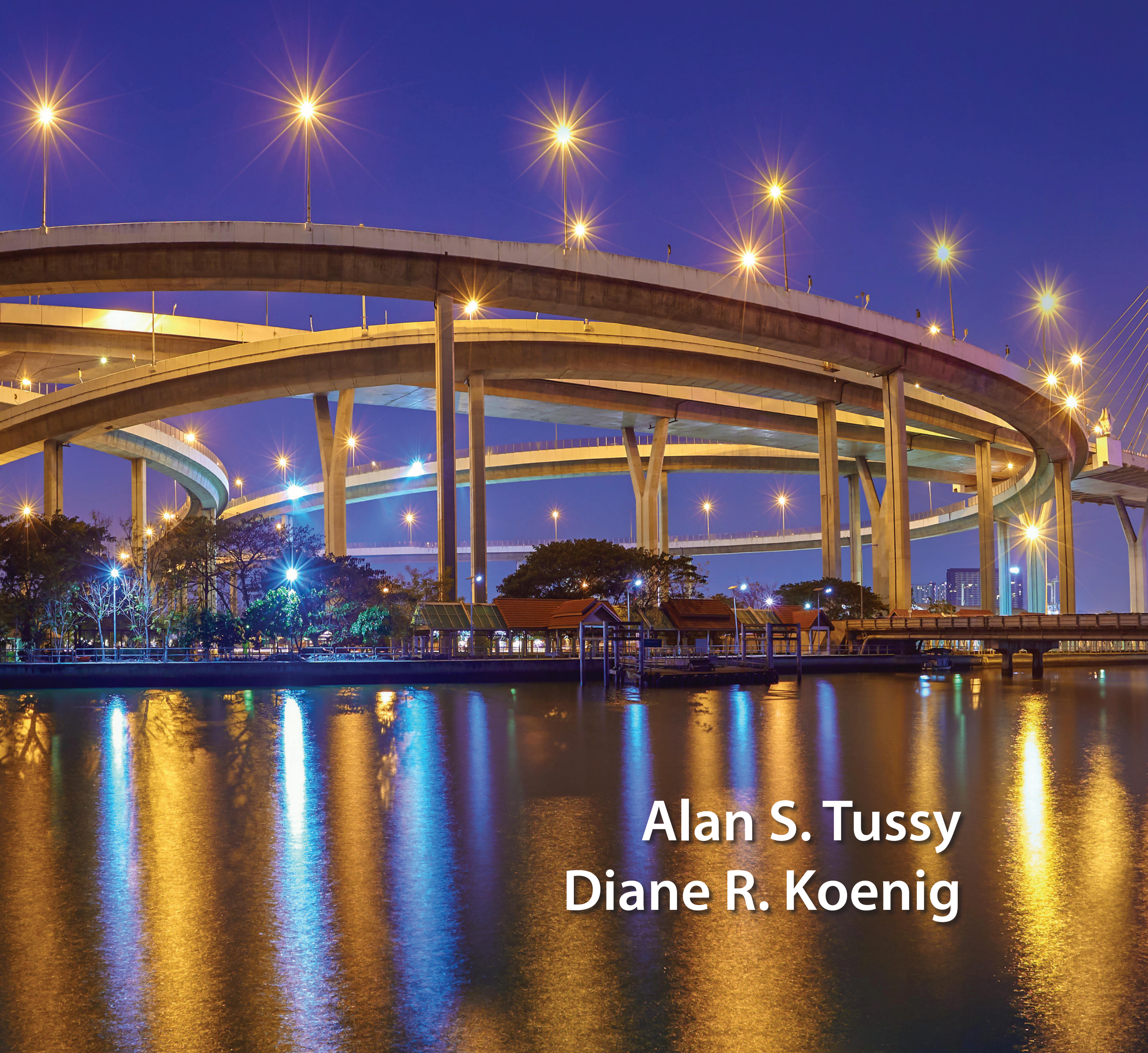
Basic Mathematics
with Early Integers
SIXTH
EDITION



Alan S. Tussy
Diane R. Koenig

6e

Basic Mathematics
with Early Integers

Alan S. Tussy
Citrus College

Diane R. Koenig
Rock Valley College

CENGAGE

Australia • Brazil • Mexico • Singapore • United Kingdom • United States

To my daughters, Ashley, Brianna, and Carly, whom I love very much and who make me extremely proud.

—DRK

CENGAGE

***Basic Mathematics with Early Integers*,**
Sixth Edition

Alan S. Tussy, Diane R. Koenig

Product Manager: Frank Snyder

Content Developers: Samantha Gomez, Alison Duncan

Product Assistant: Abigail DeVeuve

Marketing Manager: Pamela Polk

Content Project Manager: Rebecca Charles

Manufacturing Planner: Becky Cross

IP Analyst: Ashley Maynard

IP Project Manager: Reba Frederics

Senior Art Director: Vernon Boes

Text and Cover Designer: Terri Wright

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PREFACE

We are excited to present the Sixth Edition of *Basic Mathematics with Early Integers*, and are confident that the revision process has produced an even stronger experience for students and teachers. A new instructional feature, Look Alikes, has been added to the Study Sets. The objective of Look Alikes is to improve students' problem-recognition skills. Furthermore, all data in the real-life application problems have been updated, and some new, vocationally focused problems have been added.

To strengthen the online experience, the quantity and types of problems available in Enhanced Web Assign® have been expanded. The digital package that accompanies this text introduces a new customized learning tool, MindTap reader.

We want to thank all of you across the country who provided suggestions and input about the previous edition. Your insights have proven invaluable. Throughout the revision process, our fundamental belief has remained the same: Mathematics is a language in its own right. As always, the prime objective of this textbook (and its accompanying ancillaries) is to teach students how to read, write, speak, and think using the language of mathematics.

About the Authors

ALAN S. TUSSY

Alan Tussy grew up in West Covina, California, and in the early 1970's attended the University of Redlands where he majored in mathematics. After taking a fifth year of education courses and completing student teaching requirements, he was hired by the Arcadia Unified School District to teach junior high school math. He later transferred to Arcadia High School where he taught AP Calculus, served as department chair, and coached varsity football. During that time, Alan took evening classes at California State University, Los Angeles, and eventually earned a master's degree in applied mathematics. The master's degree led to a weeknight adjunct assignment at Citrus College in Glendora, California. He was soon hired full-time at Citrus, where he has taught mathematics for 25 years. He has taught up and down the community college curriculum from Arithmetic Fundamentals to Differential Equations, paying special attention to developmental math courses. Alan has written nine math books—a paperback series and a hardcover series. He is a creative and visionary teacher who maintains a keen focus on his students' greatest challenges. He is an extraordinary author dedicated to his students' success. His latest (and very rewarding) endeavor is mentoring and counseling a large number of adjunct math instructors at his school.

DIANE R. KOENIG

A nationally recognized educator and author, Diane Koenig actively shaped several textbooks, ancillaries, and series. Since 1982 when she helped develop the Gustafson/Frisk series to her work on the Tussy/Koenig/Gustafson series, Diane's writing continues to reflect the expertise she gains from working with students in her Mathematics courses. Throughout her work, she integrates research-based strategies in Mathematics education. She earned a Bachelor of Science degree in Secondary Math Education from Illinois State University in 1980, and began her career at Rock Valley College in 1981, when she became the Math Supervisor for a newly formed Personalized Learning Center.

Earning her Master's Degree in Applied Mathematics from Northern Illinois University in 1984, Diane enjoys the distinction of being the first woman to become a full-time faculty member in the Mathematics department for Rock Valley College. In addition to being awarded AMATYC's Excellence in Teaching Award in 2015, she was chosen as the Rock Valley College Faculty of the Year by her peers in 2005, and the next year she was awarded the NISOD Teaching Excellence Award and the Illinois Mathematics Association of Community Colleges Award for Teaching Excellence. In addition to her teaching, she has been an active member of the Illinois Mathematics Association of Community Colleges (IMACC), serving on the board of directors, on a state-level task force rewriting the course outlines for the Developmental Mathematics courses, and as the Association's newsletter editor.

New to This Edition

EXAMPLE 1

The authors present a comprehensive revision to a classic text that updates the real-world data appearing in worked examples and Study Sets.

APPLICATIONS

New application problems highlighting a variety of occupations and vocations have been added to the Study Sets. These problems provide students with practical context for the mathematical topics they are studying.

LOOK ALIKES

A new *Look Alike* feature builds student problem-recognition skills. *Look Alike* problems require students to distinguish between problems that at first glance appear similar, but in actuality call for different strategies to solve them.

THE LANGUAGE OF ALGEBRA

Success Tip

Caution!

Additional displays, diagrams, and explanations have been added to assist students who are visual learners. A new page design places the *Language of Algebra*, *Success Tip*, and *Caution* features in the margin for easier reading.

Ancillaries

For the Student	For the Instructor
Online Student Solutions Manual (ISBN: 978-1-337-61582-2) The Online Student Solutions Manual provides worked-out solutions to all of the odd-numbered exercises in the text.	**Online Complete Solutions Manual** (ISBN: 978-1-337-61583-9) The Online Complete Solutions Manual provides worked-out solutions to all of the problems in the text.
	Instructor's Companion Website Everything you need for your course in one place! Access and download a helpful Instructor Manual that paces the chapters, provides a how-to approach and additional opportunities for in-class practice and homework. In addition, you can find the online appendix, PowerPoint presentations, and more on the companion site. This collection of book-specific lecture and class tools is available online via www.cengage.com/login.
WEBASSIGN From Cengage **www.webassign.com/cengage** (Printed Access Card ISBN: 978-1-337-61831-1, Online Access Code ISBN: 978-1-337-61589-1) Prepare for class with confidence using WebAssign from Cengage for Tussy, *Basic Mathematics with Early Integers,* 6e. This online learning platform, which includes an interactive ebook, fuels practice, so you truly absorb what you learn—and are better prepared come test time. Videos and tutorials walk you through concepts and deliver instant feedback and grading, so you always know where you stand in class. Focus your study time and get extra practice where you need it most. Study smarter with WebAssign! Ask your instructor today how you can get access to WebAssign, or learn about self-study options at www.webassign.com.	WEBASSIGN From Cengage **www.webassign.com/cengage** (Printed Access Card ISBN: 978-1-337-61831-1, Online Access Code ISBN: 978-1-337-61830-4) WebAssign from Cengage for *Basic Mathematics with Early Integers*, 6e is a fully customizable online solution, including an interactive ebook, for STEM disciplines that empowers you to help your students learn, not just do homework. Insightful tools save you time and highlight exactly where your students are struggling. Decide when and what type of help students can access while working on assignments—and incentivize independent work so help features aren't abused. Meanwhile, your students get an engaging experience, instant feedback, and better outcomes. A total win-win! To try a sample assignment, learn about LMS integration, or connect with our digital course support visit www.webassign.com/cengage.

Acknowledgments

Authoring a textbook is a tremendous undertaking. A revision of this scale would not have been possible without the thoughtful feedback and support from our fellow colleagues. Your contributions to this edition have shaped this revision in countless ways.

We would also like to express our thanks to the editorial, marketing, and production staff of Cengage—Frank Snyder, Rebecca Charles, Samantha Gomez, Alison Duncan, Pamela Polk, and Abigail DeVeuve—for helping us to craft this new edition. Thanks also to Vernon Boes for his work on the design and art program. In addition, our gratitude goes to Beth Asselin and the entire SPi Global team for their copyediting and proofreading expertise.

We want to express our gratitude to those who helped with this project: Brenda Keller, Rhoda Oden, Steve Odrich, Mary Lou Wogan, Paul McCombs, Maria H. Andersen, Sheila Pisa, Laurie McManus, Alexander Lee, Ed Kavanaugh, Karl Hunsicker, Cathy Gong, Dave Ryba, Terry Damron, Marion Hammond, Lin Humphrey, Doug Keebaugh, Robin Carter, Tanja Rinkel, Jeff Cleveland, Jo Morrison, Sheila White, Jim McClain, Paul Swatzel, Matt Stevenson, Carole Carney, Joyce Low, Rob Everest, David Casey, Heddy Paek, Ralph Tippins, Mo Trad, Eagle Zhuang, Chris Scott, Victoria Dominguez, Esme Medrano, Sam M. Ditzion, Lisa Brown, Elaine Tucker, Laura McInerney, Solomon Willis, Tracy Nehnevaji, Philomena Sefranek, and the Citrus College library staff (including Barbara Rugeley) for their help with this project. Your encouragement, suggestions, and insight have been invaluable to us.

Alan S. Tussy
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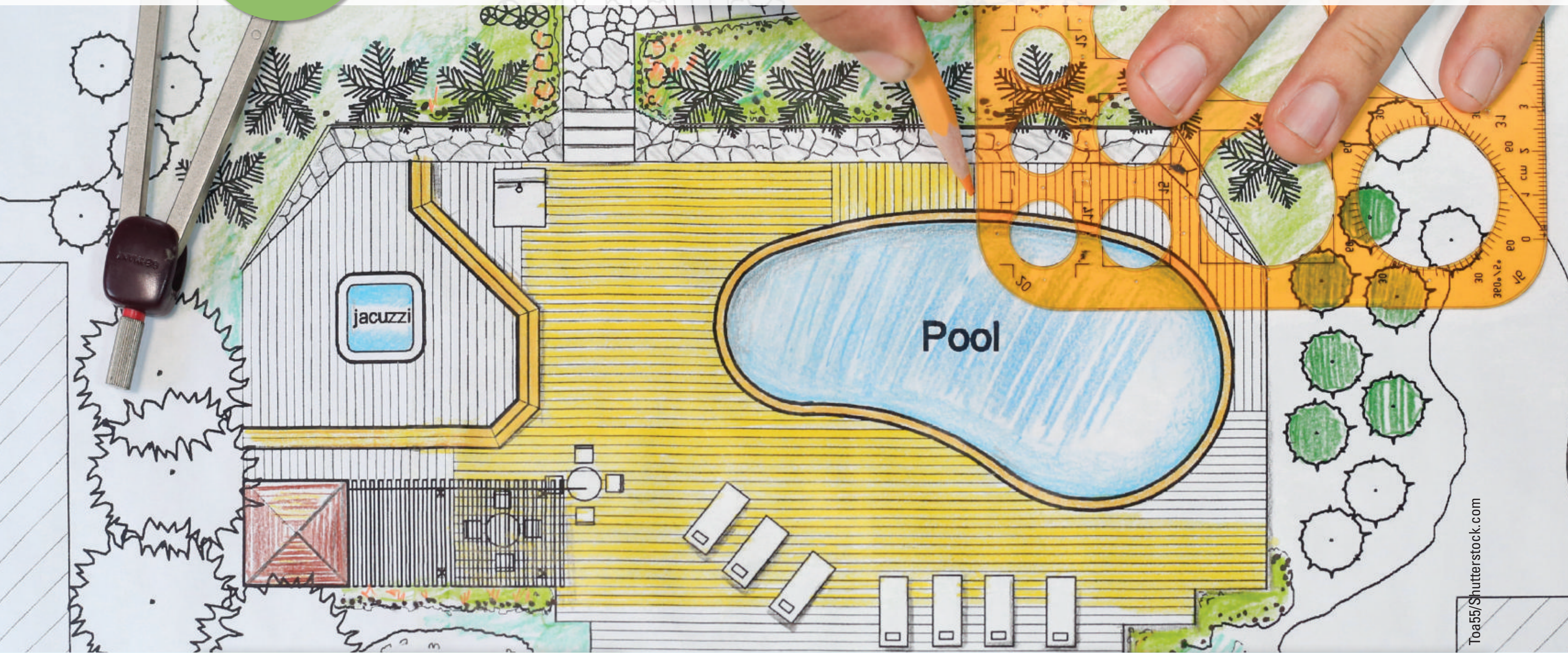
1 Whole Numbers



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from Campus to Careers

Landscape Designer

Landscape designers make outdoor places more beautiful and useful. They work on all types of projects. Some focus on yards and parks, others on land around buildings and highways. The training of a landscape designer should include botany classes to learn about plants; art classes to learn about color, line, and form; and mathematics classes to learn how to take measurements and keep business records.

In **Problem 108** of **Study Set 1.5**, you will see how a landscape designer uses division to determine the number of pine trees that are needed to form a windscreen for a flower garden. In **Problem 57** on **Study Set 1.6**, you will see how a landscape designer uses addition and multiplication of whole numbers to calculate the cost of landscaping a yard. And in **Problem 116** of **Study Set 1.9**, the rule for the order of operations is used to calculate the available planting area.

JOB TITLE:
Landscape designer

EDUCATION:
A bachelor's degree in landscape design. Most states require a license.

JOB OUTLOOK:
Expected to grow 5% from 2014 to 2024

ANNUAL EARNINGS:
The average annual salary is $68,600.

FOR MORE INFORMATION
www.thelandlovers.org/career_LandscapeDesign.asp

CHAPTER OUTLINE

SECTION 1.1 An Introduction to the Whole Numbers

OBJECTIVES

1. Identify the place value of a digit in a whole number.
2. Write whole numbers in words and in standard form.
3. Write a whole number in expanded form.
4. Compare whole numbers using inequality symbols.
5. Round whole numbers.
6. Read tables and graphs involving whole numbers.

The **whole numbers** are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and so on. They are used to answer questions such as How many?, How fast?, and How far?

- Michael Phelps earned 23 Olympic Gold Medals during his swimming career.
- The average American adult reads at a rate of 250 to 300 words per minute.
- The driving distance from New York City to Los Angeles is 2,786 miles.

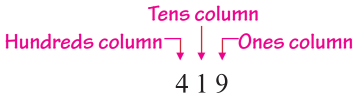
The *set of whole numbers* is written using **braces** { }, as shown below. The three dots indicate that the list continues forever—there is no largest whole number. The smallest whole number is 0.

