $\approx 2.2046$  pounds
- 1 liter  $\approx 0.2642$  gallon
- 1 milliliter  $\approx 0.0381$  fluid ounce
- 1 joule  $\approx 0.7376$  foot-pound
- 1 newton  $\approx 0.2248$  pound
- 1 lumen  $\approx 0.0015$  watt
- 1 acre = 43,560 square feet

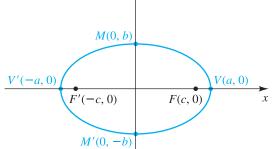
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# **CONIC SECTIONS**

#### PARABOLA

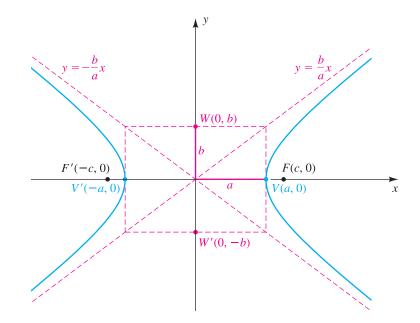






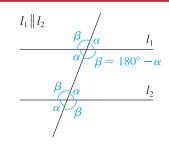
#### **HYPERBOLA**

$\frac{x^2}{a^2}$	$-\frac{y^2}{b^2} = 1$	with	$c^2 = a^2 + b^2$
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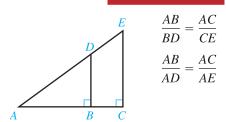


# **PLANE GEOMETRY**

#### **CONGRUENT ALTERNATE INTERIOR ANGLES**



#### SIMILAR TRIANGLES

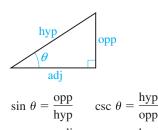


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#### TRIGONOMETRY

#### **TRIGONOMETRIC FUNCTIONS**

#### **OF ACUTE ANGLES**



$\cos \theta = \frac{\mathrm{adj}}{\mathrm{hyp}}$	$\sec \theta = \frac{\text{hyp}}{\text{adj}}$
$\tan \theta = \frac{\mathrm{opp}}{\mathrm{adj}}$	$\cot \theta = \frac{\mathrm{adj}}{\mathrm{opp}}$

#### SPECIAL RIGHT TRIANGLES

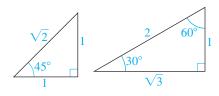
LAW OF COSINES

 $a^2 = b^2 + c^2 - 2bc \cos \alpha$ 

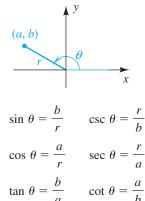
 $b^2 = a^2 + c^2 - 2ac\cos\beta$  $c^2 = a^2 + b^2 - 2ab\cos\gamma$ 

SPECIAL VALUES OF

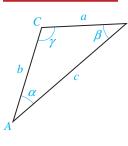
TRIGONOMETRIC FUNCTIONS



#### **OF ARBITRARY ANGLES**







#### LAW OF SINES

 $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$ 

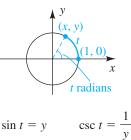
# AREA $\mathcal{A} = \frac{1}{2}bc\sin\alpha$ $\mathcal{A} = \frac{1}{2}ac\sin\beta$ $\mathcal{A} = \frac{1}{2}ab\sin\gamma$ $\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)},$

where  $s = \frac{1}{2}(a + b + c)$  (Heron's Formula)

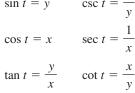
#### **GREEK ALPHABET**

θ (degrees)	θ (radians)	sin $ heta$	cos θ	tan θ	cot θ	sec θ	csc θ
0°	0	0	1	0		1	
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0		0	_	1

Letter Name Letter Name alpha Νν α nu  $\Xi \xi$ β beta xi gamma 0 0 omicron γ  $\Pi \pi$ δ delta pi **Ρ** ρ rho  $\epsilon$ epsilon ζ  $\Sigma \sigma$ zeta sigma T  $\tau$ eta tau η  $\theta$ theta Υv upsilon iota  $\Phi \phi(\varphi)$ phi L kappa Xχ chi к  $\Psi \psi$ λ lambda psi ωΩ μ mu omega



**OF REAL NUMBERS** 



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#### TRIGONOMETRY

#### **COFUNCTION FORMULAS**

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u$$
$$\cos\left(\frac{\pi}{2} - u\right) = \sin u$$
$$\tan\left(\frac{\pi}{2} - u\right) = \cot u$$
$$\cot\left(\frac{\pi}{2} - u\right) = \tan u$$
$$\sec\left(\frac{\pi}{2} - u\right) = \csc u$$
$$\csc\left(\frac{\pi}{2} - u\right) = \sec u$$