The Set of Whole Numbers

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, . . .}

OBJECTIVE 1 Identify the place value of a digit in a whole number.

When a whole number is written using the **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, it is said to be in **standard form** (also called **standard notation**). The position of a digit in a whole number determines its **place value**. In the number 419, the 9 is in the *ones column*, the 1 is in the *tens column*, and the 4 is in the *hundreds column*.

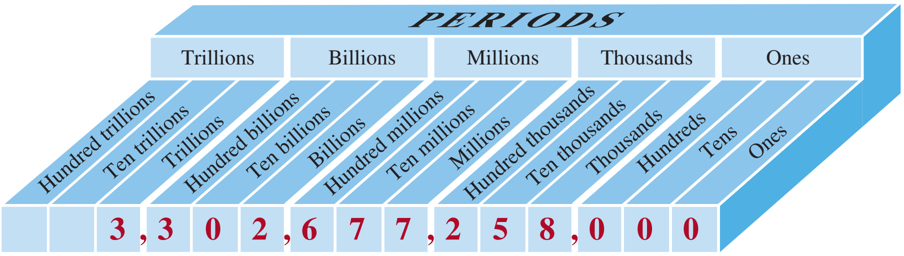


To make large whole numbers easier to read, we use commas to separate their digits into groups of three, called **periods**. Each period has a name, such as *ones*, *thousands*, *millions*, *billions*, and *trillions*. The following **place-value chart** shows the place value of each digit in the number 3,302,677,258,000, which is read as:

Three trillion, three hundred two billion, six hundred seventy-seven million, two hundred fifty-eight thousand


Elena Yakusheva/Shutterstock.com

In 2015, the federal government collected $3,302,677,258,000 in taxes.



LANGUAGE OF MATHEMATICS

As we move to the left in the chart, the place value of each column is 10 times the column directly to its right. This is why we call our number system the ***base-10 number system***.

Each of the 2's in 3,302,677,258,000 has a different place value because of its position. The place value of the red 2 is 2 *billions*. The place value of the blue 2 is 2 *hundred thousands*.

EXAMPLE 1 **Airports.** Hartsfield-Jackson Atlanta International Airport is the busiest airport in the United States, handling 101,491,106 passengers in 2015. (Source: Airports Council International–North America)

a. What is the place value of the digit 4?

b. Which digit tells the number of millions?

Strategy We will begin in the ones column of 101,491,106. Then, moving to the left, we will name each column (ones, tens, hundreds, and so on) until we reach the digit 4.

WHY It's easier to remember the names of the columns if you begin with the smallest place value and move to the columns that have larger place values.

Solution

a. 101,491,106 Say, "Ones, tens, hundreds, thousands, ten thousands, hundred thousands" as you move from column to column.

4 hundred thousands is the place value of the digit 4.

b. 101,491,106

The digit 1 is in the millions column.

Self Check 1

Cell Phones. In 2015, there were 377,921,241 cellular telephone subscriber connections in the United States. (Source: CTIA The Wireless Association)

a. What is the place value of the digit 3?

b. Which digit tells the number of hundred thousands?

Now Try Problem 23

LANGUAGE OF MATHEMATICS

Each of the worked examples in this textbook includes a *Strategy* and *Why* explanation. A ***strategy*** is a plan of action to follow to solve the given problem.

OBJECTIVE 2 Write whole numbers in words and in standard form.

Since we use whole numbers so often in our daily lives, it is important to be able to read and write them.

Reading and Writing Whole Numbers

To write a whole number in words, start from the left. Write the number in each period followed by the name of the period (except for the *ones period*, which is not used). Use commas to separate the periods.

To read a whole number out loud, follow the same procedure. The commas are read as slight pauses.

LANGUAGE OF MATHEMATICS

The word ***and*** should not be said when reading a whole number. It should only be used when reading a mixed number such as 5½ (five *and* one-half) or a decimal such as 3.9 (three *and* nine-tenths).

EXAMPLE 2 Write each number in words:

a. 63 **b.** 499 **c.** 89,015 **d.** 6,070,534

Strategy For the larger numbers in parts c and d, we will name the periods from right to left to find the *greatest* period.

WHY To write a whole number in words, we must give the name of each period (except for the ones period). Finding the largest period helps to start the process.

Solution

a. 63 is written: *sixty-three.* Use a hyphen to write whole numbers from 21 to 99 in words (except for 30, 40, 50, 60, 70, 80, and 90).

b. 499 is written: *four hundred ninety-nine.*

c. Thousands Ones

Say the names of the periods, working from right to left.

89 , 015

Eighty-nine **thousand**, **fifteen**

We do not use a hyphen to write numbers between 1 and 20, such as 15. The ones period is not written.

d. Millions Thousands Ones

Say the names of the periods, working from right to left.

6 , 0 7 0 , 5 3 4

Six **million**, seventy **thousand**, five **hundred** thirty-four.

The ones period is not written.

Self Check 2

Write each number in words:

a. 42
b. 798
c. 97,053
d. 23,000,017

Now Try Problems 31, 33, and 35

EXAMPLE 3 Write each number in standard form:

a. *Twelve thousand, four hundred seventy-two*
b. *Seven hundred one million, thirty-six thousand, six*
c. *Forty-three million, sixty-eight*

Strategy We will locate the commas in the written-word form of each number.

WHY When a whole number is written in words, commas are used to separate periods.

Solution

a. Twelve thousand , four hundred seventy-two

12,472

b. Seven hundred one million , thirty-six thousand , six

701,036,006

c. Forty-three million , sixty-eight

43,000,068

The written-word form does not mention the thousands period.

If a period is not named, three zeros hold its place.

Caution! Two numbers, 40 and 90, are often misspelled: write forty (not fourty) and ninety (not ninty).

Self Check 3

Write each number in standard form:

a. *Two hundred three thousand, fifty-two*
b. *Nine hundred forty-six million, four hundred sixteen thousand, twenty-two*
c. *Three million, five hundred seventy-nine*

Now Try Problems 39 and 45

OBJECTIVE 3 Write a whole number in expanded form.

In the number 6,352, the digit 6 is in the thousands column, 3 is in the hundreds column, 5 is in the tens column, and 2 is in the ones (or units) column. The meaning of 6,352 becomes clear when we write it in **expanded form** (also called **expanded notation**).

$$6{,}352 = 6 \text{ thousands} + 3 \text{ hundreds} + 5 \text{ tens} + 2 \text{ ones}$$

or

$$6{,}352 = 6{,}000 + 300 + 50 + 2$$

Success Tip Four-digit whole numbers are sometimes written without a comma. For example, we may write 3,911 or 3911 to represent three thousand, nine hundred eleven.

EXAMPLE 4 Write each number in expanded form:

a. 85,427 **b.** 1,251,609

Strategy Working from left to right, we will give the place value of each digit and combine them with + symbols.

WHY The term *expanded form* means to write the number as an addition of the place values of each of its digits.

Solution

a. The expanded form of 85,427 is:

8 ten thousands + 5 thousands + 4 hundreds + 2 tens + 7 ones

which can be written as:

80,000 + 5,000 + 400 + 20 + 7

b. The expanded form of 1,251,609 is:

1 million + 2 hundred thousands + 5 ten thousands + 1 thousand + 6 hundreds + 0 tens + 9 ones

Since 0 tens is zero, the expanded form can also be written as:

1 million + 2 hundred thousands + 5 ten thousands + 1 thousand + 6 hundreds + 9 ones

which can be written as:

1,000,000 + 2,00,000 + 50,000 + 1,000 + 600 + 9

Self Check 4

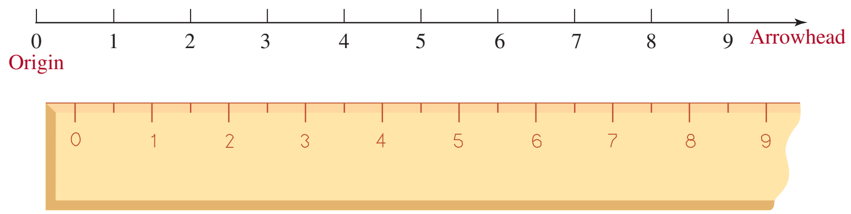
Write 708,413 in expanded form.

Now Try Problems 49, 53, and 57

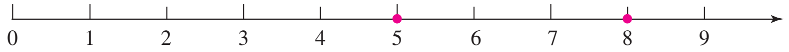
OBJECTIVE 4 Compare whole numbers using inequality symbols.

Whole numbers can be shown by drawing points on a **number line**. Like a ruler, a number line is straight and has uniform markings. To construct a number line, we begin on the left with a point on the line representing the number 0. This point is called the **origin**. We then move to the right, drawing equally spaced marks and labeling them with whole numbers that increase in value. The arrowhead at the right indicates that the number line continues forever.

A number line



Using a process known as **graphing**, we can represent a single number or a set of numbers on a number line. **The graph of a number** is the point on the number line that corresponds to that number. *To graph a number* means to locate its position on the number line and highlight it with a heavy dot. The graphs of 5 and 8 are shown on the number line below.



As we move to the right on the number line, the numbers increase in value. Because 8 lies to the right of 5, we say that 8 is greater than 5. The **inequality symbol** > ("is greater than") can be used to write this fact:

$8 > 5$ Read as "8 is greater than 5."

Since $8 > 5$, it is also true that $5 < 8$. We read this as "5 is less than 8."

Success Tip To tell the difference between these two inequality symbols, remember that they always point to the smaller of the two numbers involved.

$8 > 5$ $5 < 8$

Points to the smaller number

Inequality Symbols

$>$ means *is greater than*
$<$ means *is less than*

EXAMPLE 5 Place an $<$ or an $>$ symbol in the box to make a true statement: **a.** 3 ☐ 7 **b.** 18 ☐ 16

Strategy To pick the correct inequality symbol to place between a pair of numbers, we need to determine the position of each number on the number line.

WHY For any two numbers on a number line, the number to the *left* is the smaller number and the number to the *right* is the larger number.

Solution

a. Since 3 is to the left of 7 on the number line, we have $3 < 7$.

b. Since 18 is to the right of 16 on the number line, we have $18 > 16$.

Self Check 5

Place an $<$ or an $>$ symbol in the box to make a true statement:

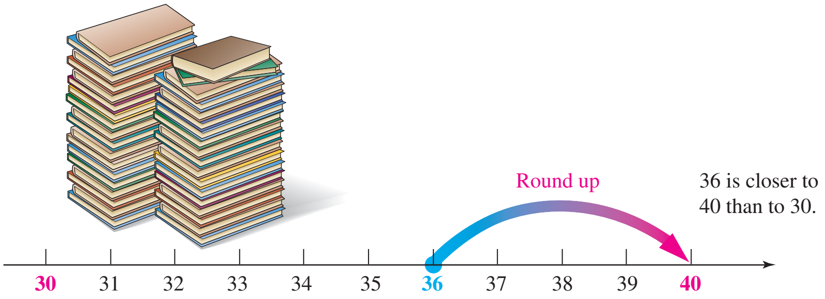
a. 12 ☐ 4

b. 7 ☐ 10

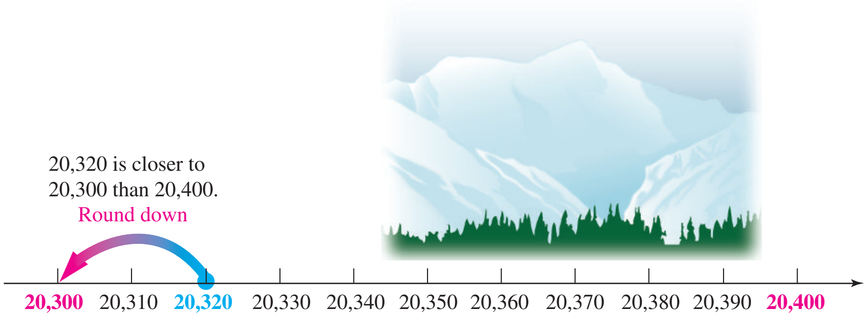
Now Try Problems 59 and 61

OBJECTIVE 5 Round whole numbers.

When we don't need exact results, we often round numbers. For example, when a teacher with 36 students orders 40 textbooks, he has rounded the actual number to the *nearest ten*, because 36 is closer to 40 than it is to 30. We say 36, rounded to the nearest 10, is 40. This process is called **rounding up**.



When a geologist says that the height of Alaska's Mount McKinley is "about 20,300 feet," she has rounded to the *nearest hundred*, because its actual height of 20,320 feet is closer to 20,300 than it is to 20,400. We say that 20,320, rounded to the nearest hundred, is 20,300. This process is called **rounding down**.



To round a whole number, we follow an established set of rules. To round a number to the nearest ten, for example, we locate the **rounding digit** in the tens column. If the **test digit** to the right of that column (the digit in the ones column) is 5 or greater, we *round up* by increasing the tens digit by 1 and replacing the test digit with 0. If the test digit is less than 5, we *round down* by leaving the tens digit unchanged and replacing the test digit with 0.

LANGUAGE OF MATHEMATICS

When we ***round*** a whole number, we are finding an approximation of the number. An approximation is close to, but not the same as, the exact value.

EXAMPLE 6 Round each number to the nearest ten: **a.** 3,761 **b.** 12,087

Strategy We will find the digit in the tens column and the digit in the ones column.

WHY To round to the nearest ten, the digit in the tens column is the rounding digit and the digit in the ones column is the test digit.

Solution

a. We find the rounding digit in the tens column, which is 6. Then we look at the test digit to the right of 6, which is the 1 in the ones column. Since $1 < 5$, we round down by leaving the 6 unchanged and replacing the test digit with 0.

Rounding digit: tens column
3,761
Test digit: 1 is less than 5.

Keep the rounding digit: Do not add 1.
3,761
Replace with 0.

Thus, 3,761 rounded to the nearest ten is 3,760.

b. We find the rounding digit in the tens column, which is 8. Then we look at the test digit to the right of 8, which is the 7 in the ones column. Because 7 is 5 or greater, we round up by adding 1 to 8 and replacing the test digit with 0.

Rounding digit: tens column
12,087
Test digit: 7 is 5 or greater.

Add 1.
12,087
Replace with 0.

Thus, 12,087 rounded to the nearest ten is 12,090.

Self Check 6

Round each number to the nearest ten:
a. 35,642
b. 9,756

Now Try ➔ **Problem 63**

A similar method is used to round numbers to the nearest hundred, the nearest thousand, the nearest ten thousand, and so on.

Rounding a Whole Number

1. To round a number to a certain place value, locate the **rounding digit** in that place.
2. Look at the **test digit**, which is directly to the right of the rounding digit.
3. If the test digit is 5 or greater, round up by adding 1 to the rounding digit and replace all of the digits to its right with 0.

   If the test digit is less than 5, replace it and all of the digits to its right with 0.

EXAMPLE 7 Round each number to the nearest hundred:
a. 18,349 **b.** 7,960

Strategy We will find the rounding digit in the hundreds column and the test digit in the tens column.

WHY To round to the nearest hundred, the digit in the hundreds column is the rounding digit and the digit in the tens column is the test digit.

Caution! To round a number, use *only* the test digit directly to the right of the rounding digit to determine whether to round up or round down.

Solution

a. First, we find the rounding digit in the hundreds column, which is 3. Then we look at the test digit 4 to the right of 3 in the tens column. Because 4 < 5, we round down and leave the 3 in the hundreds column. We then replace the two rightmost digits with 0's.

Rounding digit: hundreds column

18,349

Test digit: 4 is less than 5.

Keep the rounding digit: Do not add 1.

18,349

Replace with 0's.

Thus, 18,349 rounded to the nearest hundred is 18,300.

b. First, we find the rounding digit in the hundreds column, which is 9. Then we look at the test digit 6 to the right of 9. Because 6 is 5 or greater, we round up and increase 9 in the hundreds column by 1. Since the 9 in the hundreds column represents 900, increasing 9 by 1 represents increasing 900 to 1,000. Thus, we replace the 9 with a 0 and add 1 to the 7 in the thousands column. Finally, we replace the two rightmost digits with 0's.

Rounding digit: hundreds column

7,960

Test digit: 6 is 5 or greater.

Add 1. Since 9 + 1 = 10, write 0 in this column and carry 1 to the next column.

$\overset{7+1}{\cancel{7}},\overset{0}{\cancel{9}}60$

Replace with 0's.

Thus, 7,960 rounded to the nearest hundred is 8,000.

Self Check 7

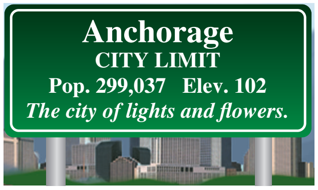
Round 365,283 to the nearest hundred.

Now Try ➲ **Problems 69 and 71**

EXAMPLE 8 **U.S. cities.** Anchorage is the largest city in Alaska. Round the 2017 population of Anchorage shown in the sign to:

a. the nearest thousand

b. the nearest ten thousand



Strategy In each case, we will identify the rounding digit and the test digit.

WHY We need to know the value of the test digit to determine whether we round the population up or down.

Solution

a. The rounding digit in the thousands column is 9. Since the test digit 0 is less than 5, we round down.

Rounding digit Test digit

299,037

To the nearest thousand, Anchorage's population in 2017 was 299,000.

b. The rounding digit in the ten thousands column is 9. Since the test digit 9 is 5 or greater, we round up by writing 0 in the ten thousands column and carrying a 1 to the hundred thousands column.

Rounding digit Test digit

299,037

To the nearest ten thousand, Anchorage's population in 2017 was 300,000.

Self Check 8

U.S. cities. In 2015, San Antonio, Texas, with a population of 1,469,845, was the nation's 7th largest city. Round the population of San Antonio to:

a. the nearest thousand

b. the nearest million

Now Try ➲ **Problems 75 and 79**

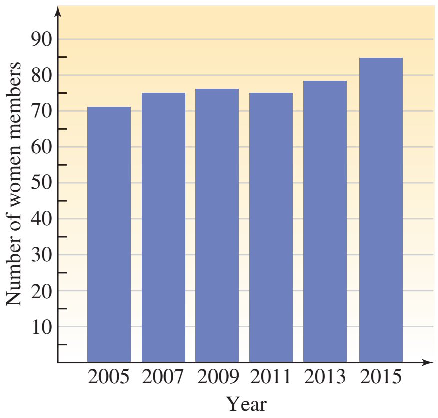
OBJECTIVE 6 Read tables and graphs involving whole numbers.

The following table is an example of the use of whole numbers. It shows the number of women members of the U.S. House of Representatives for the years 2005 through 2015.

Year	Number of women members
2005	71
2007	75
2009	76
2011	75
2013	78
2015	84

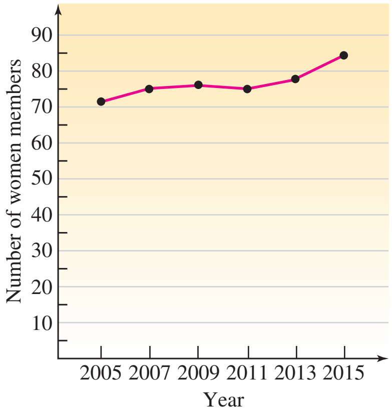
Source: Data from http://www.cawp.rutgers.edu/women-us-congress-2015

Bar graph



(a)

Line graph



(b)

In figure (a), the information in the table is presented in a **bar graph**. The *horizontal* scale is labeled "Year," and units of 2 years are used. The *vertical* scale is labeled "Number of women members," and units of 10 are used. The bar directly over each year extends to a height that shows the number of women members of the House of Representatives that year.

Another way to present the information in the table is with a **line graph**. Instead of using a bar to represent the number of women members, we use a dot drawn at the correct height. After drawing data points for 2005, 2007, 2009, 2011, 2013, and 2015, we connect the points to create the line graph in figure (b).

LANGUAGE OF MATHEMATICS

Horizontal is a form of the word *horizon.* Think of the sun setting over the *horizon.* ***Vertical*** means in an upright position. Pro basketball player LeBron James' *vertical* leap measures more than 49 inches.

Think it Through • RE-ENTRY STUDENTS

"A re-entry student is considered one who is the age of 25 or older, or those students that have had a break in their academic work for 5 years or more. Nationally, this group of students is growing at an astounding rate."

—Student Life and Leadership Department, University Union, Cal Poly University, San Luis Obispo

Some common concerns expressed by adult students considering returning to school are listed below in Column I. Match each concern to an encouraging reply in Column II.

Column I

1. I'm too old to learn.

2. I don't have the time.

3. I didn't do well in school the first time around. I don't think a college would accept me.

4. I'm afraid I won't fit in.

5. I don't have the money to pay for college.

Column II

a. Many students qualify for some type of financial aid.

b. Taking even a single class puts you one step closer to your educational goal.

c. There's no evidence that older students can't learn as well as younger ones.

d. More than 41% of the students in college are older than 25.

e. Typically, community colleges and career schools have an open admissions policy.

Source: Adapted from *Common Concerns for Adult Students,* Minnesota Higher Education Services Office

Answers to Self Checks

1. a. 3 hundred millions **b.** 9 **2. a.** forty-two **b.** seven hundred ninety-eight **c.** ninety-seven thousand, fifty-three **d.** twenty-three million, seventeen **3. a.** 203,052 **b.** 946,416,022 **c.** 3,000,579 **4.** 700,000 + 8,000 + 400 + 10 + 3 **5. a.** > **b.** < **6. a.** 35,640 **b.** 9,760 **7.** 365,300 **8. a.** 1,470,000 **b.** 1,000,000

SECTION 1.1 STUDY SET

VOCABULARY

Fill in the blanks.

1. The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are the ______.

2. The set of ______ numbers is {0, 1, 2, 3, 4, 5, …}.

3. When we write five thousand eighty-nine as 5,089, we are writing the number in _________ form.

4. To make large whole numbers easier to read, we use commas to separate their digits into groups of three, called _______.

5. When 297 is written as 200 + 90 + 7, we are writing 297 in _________ form.

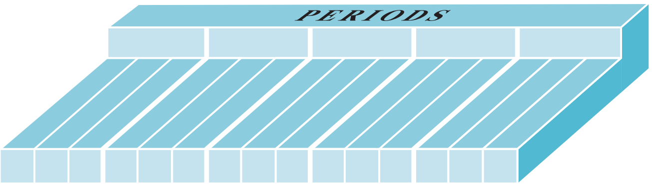
6. Using a process called *graphing*, we can represent whole numbers as points on a ________ line.

7. The symbols > and < are _________ symbols.

8. If we ______ 627 to the nearest ten, we get 630.

CONCEPTS

9. Copy the following place-value chart. Then enter the whole number 1,342,587,200,946 and fill in the place value names and the periods.



10. a. Insert commas in the proper positions for the following whole number written in standard form: 5467010

b. Insert commas in the proper positions for the following whole number written in words: *seventy-two million four hundred twelve thousand six hundred thirty-five*

11. Write each number in words.

a. 40 **b.** 90

c. 68 **d.** 15

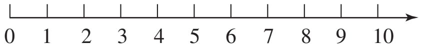
12. Write each number in standard form.

a. 8 ten thousands + 1 thousand + 6 hundreds + 9 tens + 2 ones

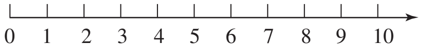
b. 900,000 + 60,000 + 5,000 + 300 + 40 + 7

Graph the following numbers on a number line.

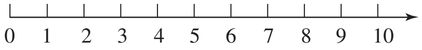
13. 1, 3, 5, 7



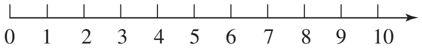
14. 0, 2, 4, 6, 8



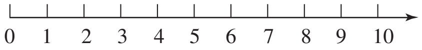
15. 2, 4, 5, 8



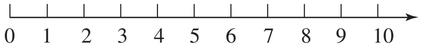
16. 2, 3, 5, 7, 9



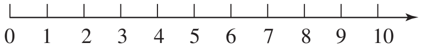
17. the whole numbers less than 6



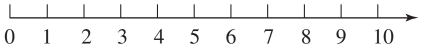
18. the whole numbers less than 9



19. the whole numbers between 2 and 8



20. the whole numbers between 0 and 6



NOTATION

Fill in the blanks.

21. The symbols { }, called _______, are used when writing a set.

22. The symbol > means ___ ________ _____, and the symbol < means ___ _____ _____.

GUIDED PRACTICE

Find the place values. **See Example 1.**

23. Consider the number 57,634.

a. What is the place value of the digit 3?

b. What digit is in the thousands column?

c. What is the place value of the digit 6?

d. What digit is in the ten thousands column?

24. Consider the number 128,940.

a. What is the place value of the digit 8?

b. What digit is in the hundreds column?

c. What is the place value of the digit 2?

d. What digit is in the hundred thousands column?

25. World hunger. On the website Freerice.com, sponsors donate grains of rice to feed the hungry. From 2007 through 2017, there have been 96,128,453,798 grains of rice donated.

- **a.** What is the place value of the digit 2?
- **b.** What digit is in the billions place?
- **c.** What is the place value of the digit 3?
- **d.** What digit is in the ten billions place?

26. YouTube views. According to the counter on YouTube, as of January 31, 2017, the video GANGNAM STYLE by PSY has been viewed 2,739,387,518 times.

- **a.** What is the place value of the digit 5?
- **b.** What digit is in the ten thousands place?
- **c.** What is the place value of the digit 2?
- **d.** What digit is in the hundred millions place?

Write each number in words. See Example 2.

27. 93
28. 48
29. 732
30. 259
31. 154,302
32. 615,019
33. 14,432,500
34. 104,052,005
35. 970,031,500,104
36. 5,800,010,700
37. 82,000,415
38. 51,000,201,078

Write each number in standard form. See Example 3.

39. Three thousand, seven hundred thirty-seven

40. Fifteen thousand, four hundred ninety-two

41. Nine hundred thirty

42. Six hundred forty

43. Seven thousand, twenty-one

44. Four thousand, five hundred

45. Twenty-six million, four hundred thirty-two

46. Ninety-two billion, eighteen thousand, three hundred ninety-nine

Write each number in expanded form. See Example 4.

47. 245
48. 518
49. 3,609
50. 3,961
51. 72,533
52. 73,009
53. 104,401
54. 570,003
55. 8,403,613
56. 3,519,807
57. 26,000,156
58. 48,000,061

Place an $<$ or an $>$ symbol in the box to make a true statement. See Example 5.

59. a. 11 ☐ 8 **b.** 29 ☐ 54

60. a. 410 ☐ 609 **b.** 3,206 ☐ 3,231

61. a. 12,321 ☐ 12,209 **b.** 23,223 ☐ 23,231

62. a. 178,989 ☐ 178,898 **b.** 850,234 ☐ 850,342

Round to the nearest ten. See Example 6.

63. 98,154
64. 26,742
65. 512,967
66. 621,116

Round to the nearest hundred. See Example 7.

67. 8,352
68. 1,845
69. 32,439
70. 73,931
71. 65,981
72. 5,346,975
73. 2,580,952
74. 3,428,961

Round each number to the nearest thousand and then to the nearest ten thousand. See Example 8.

75. 52,867
76. 85,432
77. 76,804
78. 34,209
79. 816,492
80. 535,600
81. 296,500
82. 498,903

TRY IT YOURSELF

83. Round 79,593 to the nearest …
a. ten **b.** hundred
c. thousand **d.** ten thousand

84. Round 5,925,830 to the nearest …
a. thousand **b.** ten thousand
c. hundred thousand **d.** million

85. Round $419,161 to the nearest …
a. $10 **b.** $100
c. $1,000 **d.** $10,000

86. Round 5,436,483 ft to the nearest …
a. 10 ft **b.** 100 ft
c. 1,000 ft **d.** 10,000 ft

Write each number in standard notation.

87. 4 ten thousands + 2 tens + 5 ones

88. 7 millions + 7 tens + 7 ones

89. 200,000 + 2,000 + 30 + 6

90. 7,000,000,000 + 300 + 50

91. Twenty-seven thousand, five hundred ninety-eight

92. Seven million, four hundred fifty-two thousand, eight hundred sixty

93. Ten million, seven hundred thousand, five hundred six

94. Eighty-six thousand, four hundred twelve

LOOK ALIKES

Write each number in standard notation.

95. a. One trillion, six hundred million
b. One billion, six hundred thousand
c. One million, six hundred

96. **a.** Ninety-nine billion, ninety-nine
  b. Eighty-eight million, eighty-eight
  c. Seventy-seven thousand, seventy-seven

97. **a.** 9 billion
  b. 9,000 million

98. **a.** 1 billion + 1 million + 1 thousand + 1 one
  b. 1,000,000,000 + 1,000,000 + 1,000 + 1

APPLICATIONS

99. Game shows. On *The Price Is Right* television show, the winning contestant is the person who comes closest to (without going over) the price of the item up for bid. Which contestant shown below will win if they are bidding on a bedroom set that has a suggested retail price of $4,745?

100. Presidents. The following list shows the ten youngest U.S. presidents and their ages (in years/days) when they took office. Construct a two-column table that presents the data in order, beginning with the youngest president.

J. Polk 49 yr/122 days	U. Grant 46 yr/236 days
G. Cleveland 47 yr/351 days	J. Kennedy 43 yr/236 days
W. Clinton 46 yr/154 days	F. Pierce 48 yr/101 days
M. Filmore 50 yr/184 days	B. Obama 47 yr/169 days
J. Garfield 49 yr/105 days	T. Roosevelt 42 yr/322 days

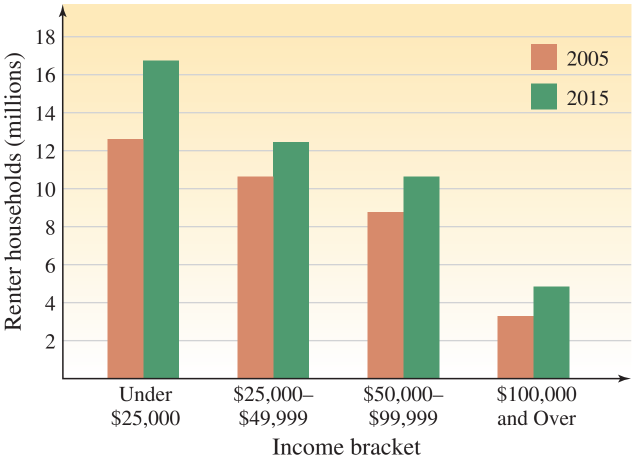
101. Renters. The number of renter households in the United States increased for all income brackets from 2005 to 2015. Use the graph in the next column to answer the following questions.

a. Which income bracket had the greatest number of renter households?

b. Which income bracket had the least number of renter households?

c. In 2015, for the income bracket $25,000–$49,999, estimate the number of renter households rounded to the nearest million.

d. In 2005, for the income bracket $100,000 and over, estimate the number of renter households rounded to the nearest million.



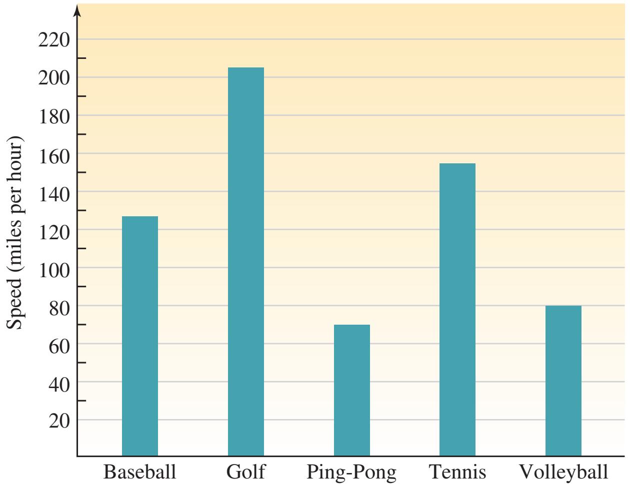
Source: JCHS tabulations of U.S. Census Bureau, Current Population Surveys

102. Sports. The graph shows the maximum recorded ball speeds for five sports.

a. Which sport had the fastest recorded maximum ball speed? Estimate the speed.

b. Which sport had the slowest maximum recorded ball speed? Estimate the speed.

c. Which sport had the second fastest maximum recorded ball speed? Estimate the speed.

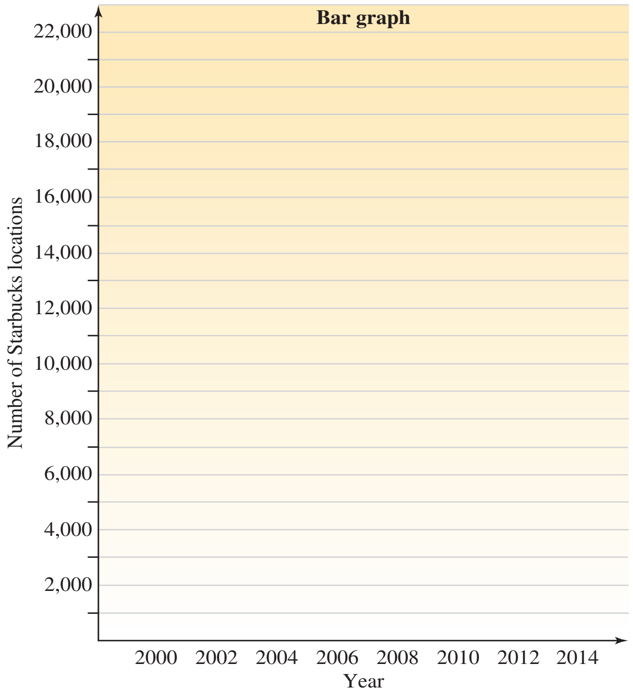


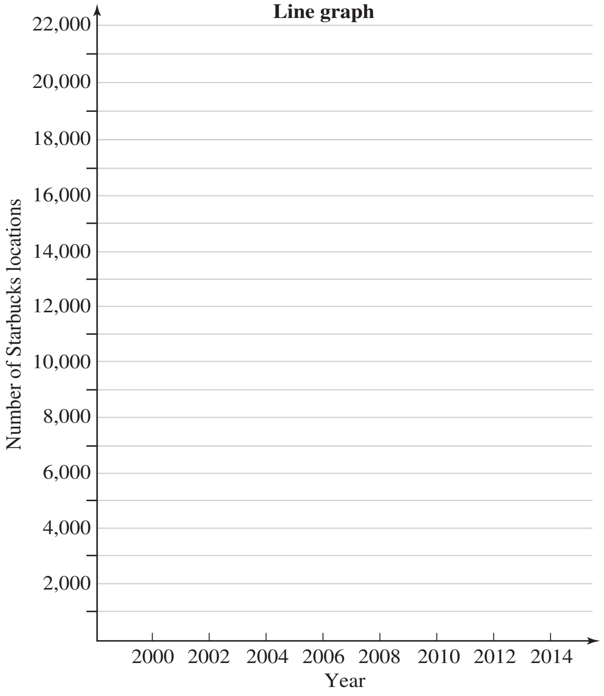
103. Coffee. Complete the bar graph and line graph on the next page using the data in the table.