#### FORMULAS FOR NEGATIVES

$\sin\left(-t\right)=-\sin t$
$\cos\left(-t\right)=\cos t$
$\tan\left(-t\right) = -\tan t$
$\cot\left(-t\right) = -\cot t$
$\sec(-t) = \sec t$
$\csc\left(-t\right) = -\csc t$

# DOUBLE-ANGLE FORMULAS

$$\sin 2u = 2 \sin u \cos u$$
$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= 1 - 2 \sin^2 u$$
$$= 2 \cos^2 u - 1$$
$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

#### ADDITION FORMULAS

$\sin (u + v) = \sin u \cos v + \cos u \sin v$
$\cos(u+v) = \cos u \cos v - \sin u \sin v$
$\tan (u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$

#### HALF-ANGLE IDENTITIES

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

#### **PRODUCT-TO-SUM FORMULAS**

 $\sin u \cos v = \frac{1}{2} [\sin (u + v) + \sin (u - v)]$  $\cos u \sin v = \frac{1}{2} [\sin (u + v) - \sin (u - v)]$  $\cos u \cos v = \frac{1}{2} [\cos (u + v) + \cos (u - v)]$  $\sin u \sin v = \frac{1}{2} [\cos (u - v) - \cos (u + v)]$ 

#### SUBTRACTION FORMULAS

 $\sin (u - v) = \sin u \cos v - \cos u \sin v$  $\cos (u - v) = \cos u \cos v + \sin u \sin v$  $\tan (u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$ 

#### **HALF-ANGLE FORMULAS**

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$
$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$
$$u = 1 = \cos u$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

#### SUM-TO-PRODUCT FORMULAS

$$\sin u + \sin v = 2 \sin \left(\frac{u+v}{2}\right) \cos \left(\frac{u-v}{2}\right)$$
$$\sin u - \sin v = 2 \cos \left(\frac{u+v}{2}\right) \sin \left(\frac{u-v}{2}\right)$$
$$\cos u + \cos v = 2 \cos \left(\frac{u+v}{2}\right) \cos \left(\frac{u-v}{2}\right)$$
$$\cos u - \cos v = -2 \sin \left(\frac{u+v}{2}\right) \sin \left(\frac{u-v}{2}\right)$$

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#### FUNDAMENTAL IDENTITIES

$\csc t = \frac{1}{\sin t}$
$\sec t = \frac{1}{\cos t}$
$\cot t = \frac{1}{\tan t}$
$\tan t = \frac{\sin t}{\cos t}$
$\cot t = \frac{\cos t}{\sin t}$
$\sin^2 t + \cos^2 t = 1$
$1 + \tan^2 t = \sec^2 t$

 $1 + \cot^2 t = \csc^2 t$ 

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THIRTEENTH EDITION

# **PRECALCULUS** FUNCTIONS AND GRAPHS

# EARL W. SWOKOWSKI JEFFERY A. COLE

Anoka-Ramsey Community College



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TO THE MEMORY OF EARL W. SWOKOWSKI

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# List of Graphing Calculator Topics

There are many other places where a graphing calculator is used—these are the ones that include specific keystrokes.

#### **CHAPTER 1** Topics from Algebra

Storing Values and Evaluating Expressions 4 Reciprocals 6 Subtraction and Negatives 6 Testing Inequalities and the Trichotomy Law 8 Absolute Value 10 Scientific Form 13 Exponential Notation 16 Principal *n*th Root 20 Rational Exponents 24 Checking a Factoring Result 32 Finding the LCM 34 Adding Fractions 34 Creating a Table 35 Checking Equations 47 Complex Number Operations 60 Complex Number Operations 61

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# Preface

The thirteenth edition of *Precalculus: Functions and Graphs* includes over 450 new exercises and 35 new examples, many of these resulting from suggestions of users and reviewers of previous editions. The most important change in this edition is the addition of Chapter 11, "Limits of Functions." Accompanying Chapter 11 is Appendix V, "Theorems on Limits," which contains proofs for some of the theorems in Chapter 11. These changes have been incorporated without sacrificing the mathematical soundness that has been paramount to the success of this text.

The inclusion of graphing calculator examples and inserts, which feature specific color-coded keystrokes and screens for the TI-83/4 Plus, has proven to give added VALUE to the text for students—especially those who are working with a graphing calculator for the first time. It also gives professors more flexibility in terms of the way they approach a solution. The design of the text makes the technology inserts easily identifiable, and they are listed in a separate table of contents to make looking them up easier.

Below is a brief overview of the chapters, followed by a short description of the College Algebra course that I taught at Anoka-Ramsey Community College and then a list of the general features of the text.