Starbucks Locations

Year	Number
2000	3,501
2002	5,886
2004	8,569
2006	12,440
2008	16,680
2010	16,858
2012	17,651
2014	21,366

Source: Starbucks Company



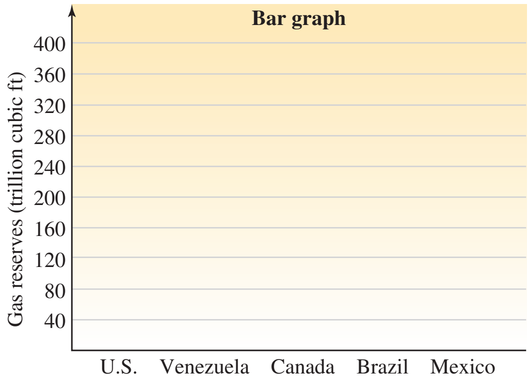


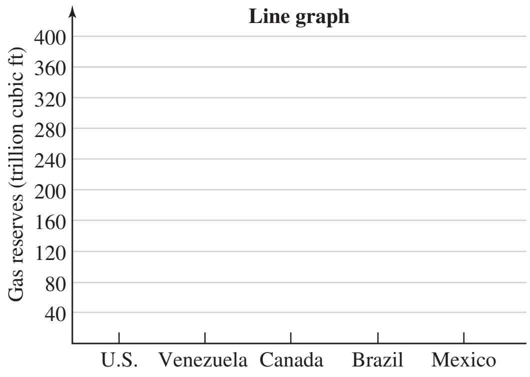
104. Energy reserves. Complete the bar graph and line graph in the next column using the data in the table.

Natural Gas Reserves, 2015 Estimates (in Trillion Cubic Feet)

United States	369
Venezuela	198
Canada	70
Brazil	15
Mexico	11

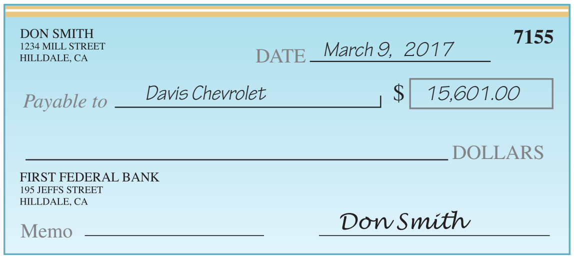
Source: BP Statistical Review of World Energy, 2015





105. Checking accounts. Complete each check by writing the amount in words on the proper line.

a.



DON SMITH
1234 MILL STREET
HILLDALE, CA

7155

DATE March 9, 2017

Payable to Davis Chevrolet $ 15,601.00

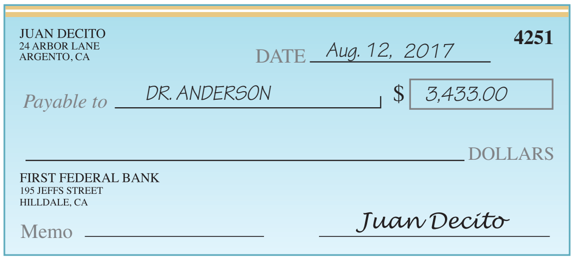
DOLLARS

FIRST FEDERAL BANK
195 JEFFS STREET
HILLDALE, CA

Memo

Don Smith

b.



JUAN DECITO
24 ARBOR LANE
ARGENTO, CA

4251

DATE Aug. 12, 2017

Payable to DR. ANDERSON $ 3,433.00

DOLLARS

FIRST FEDERAL BANK
195 JEFFS STREET
HILLDALE, CA

Memo

Juan Decito

106. Announcements. One style used when printing formal invitations and announcements is to write all numbers in words. Use this style to write each of the following phrases.

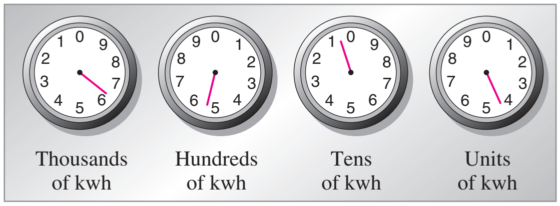
a. This diploma awarded this 27th day of June, 2017.

b. The suggested contribution for the fundraiser is $850 a plate, or an entire table may be purchased for $5,250.

107. Copyediting. Edit this excerpt from a history text by circling all numbers written in words and rewriting them in standard form using digits.

Abraham Lincoln was elected with a total of one million, eight hundred sixty-five thousand, five hundred ninety-three votes—four hundred eighty-two thousand, eight hundred eighty more than the runner-up, Stephen Douglas. He was assassinated after having served a total of one thousand, five hundred three days in office. Lincoln's Gettysburg Address, a mere two hundred sixty-nine words long, was delivered at the battle site where forty-three thousand, four hundred forty-nine casualties occurred.

108. Reading meters. The amount of electricity used in a household is measured in kilowatt-hours (kwh). Determine the reading on the meter shown below. (When the pointer is between two numbers, read the *smaller* number.)



109. Speed of light. The speed of light is 983,571,072 feet per second.

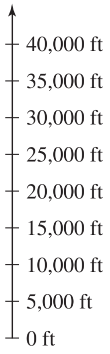
a. In what place-value column is the 5?

b. Round the speed of light to the nearest ten million. Give your answer in standard notation and in expanded notation.

c. Round the speed of light to the nearest hundred million. Give your answer in standard notation and in written-word form.

110. Clouds. Graph each cloud type given in the table at the proper altitude on the vertical number line below.

Cloud type	Altitude (ft)
Altocumulus	21,000
Cirrocumulus	37,000
Cirrus	38,000
Cumulonimbus	15,000
Cumulus	8,000
Stratocumulus	9,000
Stratus	4,000



WRITING

111. Explain how you would round 687 to the nearest ten.

112. The houses in a new subdivision are priced "in the low 130's." What does this mean?

113. A million is a thousand thousands. Explain why this is so.

114. Many television infomercials offer the viewer creative ways to make a six-figure income. What is a six-figure income? What is the smallest and what is the largest six-figure income?

115. What whole number is associated with each of the following words?

duo decade zilch a grand four score
dozen trio century a pair nil

116. Explain what is wrong by reading 20,003 as *twenty thousand and three.*

117. The words *two*, *to*, and *too* sound the same but are spelled differently and have different meanings. (Such words are called **homonyms**.) Write a sentence that contains all three words.

118. Write each statement in words.

a. $2{,}016 < 2{,}106$ **b.** $7{,}080{,}008 > 7{,}008{,}800$

SECTION 1.2 Adding Whole Numbers

OBJECTIVES

1 Add whole numbers.
2 Use properties of addition to add whole numbers.
3 Estimate sums of whole numbers.
4 Solve application problems by adding whole numbers.
5 Find the perimeter of a rectangle and a square.
6 Use a calculator to add whole numbers (optional).

Everyone uses addition of whole numbers. For example, to prepare an annual budget, an accountant adds separate line item costs. To determine the number of yearbooks to order, a principal adds the number of students in each grade level. A flight attendant adds the number of people in the first-class and economy sections to find the total number of passengers on an airplane.

OBJECTIVE 1 Add whole numbers.

To add whole numbers, think of combining sets of similar objects. For example, if a set of 4 stars is combined with a set of 5 stars, the result is a set of 9 stars.

We can write this addition problem in **horizontal** or **vertical form** using an **addition symbol +**, which is read as "plus." The numbers that are being added are called **addends**, and the answer is called the **sum** or **total**.

Horizontal form

$$\underset{\substack{\uparrow\\ \text{Addend}}}{4} \quad + \quad \underset{\substack{\uparrow\\ \text{Addend}}}{5} \quad = \quad \underset{\substack{\uparrow\\ \text{Sum}}}{9}$$

We read each form as "4 plus 5 equals (or is) 9."

Vertical form

$$\begin{array}{rl} 4 & \leftarrow \text{Addend} \\ +\ 5 & \leftarrow \text{Addend} \\ \hline 9 & \leftarrow \text{Sum} \end{array}$$

To add whole numbers that are less than 10, we rely on our understanding of basic addition facts. For example,

$$2 + 3 = 5, \qquad 6 + 4 = 10, \qquad \text{and} \qquad 9 + 7 = 16$$

If you need to review the basic addition facts, they can be found in Appendix 1 at the back of the book.

To add whole numbers that are greater than 10, we can use vertical form by stacking them with their corresponding place values lined up. Then we simply add the digits in each corresponding column.

EXAMPLE 1 Add: 421 + 123 + 245

Strategy We will write the addition in vertical form with the ones digits in a column, the tens digits in a column, and the hundreds digits in a column. Then we will add the digits, column by column, working from right to left.

WHY Like money, where pennies are only added to pennies, dimes are only added to dimes, and dollars are only added to dollars, we can only add digits with the same place value: ones to ones, tens to tens, hundreds to hundreds.

Solution

We start at the right and add the ones digits, then the tens digits, and finally the hundreds digits and write each sum below the horizontal bar.

Hundreds column
Tens column
Ones column

Vertical form

$$\begin{array}{rrrl} & 4 & 2 & 1 \\ & 1 & 2 & 3 \\ + & 2 & 4 & 5 \\ \hline & 7 & 8 & 9 & \leftarrow \text{The answer (sum)} \end{array}$$

Sum of the ones digits: Think: 1 + 3 + 5 = 9.
Sum of the tens digits: Think: 2 + 2 + 4 = 8.
Sum of the hundreds digits: Think: 4 + 1 + 2 = 7.

The sum is 789.

Self Check 1

Add: 131 + 232 + 221 + 312

Now Try ➔ **Problems 21 and 27**

If an addition of the digits in any place-value column produces a sum that is greater than 9, we must **carry**.

EXAMPLE 2 Add: 27 + 18

Strategy We will write the addition in vertical form and add the digits, column by column, working from right to left. We must watch for sums in any place-value column that are greater than 9.

WHY If the sum of the digits in any column is more than 9, we must carry.

Solution

To help you understand the process, each step of this addition is explained separately. Your solution need only look like the *last* step.

We begin by adding the digits in the ones column: 7 + 8 = 15. Because 15 = 1 ten + 5 ones, we write 5 in the ones column of the answer and carry 1 to the tens column.

$$\begin{array}{r} \overset{1}{2}\,7 \\ +\ 1\,8 \\ \hline 5 \end{array}$$

Add the digits in the ones column: 7 + 8 = 15. Carry 1 to the tens column.

Then we add the digits in the tens column.

$$\begin{array}{r} \overset{1}{2}\,7 \\ +\ 1\,8 \\ \hline 4\,5 \end{array}$$

Add the digits in the tens column: 1 + 2 + 1 = 4. Place the result of 4 in the tens column of the answer.

Your solution should look like this:

$$\begin{array}{r} \overset{1}{2}7 \\ +\ 18 \\ \hline 45 \end{array}$$

The sum is 45.

Self Check 2

Add: 35 + 47

Now Try ➡ **Problems 29 and 33**

EXAMPLE 3

Add: 9,835 + 692 + 7,275

Strategy We will write the numbers in vertical form so that corresponding place-value columns are lined up. Then we will add the digits in each column, watching for any sums that are greater than 9.

WHY If the sum of the digits in any column is more than 9, we must carry.

Solution

We write the addition in vertical form so that the corresponding digits are lined up. Each step of this addition is explained separately. Your solution need only look like the *last* step.

$$\begin{array}{r} 9,8\,\overset{1}{3}\,5 \\ 6\,9\,2 \\ +\ 7,2\,7\,5 \\ \hline 2 \end{array}$$

Add the digits in the ones column: 5 + 2 + 5 = 12. Write 2 in the ones column of the answer and carry 1 to the tens column.

$$\begin{array}{r} 9,\overset{2}{8}\,\overset{1}{3}\,5 \\ 6\,9\,2 \\ +\ 7,2\,7\,5 \\ \hline 0\,2 \end{array}$$

Add the digits in the tens column: 1 + 3 + 9 + 7 = 20. Write 0 in the tens column of the answer and carry 2 to the hundreds column.

$$\begin{array}{r} \overset{1}{9},\overset{2}{8}\,\overset{1}{3}\,5 \\ 6\,9\,2 \\ +\ 7,2\,7\,5 \\ \hline 8\,0\,2 \end{array}$$

Add the digits in the hundreds column: 2 + 8 + 6 + 2 = 18. Write 8 in the hundreds column of the answer and carry 1 to the thousands column.

$$\begin{array}{r} \overset{1}{9},\overset{2}{8}\,\overset{1}{3}\,5 \\ 6\,9\,2 \\ +\ 7,2\,7\,5 \\ \hline 17,8\,0\,2 \end{array}$$

Add the digits in the thousands column: 1 + 9 + 7 = 17. Write 7 in the thousands column of the answer. Write 1 in the ten thousands column.

Your solution should look like this:

$$\begin{array}{r} \overset{1}{9},\overset{2}{8}\overset{1}{3}5 \\ 692 \\ +\ 7,275 \\ \hline 17,802 \end{array}$$

The sum is 17,802.

Self Check 3

Add: 675 + 1,497 + 1,527

Now Try ➡ **Problems 37 and 41**

Success Tip In Example 3, the digits in each place-value column were added from *top to bottom*. To check the answer, we can instead add from *bottom to top*. Adding down or adding up should give the same result. If it does not, an error has been made and you should re-add. You will learn why the two results should be the same in Objective 2, which follows.

First add top to bottom

$$\begin{array}{r} 17{,}802 \\ \hline 9{,}835 \\ 692 \\ +\ 7{,}275 \\ \hline 17{,}802 \end{array}$$

To check, add bottom to top

OBJECTIVE 2 Use properties of addition to add whole numbers.

Have you ever noticed that two whole numbers can be added in either order because the result is the same? For example,

$$2 + 8 = 10 \qquad \text{and} \qquad 8 + 2 = 10$$

This example illustrates the **commutative property of addition.**

Commutative Property of Addition

The order in which whole numbers are added does not change their sum.

For example,

$$6 + 5 = 5 + 6$$

LANGUAGE OF MATHEMATICS

Commutative is a form of the word *commute*, meaning to go back and forth. *Commuter* trains take people to and from work.

To find the sum of three whole numbers, we add two of them and then add the sum to the third number. In the following examples, we add 3 + 4 + 7 in two ways. We will use the grouping symbols ( ), called **parentheses**, to show this. It is standard practice to perform the operations within the parentheses first. The steps of the solutions are written in horizontal form.

LANGUAGE OF MATHEMATICS

We read (3 + 4) + 7 as "The ***quantity*** of 3 plus 4," pause slightly, and then say "plus 7." We read 3 + (4 + 7) as, "3 plus the *quantity* of 4 plus 7." The word *quantity* alerts the reader to the parentheses that are used as grouping symbols.

Method 1: Group 3 and 4

$$(3 + 4) + 7 = 7 + 7$$
$$= 14$$

Because of the parentheses, add 3 and 4 first to get 7. Then add 7 and 7 to get 14.

Method 2: Group 4 and 7

$$3 + (4 + 7) = 3 + 11$$
$$= 14$$

Because of the parentheses, add 4 and 7 first to get 11. Then add 3 and 11 to get 14.

Same result

Either way, the answer is 14. This example illustrates that changing the grouping when adding numbers doesn't affect the result. This property is called the **associative property of addition**.

Associative Property of Addition

The way in which whole numbers are grouped does not change their sum.

For example,

$$(2 + 5) + 4 = 2 + (5 + 4)$$

LANGUAGE OF MATHEMATICS

Associative is a form of the word *associate*, meaning to join a group. The WNBA (Women's National Basketball *Association*) is a group of 12 professional basketball teams.

Sometimes, an application of the associative property can simplify a calculation.

EXAMPLE 4 Find the sum: $98 + (2 + 17)$

Strategy We will use the associative property to group 2 with 98.

WHY It is helpful to regroup because 98 and 2 are a pair of numbers that are easily added.

Solution

We will write the steps of the solution in horizontal form.

$$\begin{aligned} 98 + (2 + 17) &= (\mathbf{98 + 2}) + 17 && \text{Use the associative property of addition to regroup the addends.} \\ &= \mathbf{100} + 17 && \text{Do the addition within the parentheses first.} \\ &= 117 \end{aligned}$$

Self Check 4

Find the sum: $(139 + 25) + 75$

Now Try Problems 45 and 49

Whenever we add 0 to a whole number, the number is unchanged. This property is called the **addition property of 0**.

Addition Property of 0

The sum of any whole number and 0 is that whole number.

For example,

$3 + 0 = 3$, $5 + 0 = 5$, and $0 + 9 = 9$

EXAMPLE 5 Add: **a.** $3 + 5 + 17 + 2 + 3$ **b.**

$$\begin{array}{r} 201 \\ 867 \\ +\ \ 49 \\ \hline \end{array}$$

Strategy We will look for groups of two (or three numbers) whose sum is 10 or 20 or 30, and so on.

WHY This method is easier than adding unrelated numbers, and it reduces the chances of a mistake.

Solution

Together, the commutative and associative properties of addition enable us to use any order or grouping to add whole numbers.

a. We will write the steps of the solution in horizontal form.

$$\begin{aligned} \mathbf{3} + 5 + \mathbf{17} + 2 + 3 &= \mathbf{20} + 10 && \text{Think: } 3 + 17 = 20 \text{ and } 5 + 2 + 3 = 10. \\ &= 30 \end{aligned}$$

b. Each step of the addition is explained separately. Your solution should look like the last step.

$$\begin{array}{cccc} & & 1 & \\ & 2 & 0 & \mathbf{1} \\ & 8 & 6 & \mathbf{7} \\ + & & 4 & \mathbf{9} \\ \hline & & & 7 \end{array}$$

Add the bold numbers in the ones column first.
Think: $(9 + 1) + 7 = 10 + 7 = 17$.
Write the 7 and carry the 1.

$$\begin{array}{cccc} & 1 & 1 & \\ & 2 & 0 & 1 \\ & 8 & \mathbf{6} & 7 \\ + & & \mathbf{4} & 9 \\ \hline & & 1 & 7 \end{array}$$

Add the bold numbers in the tens column.
Think: $(6 + 4) + 1 = 10 + 1 = 11$.
Write the 1 and carry the 1.

$$\begin{array}{r} \overset{1}{\mathbf{2}}\;\overset{1}{0}\;1 \\ \mathbf{8}\;6\;7 \\ +\quad 4\;9 \\ \hline 1{,}1\;1\;7 \end{array}$$

Add the bold numbers in the hundreds column.
Think: (2 + 8) + 1 = 10 + 1 = 11.

The sum is 1,117.

Self Check 5

Add:

a. 14 + 7 + 16 + 1 + 2

b.

$$\begin{array}{r} 675 \\ 204 \\ +435 \\ \hline \end{array}$$

Now Try ➔ Problems 53 and 57

OBJECTIVE 3 Estimate sums of whole numbers.

Estimation is used to find an approximate answer to a problem. Estimates are helpful in two ways. First, they serve as an accuracy check that can find errors. If an answer does not seem reasonable when compared to the estimate, the original problem should be reworked. Second, some situations call for only an approximate answer rather than the exact answer.

There are several ways to estimate, but the objective is the same: Simplify the numbers in the problem so that the calculations can be made easily and quickly. One popular method of estimation is called **front-end rounding**.

Success Tip Estimates can be greater than or less than the exact answer. It depends on how often rounding up and rounding down occurs in the estimation.

EXAMPLE 6 Use front-end rounding to estimate the sum: 3,714 + 2,489 + 781 + 5,500 + 303

Strategy We will use front-end rounding to approximate each addend. Then we will find the sum of the approximations.

WHY Front-end rounding produces addends containing many 0's. Such numbers are easier to add.

Solution

Each of the addends is rounded to its *largest place value* so that all but its first digit is zero. Then we add the approximations using vertical form.

$$\begin{array}{rcrl} 3{,}714 & \longrightarrow & 4{,}000 & \text{Round to the nearest thousand.} \\ 2{,}489 & \longrightarrow & 2{,}000 & \text{Round to the nearest thousand.} \\ 781 & \longrightarrow & 800 & \text{Round to the nearest hundred.} \\ 5{,}500 & \longrightarrow & 6{,}000 & \text{Round to the nearest thousand.} \\ +\ \ 303 & \longrightarrow & +\ \ 300 & \text{Round to the nearest hundred.} \\ \cline{1-1}\cline{3-3} & & 13{,}100 & \end{array}$$

The estimate is 13,100.

If we calculate 3,714 + 2,489 + 781 + 5,500 + 303, the sum is exactly 12,787. Note that the estimate is close: It's just 313 more than 12,787. This illustrates the tradeoff when using estimation: The calculations are easier to perform and take less time, but the answers are not exact.

Self Check 6

Use front-end rounding to estimate the sum:

$$\begin{array}{r} 6{,}780 \\ 3{,}278 \\ 566 \\ 4{,}230 \\ +1{,}923 \\ \hline \end{array}$$

Now Try ➔ Problem 61

OBJECTIVE 4 Solve application problems by adding whole numbers.

Since application problems are almost always written in words, the ability to understand what you read is very important.

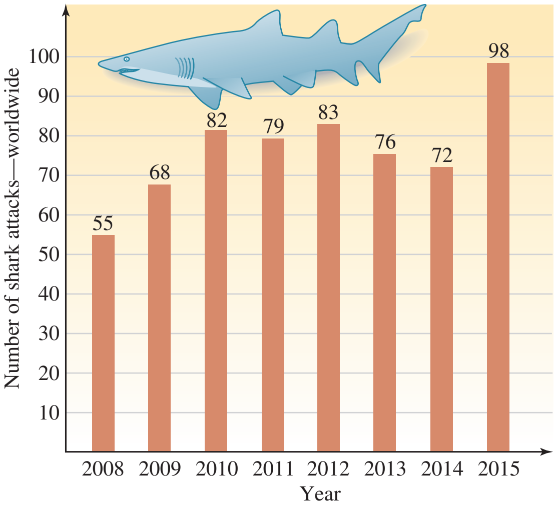
LANGUAGE OF MATHEMATICS

Here are some key words and phrases that are often used to indicate ***addition***:

gain	*total*
increase	*combined*
up	*in all*
forward	*in the future*
rise	*altogether*
more than	*extra*

EXAMPLE 7

Sharks. The graph on the right shows the number of shark attacks worldwide for the years 2008 through 2015. Find the total number of shark attacks for those years.



Source: Florida Museum of Natural History, Ichthyology Department

Strategy We will carefully read the problem looking for a key word or phrase.

WHY Key words and phrases indicate which arithmetic operation(s) should be used to solve the problem.

Solution

In the second sentence of the problem, the key word *total* indicates that we should add the number of shark attacks for the years 2008 through 2015. We can use vertical form to find the sum.

$$\begin{array}{r} {}^{6\,4}\\ 55\\ 68\\ 82\\ 79\\ 83\\ 76\\ 72\\ +\ 98\\ \hline 613 \end{array}$$

Add the digits, one column at a time, working from right to left. To simplify the calculations, we can look for groups of two or three numbers in each column whose sum is 10.

The total number of shark attacks worldwide for the years 2008 through 2015 was 613.

Self Check 7

Airline accidents. The numbers of accidents involving U.S. airlines for the years 2008 through 2015 are listed in the table below. Find the total number of accidents for those years.

Year	Accidents
2008	20
2009	26
2010	28
2011	29
2012	26
2013	19
2014	28
2015	27

Source: National Transportation Safety Board

Now Try ➡ **Problem 104**

LANGUAGE OF MATHEMATICS

To solve application problems, we must often ***translate*** the words of the problem to numbers and symbols. To translate means to change from one form to another, as in translating from Spanish to English.

EXAMPLE 8

iStock.com/kjekol

Endangered wolves. In 1989, there were 1,814 gray wolves in the western Great Lakes region of Minnesota, Wisconsin, and Michigan. By 2015, the number had increased by 1,792. Find the number of gray wolves in 2015 in that region. (Source: U.S. Fish and Wildlife Service)

Strategy We will carefully read the problem looking for key words or phrases.

WHY Key words and phrases indicate which arithmetic operation(s) should be used to solve the problem.

Solution

The phrase *increased by* indicates addition. With that in mind, we translate the words of the problem to numbers and symbols.

The number of gray wolves in 2015 is equal to the number of gray wolves in 1989 increased by 1,792.

The number of gray wolves in 2015 = 1,814 + 1,792

Use vertical form to perform the addition:

$$\begin{array}{r} {\scriptstyle 1\;1\;\;\;\;} \\ 1{,}814 \\ +\,1{,}792 \\ \hline 3{,}606 \end{array}$$

In 2015, the number of gray wolves in the western Great Lakes region was 3,606.

Self Check 8

Magazines. In 2011, the monthly circulation of *Game Informer* magazine was 5,954,884 copies. By 2016, the circulation had increased by 398,191 copies per month. What was the monthly circulation of *Game Informer* magazine in 2016? (Source: *The World Almanac and Book of Facts*, 2012, 2017)

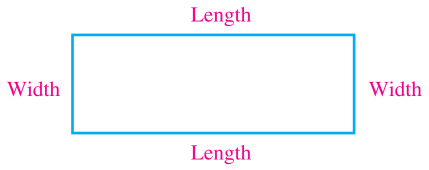
Now Try ➜ **Problem 96**

OBJECTIVE 5 Find the perimeter of a rectangle and a square.

Figure (a) below is an example of a four-sided figure called a **rectangle**. Either of the longer sides of a rectangle is called its **length** and either of the shorter sides is called its **width**. Together, the length and width are called the **dimensions** of the rectangle. For any rectangle, opposite sides have the same measure.

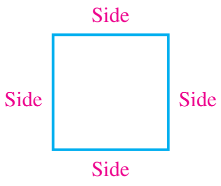
When all four of the sides of a rectangle are the same length, we call the rectangle a **square**. An example of a square is shown in figure (b).

A rectangle



(a)

A square



(b)

LANGUAGE OF MATHEMATICS

When you hear the word ***perimeter***, think of the distance around the "rim" of a flat figure.

The distance around a rectangle or a square is called its **perimeter**. To find the perimeter of a rectangle, we add the lengths of its four sides.

The perimeter of a rectangle = length + length + width + width

To find the perimeter of a square, we add the lengths of its four sides.

The perimeter of a square = side + side + side + side

EXAMPLE 9 **Money.** Find the perimeter of the dollar bill shown below.

mm stands for millimeters

Strategy We will add two lengths and two widths of the dollar bill.

WHY A dollar bill is rectangular-shaped, and this is how the perimeter of a rectangle is found.

Solution

We translate the words of the problem to numbers and symbols.

The perimeter of the dollar bill	is equal to	the length of the dollar bill	plus	the length of the dollar bill	plus	the width of the dollar bill	plus	the width of the dollar bill.
The perimeter of the dollar bill	=	156	+	156	+	65	+	65

Use vertical form to perform the addition:

$$\begin{array}{r} \overset{2\,2}{1}56 \\ 156 \\ 65 \\ +\ \ 65 \\ \hline 442 \end{array}$$

The perimeter of the dollar bill is 442 mm.

To see whether this result is reasonable, we estimate the answer. Because the rectangle is about 160 mm by 70 mm, its perimeter is approximately $160 + 160 + 70 + 70$, or 460 mm. An answer of 442 mm is reasonable.

Self Check 9

Board games. A Monopoly game board is a square with sides 19 inches long. Find the perimeter of the board.

Now Try ➲ **Problems 65 and 67**

OBJECTIVE 6 Use a calculator to add whole numbers (optional).

Calculators are useful for making lengthy calculations and checking results. They should not, however, be used until you have a solid understanding of the basic arithmetic facts. This textbook ***does not*** require you to have a calculator. Ask your instructor if you are allowed to use a calculator in the course.

The *Using Your Calculator* feature explains the keystrokes for an inexpensive scientific calculator. If you have any questions about your specific model, see your user's manual.

Using Your Calculator ▶ The Addition Key: Vehicle Production

In 2015, the top five producers of motor vehicles in the world were Toyota: 10,083,831; Volkswagen: 9,872,424; Hyundai: 7,988,479; General Motors: 7,485,587; and Ford: 6,396,369 (Source: OICA, 2015). We can find the total number of motor vehicles produced by these companies using the addition key $\boxed{+}$ on a calculator.

10083831 $\boxed{+}$ 9872424 $\boxed{+}$ 7988479 $\boxed{+}$ 7485587 $\boxed{+}$ 6396369 $\boxed{=}$ $\boxed{41826690}$

On some calculator models, the $\boxed{\text{Enter}}$ key is pressed instead of the $\boxed{=}$ for the result to be displayed. The total number of vehicles produced in 2015 by the top five automakers was 41,826,690.

Answers to Self Checks

1. 896 **2.** 82 **3.** 3,699 **4.** 239 **5. a.** 40 **b.** 1,314 **6.** 16,600 **7.** The total number of accidents for 2008–2015 was 203. **8.** The monthly circulation in 2016 was 6,353,075. **9.** The perimeter of the Monopoly board is 76 in.

SECTION 1.2 STUDY SET

VOCABULARY

Fill in the blanks.

1. In the addition problem shown below, label each *addend* and the *sum*.

$$\underset{\boxed{}}{\overset{\uparrow}{10}} + \underset{\boxed{}}{\overset{\uparrow}{15}} = \underset{\boxed{}}{\overset{\uparrow}{25}}$$

2. When using the vertical form to add whole numbers, if the addition of the digits in any one column produces a sum greater than 9, we must _____.

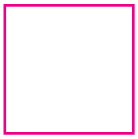
3. The __________ property of addition states that the order in which whole numbers are added does not change their sum.

4. The _________ property of addition states that the way in which whole numbers are grouped does not change their sum.

5. To see whether the result of an addition is reasonable, we can round the addends and _______ the sum.

6. The words *rise*, *gain*, *total*, and *increase* are often used to indicate the operation of _______.

7. The figure below on the left is an example of a ________. The figure on the right is an example of a ______.



8. Label the *length* and the *width* of the rectangle below. Together, the length and width of a rectangle are called its ___________.

9. When all the sides of a rectangle are the same length, we call the rectangle a ______.

10. The distance around a rectangle is called its _________.

CONCEPTS

11. Which property of addition is shown?

a. $3 + 4 = 4 + 3$

b. $(3 + 4) + 5 = 3 + (4 + 5)$

c. $(36 + 58) + 32 = 36 + (58 + 32)$

d. $319 + 507 = 507 + 319$

Fill in the blanks.

12. a. Use the commutative property of addition to complete the following:

$19 + 33 = \boxed{}$

b. Use the associative property of addition to complete the following:

$3 + (97 + 16) = \boxed{}$

13. Fill in the blank: Any number added to $\boxed{}$ stays the same.

14. Fill in the blanks. Use estimation by front-end rounding to determine if the sum shown below (14,735) seems reasonable.

$$\begin{array}{rcl} 5{,}877 & \rightarrow & \boxed{} \\ 402 & \rightarrow & \boxed{} \\ \underline{+8{,}456} & \rightarrow & \underline{+\boxed{}} \\ 14{,}735 & \rightarrow & \boxed{} \end{array}$$

NOTATION

Fill in the blanks.

15. The addition symbol + is read as " ____."

16. The symbols ( ) are called __________. It is standard practice to perform the operations within them ____.

Write each of the following addition facts in words.

17. $33 + 12 = 45$

18. $28 + 22 = 50$

Complete each step to find the sum.

19. $(36 + 11) + 5 = \boxed{} + 5$

$= \boxed{}$

20. $12 + (15 + 2) = 12 + \boxed{}$

$= \boxed{}$

GUIDED PRACTICE

Add. **See Example 1.**

21. $25 + 13$

22. $47 + 12$

23. $\begin{array}{r} 406 \\ \underline{+283} \end{array}$

24. $\begin{array}{r} 213 \\ \underline{+751} \end{array}$

25. $21 + 31 + 24$

26. $33 + 43 + 12$

27. $603 + 152 + 121$

28. $462 + 115 + 220$

Add. **See Example 2.**

29. $19 + 16$

30. $27 + 18$

31. $45 + 47$

32. $37 + 26$

33. $52 + 18$

34. $59 + 31$

35. $\begin{array}{r} 28 \\ \underline{+47} \end{array}$

36. $\begin{array}{r} 35 \\ \underline{+49} \end{array}$

Add. **See Example 3.**

37. $156 + 305$

38. $647 + 138$

39. $4{,}301 + 789 + 3{,}847$

40. $5{,}576 + 649 + 1{,}922$

41. 9,758 + 586 + 7,799

42. 9,339 + 471 + 6,883

43. $\begin{array}{r} 346 \\ 217 \\ 568 \\ +679 \\ \hline \end{array}$

44. $\begin{array}{r} 290 \\ 859 \\ 345 \\ +226 \\ \hline \end{array}$

Apply the associative property of addition to find the sum. See Example 4.

45. (9 + 3) + 7

46. (7 + 9) + 1

47. (13 + 8) + 12

48. (19 + 7) + 13

49. 94 + (6 + 37)

50. 92 + (8 + 88)

51. 125 + (75 + 41)

52. 240 + (60 + 93)

Use the commutative and associative properties of addition to find the sum. See Example 5.

53. 4 + 8 + 16 + 1 + 1

54. 2 + 1 + 28 + 3 + 6

55. 23 + 5 + 7 + 15 + 10

56. 31 + 6 + 9 + 14 + 20

57. $\begin{array}{r} 624 \\ 905 \\ +\ \ 86 \\ \hline \end{array}$

58. $\begin{array}{r} 495 \\ 76 \\ +835 \\ \hline \end{array}$

59. 457 + 97 + 653

60. 562 + 99 + 848

Use front-end rounding to estimate the sum. See Example 6.

61. 686 + 789 + 12,233 + 24,500 + 5,768

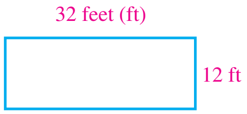
62. 404 + 389 + 11,802 + 36,902 + 7,777

63. 567,897 + 23,943 + 309,900 + 99,113

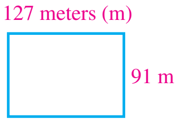
64. 822,365 + 15,444 + 302,417 + 99,010

Find the perimeter of each rectangle or square. See Example 9.

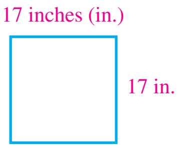
65.



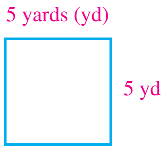
66.



67.



68.



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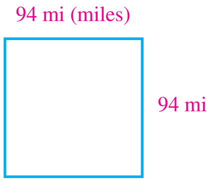

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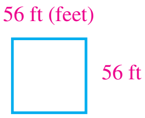
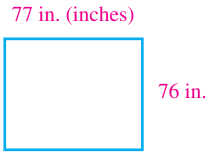


71.

72.



TRY IT YOURSELF

Add.

73. $\begin{array}{r} 8,539 \\ +7,368 \\ \hline \end{array}$

74. $\begin{array}{r} 5,799 \\ +6,879 \\ \hline \end{array}$

75. 51,246 + 578 + 37 + 4,599

76. 4,689 + 73,422 + 26 + 433

77. (45 + 16) + 4

78. 7 + (63 + 23)

79. $\begin{array}{r} 632 \\ +347 \\ \hline \end{array}$

80. $\begin{array}{r} 423 \\ +570 \\ \hline \end{array}$

81. 16,427 increased by 13,573

82. 13,567 more than 18,788

83. $\begin{array}{r} 76 \\ +\ \ 45 \\ \hline \end{array}$

84. $\begin{array}{r} 87 \\ +\ \ 56 \\ \hline \end{array}$

85. 3,156 + 1,578 + 6,578

86. 2,379 + 4,779 + 2,339

87. 12 + 1 + 8 + 4 + 9 + 16

88. 7 + 15 + 13 + 9 + 5 + 11

LOOK ALIKES

Find the answer to the problem in part a. The answer to part b should then be obvious.