### **Overview**

**CHAPTER 1** This chapter contains a summary of some basic algebra topics. Students should be familiar with much of this material, but also challenged by some of the exercises that prepare them for calculus. Graphing calculator operations are introduced and used to verify algebraic operations. Equations and inequalities are solved algebraically and numerically in this chapter with technology support; they will be solved graphically in subsequent chapters. Students will extend their knowledge of these topics; for example, they have worked with the quadratic formula, but will be asked to relate it to factoring and work with coefficients that are not real numbers (see Examples 6 and 8 in Section 1.4).

**CHAPTER 2** Two-dimensional graphs and functions are introduced in this chapter. Specific graphing calculator directions are given for most of the basic graphing features, such as finding zeros and points of intersection, as well as some of the more difficult topics, such as finding a regression model and graphing a piecewise-defined function. A favorite example of mine, Example 10 in Section 2.5, is an application (taxes) that relates tables, formulas, and graphs. Arrow notation, introduced in Section 2.2, is referred to throughout the text, preparing the student for limit notation.

**CHAPTER 3** This chapter begins with a discussion of polynomial functions and some polynomial theory. A thorough treatment of rational functions is given in Section 3.5. This is followed by a section on variation, which includes graphs of simple polynomial and rational functions.

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- **CHAPTER 4** Inverse functions is the first topic of discussion, followed by several sections that deal with exponential and logarithmic functions. A couple of examples that tie the various functions together deal with finding the inverse of a rational function (see Example 4 in Section 4.1) and finding the inverse of a hyperbolic function (Example 7 in Section 4.6).
- **CHAPTER 5** Angles is the first topic in this chapter. Next, the trigonometric functions are introduced using a right triangle approach and then defined in terms of a unit circle. Basic trigonometric identities appear throughout the chapter. The chapter concludes with sections on trigonometric graphs and applied problems.
- **CHAPTER 6** This chapter consists mostly of trigonometric identities, formulas, and equations. The last section contains definitions, properties, and applications of the inverse trigonometric functions.
- **CHAPTER 7** The law of sines and the law of cosines are used to solve oblique triangles. Vectors are then introduced and used in applications. The last two sections relate the trigonometric functions and complex numbers.
- **CHAPTER 8** Systems of inequalities and linear programming immediately follow solving systems by substitution and elimination. Next, matrices are introduced and used to solve systems. This chapter concludes with a discussion of determinants and partial fractions.
- **CHAPTER 9** This chapter begins with a discussion of sequences, and substantial technology support has been included. The formulas for the *n*th term of arithmetic and geometric sequences are generalized to find the *n*th term using any term, not just the first. Mathematical induction and the binomial theorem are next, followed by counting topics. The last section is about probability and includes topics such as odds and expected value. My favorite example introduces a different type of probability problem, and the solution can be applied to many similar problems (see Example 9 in Section 9.8).
- **CHAPTER 10** Sections on the parabola, ellipse, and hyperbola begin this chapter. Two different ways of representing functions are given in the next sections on parametric equations and polar coordinates.
- **CHAPTER 11** Limits of functions are introduced intuitively and precisely in the first two sections of this new chapter. In Section 11.2, the first example will help students become comfortable with the  $\epsilon$ - $\delta$  notation. The final two sections contain the limit theorems (including those involving infinity), providing students with a solid foundation for advancing to any beginning calculus course.

# **My Course**

At Anoka-Ramsey Community College in Coon Rapids, Minnesota, College Algebra I is a one-semester 3-credit course. For students intending to take Calculus, this course is followed by a one-semester 4-credit course, College Algebra II and Trigonometry. This course also serves as a terminal math course for many students. The sections covered in College Algebra I are

2.1–2.7, 3.1, 3.5 (part), 3.6, 4.1–4.6, 8.1–8.4, 9.1–9.3, and 9.5–9.8.

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Chapter 1 is used as review material in some classes, and the remaining sections are taught in the following course. A graphing calculator is required in some sections and optional in others.

## Features

A Separate List of Graphing Calculator Topics On pages viii and ix, there is a list of graphing calculator topics for quick reference.

**Illustrations** Brief demonstrations of the use of definitions, laws, and theorems are provided in the form of illustrations.

**Charts** Charts give students easy access to summaries of properties, laws, graphs, relationships, and definitions. These charts often contain simple illustrations of the concepts that are being introduced.

**Examples** Titled for easy reference, all examples provide detailed solutions of problems similar to those that appear in exercise sets. Many examples include graphs, charts, or tables to help the student understand procedures and solutions.

**Step-by-Step Explanations** In order to help students follow them more easily, many of the solutions in examples contain step-by-step explanations, printed in red.

**Checks** The solutions to some examples are explicitly checked, to remind students to verify that their solutions satisfy the conditions of the problems.



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**Graphing Calculator Examples** Wherever appropriate, examples requiring the use of a graphing utility have been added to the text. These are designated by a calculator icon (shown to the left) and illustrated with a figure reproduced from a graphing calculator screen.