89. **a.** 299 increased by 99
b. 99 increased by 299

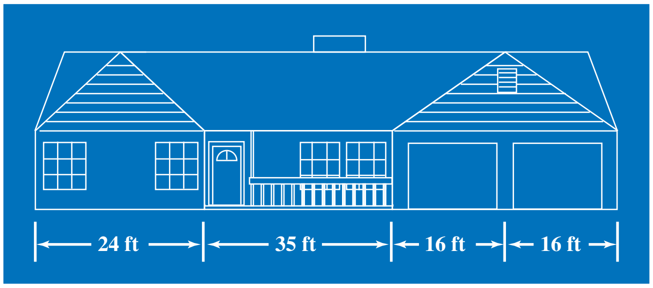
90. **a.** 3,068 more than 368
b. 368 more than 3,068

91. **a.** $\begin{array}{r} 747 \\ +\ \ 252 \\ \hline \end{array}$ **b.** $\begin{array}{r} 252 \\ +\ \ 747 \\ \hline \end{array}$

92. **a.** (913 + 87) + 688
b. 913 + (87 + 688)

APPLICATIONS

93. Dimensions of a house. Find the length of the house shown in the blueprint.



94. Rockets. A Saturn V rocket was used to launch the crew of *Apollo 11* to the moon. The first stage of the rocket was 138 feet tall, the second stage was 98 feet tall, and the third stage was 46 feet tall. Atop the third stage sat the 54-foot-tall lunar module and a 28-foot-tall escape tower. What was the total height of the spacecraft?

95. Fast food. Find the total number of calories in the following lunch from McDonald's: Big Mac (540 calories), French fries (230 calories), Fruit 'n Yogurt Parfait (150 calories), medium Coca-Cola Classic (170 calories).

96. Chief Executive Officer. In 2015, Robert A. Iger, CEO of Walt Disney Company, had a base salary of $2,548,077. He also earned an additional $42,365,536 in Stock and Option Awards, Incentive Plan compensation, pension, and other compensation. Find Robert Iger's total compensation in 2015. (Source: aflcio.org)

97. Websites. The number of persons age 15 or older who visited the Apple iTunes website at least once in June 2016 was 243,547,000. The number that visited the Alibaba website during that month was 64,511,000 greater than the iTunes site. How many visitors did the Alibaba site have? (Source: *The World Almanac and Book of Facts, 2017*)

98. Ice cream. Baskin-Robbins is the world's largest chain of ice cream specialty shops. In 2016, there were 2,524 stores in the United States, and 5,198 stores in 50 other countries of the world. Find the total number of Baskin-Robbins stores in 2016. (Source: entrepreneur.com)

99. Bridge safety. The results of a 2017 report of the condition of U.S. highway bridges is shown below. Each bridge was classified as either *safe*, *in need of repair*, or *should be replaced.* Complete the table.

Number of safe bridges	Number of bridges that need repair	Number of outdated bridges that should be replaced	Total number of bridges
464,859	61,365	84,525	

Source: Bureau of Transportation Statistics

100. Imports. The table below shows the number of new and used passenger cars imported into the United States from various countries in 2015. Find the total number of cars the United States imported from these countries.

Country	Number of passenger cars
Canada	1,969,466
France	28,024
Germany	639,838
Italy	132,316
Japan	1,609,597
Mexico	1,438,840
South Korea	1,065,971
Sweden	37,789
United Kingdom	134,367

Source: Economic Indicators Division, U.S. Census Bureau

101. Weddings. The average wedding costs for 2015 are listed in the table below. Find the total cost of a wedding.

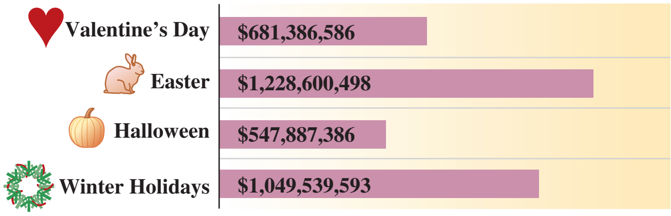
Clothing/hair/makeup	$1,947
Ceremony/music/flowers	$5,358
Photography/video	$4,442
Favors/gifts/invitations	$712
Jewelry	$5,871
Reception	$12,540
Honeymoon	$3,882

Source: jennlanedesign.com, prettypracticalbride.com

102. Budgets. A department head in a company prepared an annual budget with the line items shown. Find the projected number of dollars to be spent.

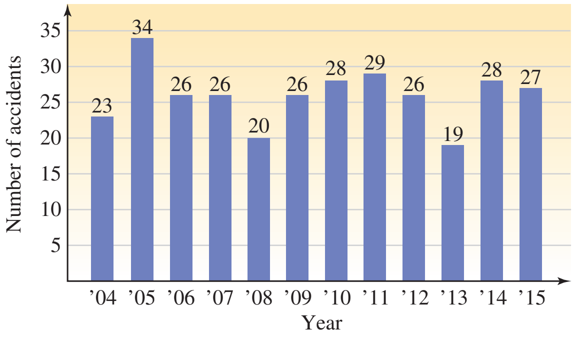
Line item	Amount
Equipment	$17,242
Utilities	$5,443
Travel	$2,775
Supplies	$10,553
Development	$3,225
Maintenance	$1,075

103. Candy. The graph below shows U.S. candy sales in 2016 during four holiday periods. Find the sum of these seasonal candy sales.



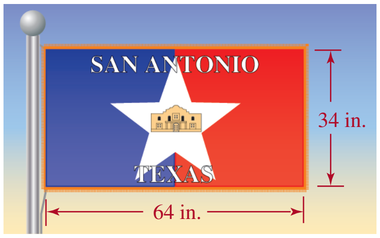
Source: Nielsen AOD

104. Airline safety. The following graph shows the U.S. passenger airlines accident report for the years 2004–2015. How many accidents were there in this 12-year time span?



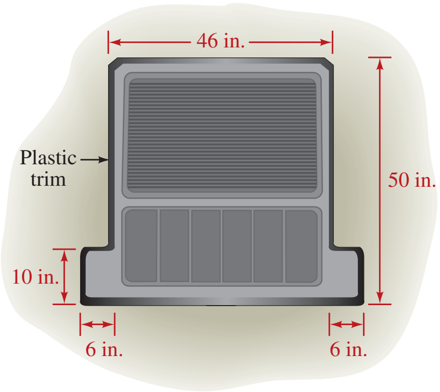
Source: National Transportation Safety Board

105. Flags. To decorate a city flag, yellow fringe is to be sewn around its outside edges, as shown. The fringe is sold by the inch. How many inches of fringe must be purchased to complete the project?



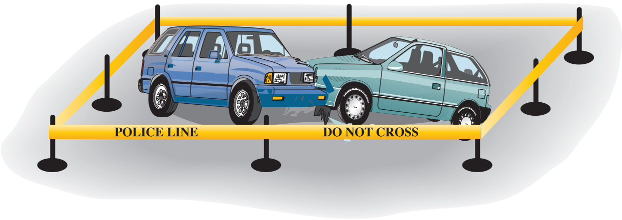
106. Decorating. A child's bedroom is rectangular in shape with dimensions 15 feet by 11 feet. How many feet of wallpaper border are needed to wrap around the entire room?

107. Floor mats. Estimate the amount of plastic trim used around the floor mat shown below.



108. Fences. A square piece of land measuring 209 feet on all four sides is approximately one *acre*. How many feet of chain-link fencing are needed to enclose a piece of land this size?

109. Traffic accidents. Police used an entire roll of yellow "DO NOT CROSS" barricade tape to seal off a rectangular region around an automobile accident, as shown below. The width of the rectangle was 50 feet and the length was 25 feet more than that. How long was the roll of yellow tape?



110. Arc welding. A "bead" of welding is placed around the outside edge of a square steel plate with sides 37 inches long. How long is the entire weld "bead"?

WRITING

111. Explain why the operation of addition is commutative.

112. Explain why the operation of addition is associative.

113. In this section, it is said that estimation is a *tradeoff.* Give one benefit and one drawback of estimation.

114. A student added three whole numbers top to bottom and then bottom to top, as shown below. What does the result in red indicate? What should the student do next?

$$\begin{array}{r} 1{,}689 \\ \hline 496 \\ 315 \\ +\ \ 788 \\ \hline 1{,}599 \end{array}$$

REVIEW

115. Write each number in expanded notation.

a. 3,125 **b.** 60,037

116. Round 6,354,784 to the nearest …

a. ten **b.** hundred

c. ten thousand **d.** hundred thousand

SECTION 1.3 Subtracting Whole Numbers

Everyone uses subtraction of whole numbers. For example, to find the sale price of an item, a store clerk subtracts the discount from the regular price. To measure climate change, a scientist subtracts the high and low temperatures. A trucker subtracts odometer readings to calculate the number of miles driven on a trip.

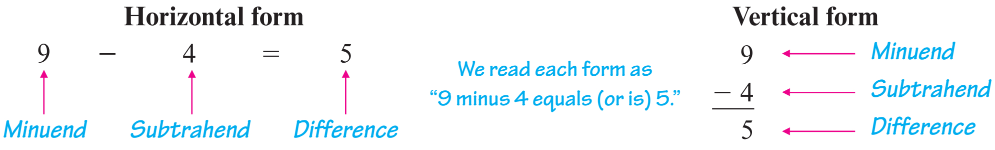
OBJECTIVES

1. Subtract whole numbers.
2. Subtract whole numbers with borrowing.
3. Check subtractions using addition.
4. Estimate differences of whole numbers.
5. Solve application problems by subtracting whole numbers.
6. Evaluate expressions involving addition and subtraction.

OBJECTIVE 1 Subtract whole numbers.

To subtract two whole numbers, think of taking away objects from a set. For example, if we start with a set of 9 stars and take away a set of 4 stars, a set of 5 stars is left.

We can write this subtraction problem in **horizontal** or **vertical form** using a **subtraction symbol** −, which is read as "minus." We call the number from which another number is subtracted the **minuend**. The number being subtracted is called the **subtrahend**, and the answer is called the **difference**.



LANGUAGE OF MATHEMATICS

The prefix ***sub*** means *below*, as in *sub*marine or *sub*way. Notice that in vertical form, the *sub*trahend is written below the minuend.

To subtract two whole numbers that are less than 10, we rely on our understanding of basic subtraction facts. For example,

$$6 - 3 = 3, \qquad 7 - 2 = 5, \qquad \text{and} \qquad 9 - 8 = 1$$

To subtract two whole numbers that are greater than 10, we can use vertical form by stacking them with their corresponding place values lined up. Then we simply subtract the digits in each corresponding column.

EXAMPLE 1 Subtract: $59 - 27$

Strategy We will write the subtraction in vertical form with the ones digits in a column and the tens digits in a column. Then we will subtract the digits in each column, working from right to left.

WHY Like money, where pennies are only subtracted from pennies and dimes are only subtracted from dimes, we can only subtract digits with the same place value—ones from ones and tens from tens.

Solution

We start at the right and subtract the ones digits and then the tens digits, and write each difference below the horizontal bar.

Tens column

Ones column

Vertical form

$$\begin{array}{r} 5\;9 \\ -\;2\;7 \\ \hline 3\;2 \end{array} \leftarrow \text{The answer (difference)}$$

Difference of the ones digits: Think 9 − 7 = 2.

Difference of the tens digits: Think 5 − 2 = 3.

The difference is 32.

Self Check 1

Subtract: 68 − 31

Now Try Problems 15 and 21

EXAMPLE 2 Subtract 235 from 6,496.

Strategy We will translate the sentence to mathematical symbols and then perform the subtraction. We must be careful when translating the instruction to subtract one number *from* another number.

WHY The order of the numbers in the sentence must be reversed when we translate to symbols.

Solution

Since 235 is the number to be subtracted, it is the subtrahend.

Subtract 235 from 6,496.

6,496 − 235

To find the difference, we write the subtraction in vertical form and subtract the digits in each column, working from right to left.

$$\begin{array}{r} 6{,}496 \\ -\quad 235 \\ \hline 6{,}261 \end{array}$$

Bring down the 6 in the thousands column.

When 235 is subtracted from 6,496, the difference is 6,261.

Caution! When subtracting two numbers, it is very important that we write them in the correct order, because subtraction is *not* commutative. For instance, in Example 2, if we had incorrectly translated "*Subtract 235 from 6,496*" as 235 − 6,496, we see that the difference is not 6,261. In fact, the difference is not even a whole number.

Self Check 2

Subtract: 817 from 1,958.

Now Try Problem 23

OBJECTIVE 2 Subtract whole numbers with borrowing.

If the subtraction of the digits in any place-value column requires that we subtract a larger digit from a smaller digit, we must **borrow** or **regroup**.

EXAMPLE 3 Subtract:
$$\begin{array}{r} 32 \\ -15 \\ \hline \end{array}$$

Strategy As we prepare to subtract in each column, we will compare the digit in the subtrahend (bottom number) to the digit directly above it in the minuend (top number).

WHY If a digit in the subtrahend is greater than the digit directly above it in the minuend, we must borrow (regroup) to subtract in that column.

Solution

To help you understand the process, each step of this subtraction is explained separately. Your solution need only look like the *last* step.

We write the subtraction in vertical form to line up the tens digits and line up the ones digits.

$$\begin{array}{r} 32 \\ -15 \\ \hline \end{array}$$

Since 5 in the ones column of 15 is greater than 2 in the ones column of 32, we cannot immediately subtract in that column because 2 − 5 is *not* a whole number. To subtract in the ones column, we must regroup by borrowing 1 ten from 3 in the tens column. In this regrouping process, we use the fact that 1 ten = 10 ones.

$$\begin{array}{rr} \overset{2}{\cancel{3}} & \overset{12}{\cancel{2}} \\ -\,1 & 5 \\ \hline & 7 \end{array}$$

Borrow 1 ten from 3 in the tens column and change the 3 to 2. Add the borrowed 10 to the digit 2 in the ones column of the minuend to get 12. This step is called regrouping. Then subtract in the ones column: 12 − 5 = 7.

$$\begin{array}{rr} \overset{2}{\cancel{3}} & \overset{12}{\cancel{2}} \\ -\,1 & 5 \\ \hline 1 & 7 \end{array}$$

Subtract in the tens column: 2 − 1 = 1.

Your solution should look like this:
$$\begin{array}{rr} \overset{2}{\cancel{3}} & \overset{12}{\cancel{2}} \\ -\,1 & 5 \\ \hline 1 & 7 \end{array}$$

The difference is 17.

Self Check 3

Subtract:
$$\begin{array}{r} 83 \\ -\ 36 \\ \hline \end{array}$$

Now Try ➡ **Problem 27**

Some subtractions require borrowing from two (or more) place-value columns.

EXAMPLE 4 Subtract: 9,927 − 568

Strategy We will write the subtraction in vertical form and subtract as usual. In each column, we must watch for a digit in the subtrahend that is greater than the digit directly above it in the minuend.

WHY If a digit in the subtrahend is greater than the digit above it in the minuend, we need to borrow (regroup) to subtract in that column.

Solution

We write the subtraction in vertical form so that the corresponding digits are lined up. Each step of this subtraction is explained separately. Your solution should look like the *last* step.

$$\begin{array}{r} 9{,}927 \\ -\quad 568 \\ \hline \end{array}$$

Since 8 in the ones column of 568 is greater than 7 in the ones column of 9,927, we cannot immediately subtract. To subtract in that column, we must regroup by borrowing 1 ten from 2 in the tens column. In this process, we use the fact that 1 ten = 10 ones.

$$\begin{array}{r} 9,9\overset{1}{\cancel{2}}\overset{17}{\cancel{7}} \\ -\quad 568 \\ \hline 9 \end{array}$$

Borrow 1 ten from 2 in the tens column and change the 2 to 1. Add the borrowed 10 to the digit 7 in the ones column of the minuend to get 17. Then subtract in the ones column: 17 − 8 = 9.

Since 6 in the tens column of 5**6**8 is greater than 1 in the tens column directly above it, we cannot immediately subtract. To subtract in that column, we must regroup by borrowing 1 hundred from 9 in the hundreds column. In this process, we use the fact that 1 hundred = 10 tens.

$$\begin{array}{r} 9,\overset{8}{\cancel{9}}\overset{\overset{11}{\cancel{1}}}{\cancel{2}}\overset{17}{\cancel{7}} \\ -\quad 568 \\ \hline 59 \end{array}$$

Borrow 1 hundred from 9 in the hundreds column and change the 9 to 8. Add the borrowed 10 to the digit 1 in the tens column of the minuend to get 11. Then subtract in the tens column: 11 − 6 = 5.

Complete the solution by subtracting in the hundreds column (8 − 5 = 3) and bringing down the 9 in the thousands column.

$$\begin{array}{r} 9,\overset{8}{\cancel{9}}\overset{\overset{11}{\cancel{1}}}{\cancel{2}}\overset{17}{\cancel{7}} \\ -\quad 568 \\ \hline 9,359 \end{array}$$

Your solution should look like this:

$$\begin{array}{r} 9,\overset{8}{\cancel{9}}\overset{\overset{11}{\cancel{1}}}{\cancel{2}}\overset{17}{\cancel{7}} \\ -\quad 568 \\ \hline 9,359 \end{array}$$

The difference is 9,359.

The borrowing process is more difficult when the minuend contains one or more zeros.

Self Check 4

Subtract: 6,734 − 356

Now Try ➡ **Problem 33**

EXAMPLE 5 Subtract: 42,403 − 1,675

Strategy We will write the subtraction in vertical form. To subtract in the ones column, we will borrow from the hundreds column of the minuend 42,403.

WHY Since the digit in the tens column of 42,4**0**3 is 0, it is not possible to borrow from that column.

Solution

We write the subtraction in vertical form so that the corresponding digits are lined up. Each step of this subtraction is explained separately. Your solution should look like the *last* step.

$$\begin{array}{r} 42,403 \\ -\quad 1,675 \\ \hline \end{array}$$

Since 5 in the ones column of 1,67**5** is greater than 3 in the ones column of 42,40**3**, we cannot immediately subtract. It is not possible to borrow from the digit 0 in the tens column of 42,4**0**3. We can, however, borrow from the hundreds column to regroup in the tens column, as shown below. In this process, we use the fact that 1 hundred = 10 tens.

$$\begin{array}{r} 42,\overset{3}{\cancel{4}}\overset{10}{\cancel{0}}3 \\ -\quad 1,675 \\ \hline \end{array}$$

Borrow 1 hundred from 4 in the hundreds column and change the 4 to 3. Add the borrowed 10 to the digit 0 in the tens column of the minuend to get 10.

Now we can borrow from the 10 in the tens column to subtract in the ones column.

$$\begin{array}{r} 42,\overset{3}{\cancel{4}}\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\overset{13}{\cancel{3}} \\ -\quad 1,675 \\ \hline 8 \end{array}$$

Borrow 1 ten from 10 in the tens column and change the 10 to 9. Add the borrowed 10 to the digit 3 in the ones column of the minuend to get 13. Then subtract in the ones column: 13 − 5 = 8.

Next, we perform the subtraction in the tens column: 9 − 7 = 2.

$$\begin{array}{cccccc} & & & & 9 & \\ & & & 3 & \cancel{10} & 13 \\ & 4 & 2, & \cancel{4} & \cancel{0} & \cancel{3} \\ - & & 1, & 6 & 7 & 5 \\ \hline & & & & 2 & 8 \end{array}$$

To subtract in the hundreds column, we borrow from the 2 in the thousands column. In this process, we use the fact that 1 thousand = 10 hundreds.

$$\begin{array}{cccccc} & & & 13 & 9 & \\ & & 1 & 3 & \cancel{10} & 13 \\ & 4 & 2, & \cancel{4} & \cancel{0} & \cancel{3} \\ - & & 1, & 6 & 7 & 5 \\ \hline & & & 7 & 2 & 8 \end{array}$$

Borrow 1 thousand from 2 in the thousands column and change the 2 to 1. Add the borrowed 10 to the digit 3 in the hundreds column of the minuend to get 13. Then subtract in the hundreds column: 13 − 6 = 7.

Complete the solution by subtracting in the thousands column (1 − 1 = 0) and bringing down the 4 in the ten thousands column.

$$\begin{array}{cccccc} & & & 13 & 9 & \\ & & \cancel{1} & \cancel{3} & \cancel{10} & 13 \\ & 4 & \cancel{2}, & \cancel{4} & \cancel{0} & \cancel{3} \\ - & & 1, & 6 & 7 & 5 \\ \hline & 4 & 0, & 7 & 2 & 8 \end{array}$$

The difference is 40,728.

Your solution should look like this:

$$\begin{array}{cccccc} & & & 13 & 9 & \\ & & 1 & \cancel{3} & \cancel{10} & 13 \\ & 4 & \cancel{2}, & \cancel{4} & \cancel{0} & \cancel{3} \\ - & & 1, & 6 & 7 & 5 \\ \hline & 4 & 0, & 7 & 2 & 8 \end{array}$$

Self Check 5

Subtract: 65,304 − 1,445

Now Try ➜ **Problem 35**

OBJECTIVE 3 Check subtractions using addition.

Every subtraction has a **related addition statement**. For example,

$9 - 4 = 5$ because $5 + 4 = 9$
$25 - 15 = 10$ because $10 + 15 = 25$
$100 - 1 = 99$ because $99 + 1 = 100$

These examples illustrate how we can check subtractions. If a subtraction is done correctly, *the sum of the difference and the subtrahend will always equal the minuend*:

Difference + subtrahend = minuend

EXAMPLE 6 Check the following subtraction using addition:

$$\begin{array}{r} 3{,}682 \\ -1{,}954 \\ \hline 1{,}728 \end{array}$$

Strategy We will add the difference (1,728) and the subtrahend (1,954) and compare that result to the minuend (3,682).

WHY If the sum of the difference and the subtrahend gives the minuend, the subtraction checks.

Solution

The subtraction to check ***Its related addition statement***

$$\begin{array}{r} 3{,}682 \\ -1{,}954 \\ \hline 1{,}728 \end{array} \qquad \begin{array}{r} \text{difference} \\ +\,\text{subtrahend} \\ \hline \text{minuend} \end{array} \longrightarrow \begin{array}{r} \overset{1}{1}{,}\overset{1}{7}28 \\ +1{,}954 \\ \hline 3{,}682 \end{array}$$

Since the sum of the difference and the subtrahend is the minuend, the subtraction is correct.

Self Check 6

Check the following subtraction using addition:

$$\begin{array}{r} 9{,}784 \\ -4{,}792 \\ \hline 4{,}892 \end{array}$$

Now Try ➜ **Problem 39**

OBJECTIVE 4 Estimate differences of whole numbers.

Estimation is used to find an approximate answer to a problem.

EXAMPLE 7 Estimate the difference: 89,070 − 5,431

Strategy We will use front-end rounding to approximate the 89,070 and 5,431. Then we will find the difference of the approximations.

WHY Front-end rounding produces whole numbers containing many 0's. Such numbers are easier to subtract.

Solution

Both the minuend and the subtrahend are rounded to their *largest place value* so that all but their first digit is zero. Then we subtract the approximations using vertical form.

$$\begin{array}{rcrl} 89{,}070 & \rightarrow & 90{,}000 & \text{Round to the nearest ten thousand.} \\ -\ 5{,}431 & \rightarrow & -\ 5{,}000 & \text{Round to the nearest thousand.} \\ \cline{1-1}\cline{3-3} & & 85{,}000 & \end{array}$$

The estimate is 85,000. If we calculate 89,070 − 5,431, the difference is exactly 83,639. Note that the estimate is close: It's only 1,361 more than 83,639.

Self Check 7

Estimate the difference:
64,259 − 7,604

Now Try ➲ **Problem 43**

OBJECTIVE 5 Solve application problems by subtracting whole numbers.

To answer questions about *how much more* or *how many more*, we use subtraction.

EXAMPLE 8 **Horses.** Big Jake, the world's largest horse, weighs 2,600 pounds. Thumbelina, the world's smallest horse, weighs 57 pounds. How much more does Big Jake weigh than Thumbelina? (Source: *Guinness Book of World Records*, 2013)

AP Images/Carrie Antfinger

Brad Barket/Getty Images

Strategy We will carefully read the problem, looking for a key word or phrase.

WHY Key words and phrases indicate which arithmetic operation(s) should be used to solve the problem.

Solution

In the second sentence of the problem, the phrase *How much more* indicates that we should subtract the weights of the horses. We translate the words of the problem to numbers and symbols.

The number of pounds more that Big Jake weighs	is equal to	the weight of Big Jake	minus	the weight of Thumbelina.
The number of pounds more that Big Jake weighs	=	2,600	−	57

Use vertical form to perform the subtraction:

$$\begin{array}{r} 2{,}\overset{5}{\cancel{6}}\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\overset{10}{\cancel{0}} \\ -\quad 57 \\ \hline 2{,}543 \end{array}$$

Big Jake weighs 2,543 pounds more than Thumbelina.

Self Check 8

Elephants. An average male African elephant weighs 13,000 pounds. An average male Asian elephant weighs 11,900 pounds. How much more does an African elephant weigh than an Asian elephant?

Now Try ➲ **Problem 87**

EXAMPLE 9 **Radio stations.** In 2005, there were 763 oldies radio stations in the United States. By 2016, there were 412 fewer. How many oldies radio stations were there in 2016? (Source: *The World Almanac and Book of Facts*, 2017)

Strategy We will carefully read the problem, looking for a key word or phrase.

WHY Key words and phrases indicate which arithmetic operation(s) should be used to solve the problem.

Solution
The key phrase 412 *fewer* indicates subtraction. We translate the words of the problem to numbers and symbols.

The number of oldies radio stations in 2016	is	412	fewer than	the number of oldies radio stations in 2005.
The number of oldies radio stations in 2016	=	763	−	412

Use vertical form to perform the subtraction:

$$\begin{array}{r}763\\-412\\\hline 351\end{array}$$

In 2016, there were 351 oldies radio stations in the United States.

LANGUAGE OF MATHEMATICS

Here are some more key words and phrases that often indicate ***subtraction***:

loss
down
fell
fewer
remove
in the past
declined
decrease
backward
less than
reduce
debit
remains
take away

Self Check 9

Healthy diets. When Dwayne "The Rock" Johnson began to help Nina Gibson lose weight, she weighed 280 pounds. With diet and exercise, she eventually dropped 115 pounds. What was her weight then? (Source: today.com)

Now Try ➜ **Problem 99**

Using Your Calculator ▶ The Subtraction Key: High School Sports

In the 2015–2016 school year, the number of boys who participated in high school sports was 4,544,574 and the number of girls was 3,324,326. (Source: National Federation of State High School Associations) We can use the subtraction key $\boxed{-}$ on a calculator to determine how many more boys than girls participated in high school sports that year.

4544574 $\boxed{-}$ 3324326 $\boxed{=}$ $\boxed{1220248}$

On some calculator models, the $\boxed{\text{ENTER}}$ key is pressed instead of $\boxed{=}$ for the result to be displayed.

In the 2015–2016 school year, 1,220,248 more boys than girls participated in high school sports.

OBJECTIVE 6 Evaluate expressions involving addition and subtraction.

In arithmetic, numbers are combined with the operations of addition, subtraction, multiplication, and division to create **expressions**. For example,

$$15 + 6, \qquad 873 - 99, \qquad 6{,}512 \times 24, \qquad \text{and} \qquad 42 \div 7$$

are expressions.

Expressions can contain more than one operation. That is the case for the expression $27 - 16 + 5$, which contains addition *and* subtraction. To **evaluate** (find the value of) expressions written in horizontal form that involve addition and subtraction, we perform the operations as they occur *from left to right.*

Caution! When making the calculation in Example 10, we must perform the subtraction first because it occurs first when reading left to right. If the addition is done first, we get the incorrect answer 6.

$$\xcancel{\begin{aligned}27 - 16 + 5 &= 27 - 21\\ &= 6\end{aligned}}$$

EXAMPLE 10 Evaluate: $27 - 16 + 5$

Strategy We will perform the subtraction first and add 5 to that result.

WHY The operations of addition and subtraction must be performed as they occur from left to right.

Solution

We will write the steps of the solution in horizontal form.

$$27 - 16 + 5 = 11 + 5$$ Working left to right, do the subtraction first: $27 - 16 = 11$.

$$= 16$$ Now do the addition.

Self Check 10

Evaluate: $75 - 29 + 8$

Now Try Problems 47 and 51

Answers to Self Checks

1. 37 **2.** 1,141 **3.** 47 **4.** 6,378 **5.** 63,859 **6.** The subtraction is incorrect. **7.** 52,000 **8.** An African elephant weighs 1,100 lb more than an Asian elephant. **9.** After the dieting and exercise program, Nina weighed 165 lb. **10.** 54

SECTION 1.3 STUDY SET

VOCABULARY

Fill in the blanks.

1. In the subtraction problem shown below, label the *minuend*, the *subtrahend*, and the *difference*.

$$\begin{array}{ccccc} 25 & - & 10 & = & 15 \\ \uparrow & & \uparrow & & \uparrow \\ ☐ & & ☐ & & ☐ \end{array}$$

2. If the subtraction of the digits in any place-value column requires that we subtract a larger digit from a smaller digit, we must _______ or *regroup*.

3. The words *fall*, *lose*, *reduce*, and *decrease* often indicate the operation of _________.

4. Every subtraction has a _______ addition statement. For example,

$$7 - 2 = 5 \text{ because } 5 + 2 = 7$$

5. To see whether the result of a subtraction is reasonable, we can round the minuend and subtrahend and _______ the difference.

6. To *evaluate* an expression such as $58 - 33 + 9$ means to find its _____.

CONCEPTS

Fill in the blanks.

7. The subtraction $7 - 3 = 4$ is related to the addition statement ☐ + ☐ = ☐.

8. The operation of _______ can be used to check the result of a subtraction: If a subtraction is done correctly, *the* ____ *of the difference and the subtrahend will always equal the minuend.*

9. To *evaluate* (find the value of) an expression that contains both addition and subtraction, we perform the operations as they occur from ___ to ____.

10. To answer questions about *how much more* or *how many more*, we can use _________.

NOTATION

11. Fill in the blank: The subtraction symbol − is read as "______ ."

12. Write the following subtraction fact in words:

$$28 - 22 = 6$$

13. Which expression is the correct translation of the sentence: *Subtract* 30 *from* 83.

$$83 - 30 \quad \text{or} \quad 30 - 83$$

14. Fill in the blanks to complete each step:

$$36 - 11 + 5 = ☐ + 5$$
$$= ☐$$

GUIDED PRACTICE

Subtract. See Example 1.

15. $37 - 14$

16. $42 - 31$

17. $\begin{array}{r} 89 \\ -28 \\ \hline \end{array}$

18. $\begin{array}{r} 95 \\ -32 \\ \hline \end{array}$

19. $596 - 372$

20. $869 - 425$

21. $\begin{array}{r} 674 \\ -371 \\ \hline \end{array}$

22. $\begin{array}{r} 257 \\ -155 \\ \hline \end{array}$

Subtract. See Example 2.

23. 347 from 7,989

24. 283 from 9,799

25. 405 from 2,967

26. 304 from 1,736

Subtract. See Example 3.

27. $\begin{array}{r} 53 \\ -17 \\ \hline \end{array}$

28. $\begin{array}{r} 42 \\ -19 \\ \hline \end{array}$

29. $\begin{array}{r} 96 \\ -48 \\ \hline \end{array}$

30. $\begin{array}{r} 94 \\ -37 \\ \hline \end{array}$

Subtract. See Example 4.

31. 8,746 − 289

32. 7,531 − 276

33. $\begin{array}{r} 6{,}961 \\ -\ \ 478 \\ \hline \end{array}$

34. $\begin{array}{r} 4{,}823 \\ -\ \ 667 \\ \hline \end{array}$

Subtract. See Example 5.

35. 54,506 − 2,829

36. 69,403 − 4,635

37. $\begin{array}{r} 48{,}402 \\ -\ 3{,}958 \\ \hline \end{array}$

38. $\begin{array}{r} 39{,}506 \\ -\ 1{,}729 \\ \hline \end{array}$

Check each subtraction using addition. See Example 6.

39. $\begin{array}{r} 298 \\ -175 \\ \hline 123 \end{array}$

40. $\begin{array}{r} 469 \\ -237 \\ \hline 132 \end{array}$

41. $\begin{array}{r} 4{,}539 \\ -3{,}275 \\ \hline 1{,}364 \end{array}$

42. $\begin{array}{r} 2{,}698 \\ -1{,}569 \\ \hline 1{,}129 \end{array}$

Estimate each difference. See Example 7.

43. 67,219 − 4,076

44. 45,333 − 3,410

45. 83,872 − 27,281

46. 74,009 − 37,405

Evaluate each expression. See Example 10.

47. 35 − 12 + 6

48. 47 − 23 + 4

49. 56 − 31 + 12

50. 89 − 47 + 6

51. 574 + 47 − 13

52. 863 + 39 − 11

53. 966 + 143 − 61

54. 659 + 235 − 62

TRY IT YOURSELF

Perform the operations.

55. 416 − 357

56. 787 − 696

57. $\begin{array}{r} 3{,}430 \\ -\ \ 529 \\ \hline \end{array}$

58. $\begin{array}{r} 2{,}470 \\ -\ \ 863 \\ \hline \end{array}$

59. Subtract 199 from 301.

60. Subtract 78 from 2,047.

61. $\begin{array}{r} 367 \\ -347 \\ \hline \end{array}$

62. $\begin{array}{r} 224 \\ -122 \\ \hline \end{array}$

63. 633 − 598 + 30

64. 600 − 497 + 60

65. 420 − 390

66. 330 − 270

67. 20,007 − 78

68. 70,006 − 48

69. 852 − 695 + 40

70. 397 − 348 + 65

71. $\begin{array}{r} 17{,}246 \\ -\ 6{,}789 \\ \hline \end{array}$

72. $\begin{array}{r} 34{,}510 \\ -27{,}593 \\ \hline \end{array}$

73. $\begin{array}{r} 15{,}700 \\ -15{,}397 \\ \hline \end{array}$

74. $\begin{array}{r} 35{,}600 \\ -34{,}799 \\ \hline \end{array}$

75. Subtract 1,249 from 50,009.

76. Subtract 2,198 from 20,020.

77. 120 + 30 − 40

78. 600 + 99 − 54

79. $\begin{array}{r} 167{,}305 \\ -\ \ 23{,}746 \\ \hline \end{array}$

80. $\begin{array}{r} 393{,}001 \\ -\ \ 35{,}002 \\ \hline \end{array}$

81. 29,307 − 10,008

82. 40,012 − 19,045

LOOK ALIKES

Perform each operation.