**Graphing Calculator Inserts** In addition to the graphing calculator examples, these inserts are included to highlight some of the capabilities of graphing calculators and/or illustrate their use in performing the operations under discussion. See, for example, "Using the TI-83/4 Plus Sequence Mode" in Section 9.1.

**Graphing Calculator Exercises** Exercises specifically designed to be solved with a graphing utility are included in appropriate sections. These exercises are also designated by a calculator icon (shown to the left).

**Applications** To arouse student interest and to help students relate the exercises to current real-life situations, applied exercises have been titled. Many professors have indicated that the applications constitute one of the strongest features of the text.

**Exercises** Exercise sets begin with routine drill problems and gradually progress to more difficult problems. An ample number of exercises contain graphs and tabular data; others require the student to find a mathematical model for the given data. Many of the exercises require the student to understand the conceptual relationship of an equation and its graph.

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Applied problems generally appear near the end of an exercise set, to allow students to gain confidence in working with the new ideas that have been presented before they attempt problems that require greater analysis and synthesis of these ideas. Review exercises at the end of each chapter may be used to prepare for examinations.

**Discussion Exercises** Each chapter ends with several exercises that are suitable for small-group discussions. These exercises range from easy to difficult and from theoretical to application-oriented.

**Chapter Tests** These tests contain questions that are representative of exercises in the sections, as well as unique concept questions. I hope that professors will share their favorite test questions—please send them to me.

**Guidelines** Boxed guidelines enumerate the steps in a procedure or technique to help students solve problems in a systematic fashion.

**Warnings** Interspersed throughout the text are warnings to alert students to common mistakes.

**Text Art** Forming a total art package that is second to none, figures and graphs have been computer-generated for accuracy, using the latest technology. Colors are employed to distinguish between different parts of figures. For example, the graph of one function may be shown in blue and that of a second function in red. Labels are the same color as the parts of the figure they identify.

**Text Design** The text has been designed to ensure that discussions are easy to follow and important concepts are highlighted. Color is used pedagogically to clarify complex graphs and to help students visualize applied problems. Previous adopters of the text have confirmed that the text strikes a very appealing balance in terms of color use.

**Inside Covers and Inserts** The inside covers and inserts in the front and back of the text provide useful summaries from algebra, geometry, and trigonometry.

**Appendixes** Appendix I, "Common Graphs and Their Equations," is a pictorial summary of graphs and equations that students commonly encounter in precalculus mathematics. Appendix II, "A Summary of Graph Transformations," is an illustrative synopsis of the basic graph transformations discussed in the text: shifting, stretching, compressing, and reflecting. Appendix III, "Graphs of Trigonometric Functions and Their Inverses," contains graphs, domains, and ranges of the six trigonometric functions of Special Angles on a Unit Circle," is a full-page reference for the most common angles on a unit circle—valuable for students who are trying to learn the basic values of the trigonometric functions. Appendix V, "Theorems on Limits," contains proofs for some theorems in Chapter 11.

**Answer Section** The answer section at the end of the text provides answers for most of the odd-numbered exercises, as well as answers for all chapter

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review exercises. Considerable thought and effort were devoted to making this section a learning device for the student instead of merely a place to check answers. For instance, proofs are given for mathematical induction problems. Numerical answers for many exercises are stated in both an exact and an approximate form. Graphs, proofs, and hints are included whenever appropriate. Author-prepared solutions and answers ensure a high degree of consistency among the text, the solutions manuals, and the answers.

# **Teaching Tools for the Instructor**

**Annotated Instructor's Edition** (ISBN-13: 978-1-337-55234-9) The Annotated Instructor's Edition provides short answers in blue type by the exercises. Answers requiring more space, such as graphs, are provided in the Additional Answers located immediately after the Index.



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**Instructor's Solutions Manual** by Jeffery A. Cole This author-prepared manual includes answers to all exercises and detailed solutions to most exercises. The manual has been thoroughly reviewed for accuracy. The Instructor's Solutions Manual is available on the instructor companion website.

**Test Bank** Available on the instructor companion site, the Test Bank contains ready-to-use chapter tests. Multiple test forms are provided for each chapter.

# **Learning Tools for the Student**

**Student Solutions Manual** by Jeffery A. Cole (ISBN-13: 978-1-337-55236-3) This author-prepared manual provides solutions for all of the odd-numbered exercises, as well as strategies for solving additional exercises. Many helpful hints and warnings are also included.

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# **Other Texts in the Swokowski/Cole Series**

Fundamentals of College Algebra, Eleventh Edition Algebra and Trigonometry with Analytic Geometry, Thirteenth Edition Algebra and Trigonometry with Analytic Geometry, Classic Twelfth Edition

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Jeffery A. Cole