83. **a.** 654 subtract 127

b. 52 subtracted from 321

84. **a.** 871 subtract 192

b. 192 subtracted from 413

85. **a.** $\begin{array}{r} 302 \\ +175 \\ \hline \end{array}$ **b.** $\begin{array}{r} 302 \\ -175 \\ \hline \end{array}$

86. **a.** $\begin{array}{r} 2{,}135 \\ +\ \ 999 \\ \hline \end{array}$ **b.** $\begin{array}{r} 2{,}135 \\ -\ \ 999 \\ \hline \end{array}$

APPLICATIONS

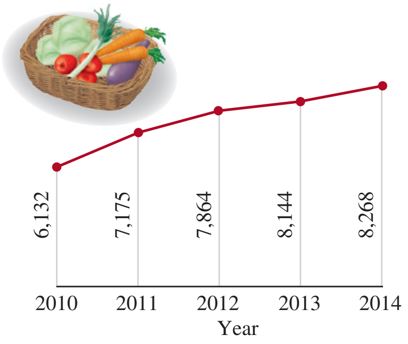
87. **World records.** The world's largest pumpkin weighed in at 2,623 pounds, and the world's largest watermelon weighed in at 351 pounds. How much more did the pumpkin weigh? (Source: ibtimes.com, guinnessworldrecords.com)

88. **Trucks.** The Nissan Titan King Cab XE weighs 5,230 pounds and the Honda Ridgeline RTL weighs 4,553 pounds. How much more does the Nissan Titan weigh?

89. **Farmer's markets.** See the graph below. How many more farmer's markets were there in 2014 compared to 2010?

90. **Farmer's markets.** See the graph below. Between what two years was there the greatest increase in the number of farmer's markets in the U.S.? What was the increase?

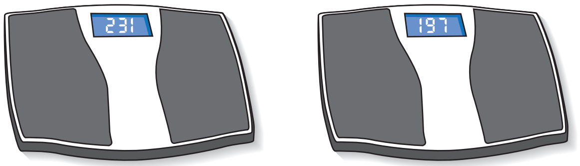
Number of Farmer's Markets in the U.S.



Source: USDA-AMS-marketing services division

91. Mileage. Find the distance (in miles) that a trucker drove on a trip from San Diego to Houston using the odometer readings shown below.

92. Diets. Use the bathroom scale readings shown below to find the number of pounds that a dieter lost.



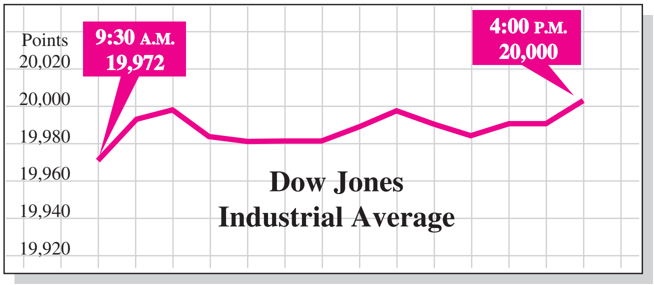
January

October

93. Renting limos. A group of high school students paid a limousine company $510 to take them to and from their prom. If that included a gratuity (tip) of $85, how much did it cost them to rent the limo?

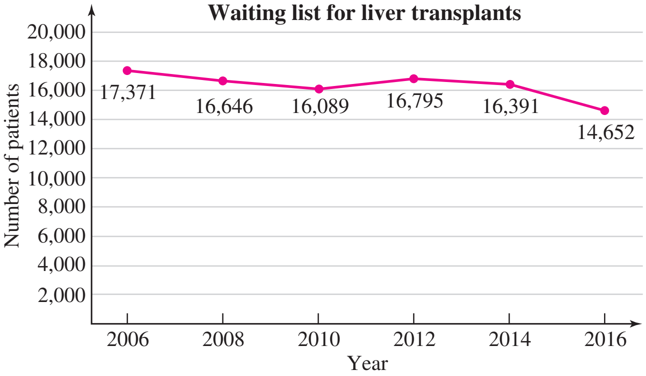
94. Magazines. In 2016, *Reader's Digest* had a circulation of 2,662,066. By what amount did this exceed *TV Guide*'s circulation of 1,571,537? (Source: *The World Almanac and Book of Facts*, 2017)

95. The stock market. How many points did the Dow Jones Industrial Average gain on the day described by the graph?



96. Transplants. See the graph below. Find the decrease in the number of patients waiting for a liver transplant from:

a. 2006 to 2008 **b.** 2012 to 2014



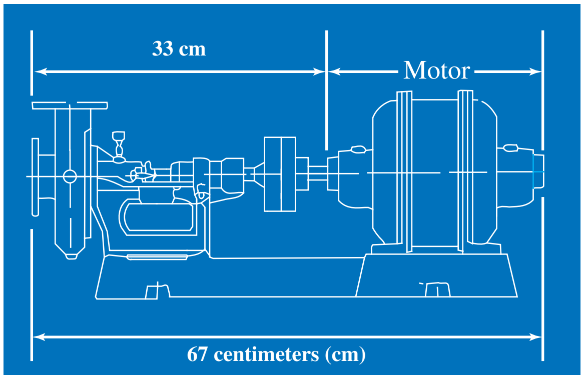
Source: Organ Procurement and Transplant Network, United Network for Organ Sharing

97. Jewelry. Gold melts at about 1,947°F. The melting point of silver is 183°F lower. What is the melting point of silver?

98. Energy costs. The electricity cost to run a 10-year-old refrigerator for 1 year is $133. A new energy-saving refrigerator costs $85 less to run for 1 year. What is the electricity cost to run the new refrigerator for 1 year?

99. Zip codes. As of 2017, the state of Florida has 1,118 fewer zip codes than California. If California has 2,590 zip codes, how many does Florida have?

100. Reading blueprints. Find the length of the motor on the machine shown in the blueprint.



101. Banking. A savings account contained $1,370. After a withdrawal of $197 and a deposit of $340, how much was left in the account?

102. Physical exams. A blood test found a man's "bad" cholesterol level to be 205. With a change of eating habits, he lowered it by 27 points in 6 months. One year later, however, the level had risen by 9 points. What was his cholesterol level then?

Refer to the teachers' salary schedule shown below. To use this table, note that a fourth-year teacher (Step 4) in Column 2 makes $52,209 per year.

103. a. What is the salary of a teacher on Step 2/Column 2?

b. How much more will that teacher make next year when she gains 1 year of teaching experience and moves down to Step 3 in that column?

104. a. What is the salary of a teacher on Step 4/Column 1?

b. How much more will that teacher make next year when he gains 1 year of teaching experience and takes enough coursework to move over to Column 2?

Teachers' Salary Schedule ABC Unified School District

Years teaching	Column 1	Column 2	Column 3
Step 1	$46,785	$48,243	$49,701
Step 2	$48,107	$49,565	$51,023
Step 3	$49,429	$50,887	$52,345
Step 4	$50,751	$52,209	$53,667
Step 5	$52,073	$53,531	$54,989

WRITING

105. Explain why the operation of subtraction is not commutative.

106. List five words or phrases that indicate subtraction.

107. Explain how addition can be used to check subtraction.

108. The borrowing process is more difficult when the minuend contains one or more zeros. Give an example and explain why.

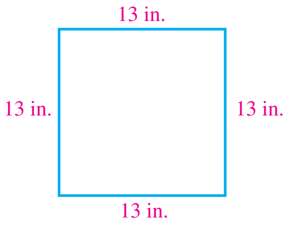
REVIEW

109. Round 5,370,645 to the indicated place value.
- **a.** Nearest ten
- **b.** Nearest ten thousand
- **c.** Nearest hundred thousand

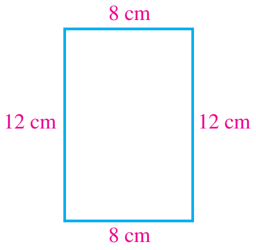
110. Write 72,001,015
- **a.** in words
- **b.** in expanded notation

Find the perimeter of the square and the rectangle.

111.



112.



Add.

113.
$$\begin{array}{r} 345 \\ 4{,}672 \\ +\ \ 513 \\ \hline \end{array}$$

114.
$$\begin{array}{r} 813 \\ 7{,}487 \\ +\ \ 654 \\ \hline \end{array}$$

SECTION 1.4 Multiplying Whole Numbers

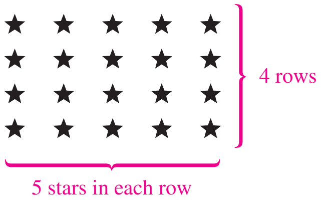
Everyone uses multiplication of whole numbers. For example, to double a recipe, a cook multiplies the amount of each ingredient by two. To determine the floor space of a dining room, a carpeting salesperson multiplies its length by its width. An accountant multiplies the number of hours worked by the hourly pay rate to calculate the weekly earnings of employees.

OBJECTIVES

1 Multiply whole numbers by one-digit numbers.
2 Multiply whole numbers that end with zeros.
3 Multiply whole numbers by two- (or more) digit numbers.
4 Use properties of multiplication to multiply whole numbers.
5 Estimate products of whole numbers.
6 Solve application problems by multiplying whole numbers.
7 Find the area of a rectangle.

OBJECTIVE 1 Multiply whole numbers by one-digit numbers.

In the following display, there are 4 rows, and each of the rows has 5 stars.



We can find the total number of stars in the display by adding: $5 + 5 + 5 + 5 = 20$.

This problem can also be solved using a simpler process called **multiplication**. Multiplication is repeated addition, and it is written using a **multiplication symbol** ×, which is read as "times." Instead of *adding* four 5's to get 20, we can *multiply* 4 and 5 to get 20.

Repeated addition		**Multiplication**	
$5 + 5 + 5 + 5$	$=$	$4 \times 5 = 20$	Read as "4 times 5 equals (or is) 20."

We can write multiplication problems in **horizontal** or **vertical form**. The numbers that are being multiplied are called **factors**, and the answer is called the **product**.

Horizontal form

$$\underset{\text{Factor}}{\underset{\uparrow}{4}} \times \underset{\text{Factor}}{\underset{\uparrow}{5}} = \underset{\text{Product}}{\underset{\uparrow}{20}}$$

Vertical form

$$\begin{array}{rl} 5 & \longleftarrow \text{Factor} \\ \underline{\times\ 4} & \longleftarrow \text{Factor} \\ 20 & \longleftarrow \text{Product} \end{array}$$

A **raised dot** · and **parentheses ( )** are also used to write multiplication in horizontal form.

Caution! In this book, we seldom use the × symbol because it can be confused with the letter *x*.

Symbols Used for Multiplication

Symbol		**Example**
×	times symbol	4×5
·	raised dot	$4 \cdot 5$
( )	parentheses	$(4)(5)$ or $4(5)$ or $(4)5$

If you need to review the basic multiplication facts, they can be found in Appendix 1 at the back of the book.

To multiply whole numbers that are less than 10, we rely on our understanding of basic multiplication facts. For example,

$$2 \cdot 3 = 6, \qquad 8(4) = 32, \qquad \text{and} \qquad 9 \times 7 = 63$$

To multiply larger whole numbers, we can use vertical form by stacking them with their corresponding place values lined up. Then we make repeated use of basic multiplication facts.

EXAMPLE 1 Multiply: 8×47

Strategy We will write the multiplication in vertical form. Then, working right to left, we will multiply each digit of 47 by 8 and carry, if necessary.

WHY This process is simpler than treating the problem as repeated addition and adding eight 47's.

Solution

To help you understand the process, each step of this multiplication is explained separately. Your solution need only look like the *last* step.

Tens column

Ones column

Vertical form $\begin{array}{r} 4\,7 \\ \underline{\times\ \ 8} \end{array}$

We begin by multiplying 7 by 8.

$$\begin{array}{r} {\scriptstyle 5}\ \ \\ 4\,7 \\ \underline{\times\ \ \ 8} \\ 6 \end{array}$$

Multiply 7 by 8. The product is 56. Write 6 in the ones column of the answer, and carry 5 to the tens column.

$$\begin{array}{r} {\scriptstyle 5}\ \ \\ 4\,7 \\ \underline{\times\ \ \ 8} \\ 3\,7\,6 \end{array}$$

Multiply 4 by 8. The product is 32. To the 32, add the carried 5 to get 37. Write 7 in the tens column and the 3 in the hundreds column of the answer.

The product is 376.

Your solution should look like this: $\begin{array}{r} {\scriptstyle 5}\ \ \\ 47 \\ \underline{\times\ \ 8} \\ 376 \end{array}$

Self Check 1

Multiply: 6×54

Now Try ➡ **Problem 19**

OBJECTIVE 2 Multiply whole numbers that end with zeros.

An interesting pattern develops when a whole number is multiplied by 10, 100, 1,000, and so on. Consider the following multiplications involving 8:

$8 \cdot 10 = 80$ There is one zero in 10. The product is 8 with one 0 attached.

$8 \cdot 100 = 800$ There are two zeros in 100. The product is 8 with two 0's attached.

$8 \cdot 1{,}000 = 8{,}000$ There are three zeros in 1,000. The product is 8 with three 0's attached.

$8 \cdot 10{,}000 = 80{,}000$ There are four zeros in 10,000. The product is 8 with four 0's attached.

These examples illustrate the following rule.

Multiplying a Whole Number by 10, 100, 1,000, and So On

To find the product of a whole number and 10, 100, 1,000, and so on, attach the number of zeros in that number to the right of the whole number.

EXAMPLE 2 Multiply: **a.** $6 \times 1{,}000$ **b.** $45 \cdot 100$ **c.** $912(10{,}000)$

Strategy For each multiplication, we will identify the factor that ends in zeros and count the number of zeros that it contains.

WHY Each product can then be found by attaching that number of zeros to the other factor.

Solution

a. $6 \times 1{,}000 = 6{,}000$ Since 1,000 has three zeros, attach three 0's after 6.

b. $45 \cdot 100 = 4{,}500$ Since 100 has two zeros, attach two 0's after 45.

c. $912(10{,}000) = 9{,}120{,}000$ Since 10,000 has four zeros, attach four 0's after 912.

Self Check 2

Multiply:

a. $9 \times 1{,}000$

b. $25 \cdot 100$

c. $875(1{,}000)$

Now Try Problems 23 and 25

We can use an approach similar to that of Example 2 for multiplication involving any whole numbers that end in zeros. For example, to find $67 \cdot 2{,}000$, we have

$67 \cdot 2{,}000 = 67 \cdot 2 \cdot 1{,}000$ Write 2,000 as $2 \cdot 1{,}000$.

$= 134 \cdot 1{,}000$ Working left to right, multiply 67 and 2 to get 134.

$= 134{,}000$ Since 1,000 has three zeros, attach three 0's after 134.

This example suggests that to find $67 \cdot 2{,}000$ we simply multiply 67 and 2 and attach three zeros to that product. This method can be extended to find products of two factors that *both* end in zeros.

EXAMPLE 3 Multiply: **a.** $14 \cdot 300$ **b.** $3{,}500 \cdot 50{,}000$

Strategy We will multiply the nonzero leading digits of each factor. To that product, we will attach the sum of the number of trailing zeros in the factors.

WHY This method is faster than the standard vertical form multiplication of factors that contain many zeros.

Solution

a. The factor 300 has two trailing zeros.

$14 \cdot 300 = 4{,}200$ Attach two 0's after 42.

Multiply 14 and 3 to get 42.

$$\begin{array}{r} \overset{1}{1}4 \\ \times\ \ 3 \\ \hline 42 \end{array}$$

b. The factors 3,500 and 50,000 have a total of six trailing zeros.

$3{,}500 \cdot 50{,}000 = 175{,}000{,}000$ Attach six 0's after 175.

Multiply 35 and 5 to get 175.

$$\begin{array}{r} \overset{2}{3}5 \\ \times\ \ 5 \\ \hline 175 \end{array}$$

Self Check 3

Multiply:

a. $15 \cdot 900$

b. $3{,}100 \cdot 7{,}000$

Now Try ➔ **Problems 29 and 33**

Success Tip Calculations that you cannot perform in your head should be shown outside the steps of your solution.

OBJECTIVE 3 Multiply whole numbers by two- (or more) digit numbers.

EXAMPLE 4 Multiply: $23 \cdot 436$

Strategy We will write the multiplication in vertical form. Then we will multiply 436 by 3 and by 20, and add those products.

WHY Since $23 = 3 + 20$, we can multiply 436 by 3 and by 20, and add those products.

Solution

Each step of this multiplication is explained separately. Your solution need only look like the *last* step.

Hundreds column

Tens column

Ones column

Vertical form

$$\begin{array}{r} 4\ 3\ 6 \\ \times\ \ \ 2\ 3 \\ \hline \end{array}$$

Vertical form multiplication is often easier if the factor with the larger number of digits is written on top.

We begin by multiplying 436 by 3.

$$\begin{array}{r} 4\ \overset{1}{3}\ 6 \\ \times\ \ \ 2\ 3 \\ \hline 8 \end{array}$$

Multiply 6 by 3. The product is 18. Write 8 in the ones column and carry 1 to the tens column.

$$\begin{array}{r} \overset{1}{4}\ \overset{1}{3}\ 6 \\ \times\ \ \ 2\ 3 \\ \hline 0\ 8 \end{array}$$

Multiply 3 by 3. The product is 9. To the 9, add the carried 1 to get 10. Write the 0 in the tens column and carry the 1 to the hundreds column.

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 436 \\ \times\quad 23 \\ \hline 1308 \end{array}$$

Multiply 4 by 3. The product is 12. Add the 12 to the carried 1 to get 13. Write 13.

We continue by multiplying 436 by 2 tens, or 20. If we think of 20 as $2 \cdot 10$, then we simply multiply 436 by 2 and attach one zero to the result.

$$\begin{array}{r} {\scriptstyle 1} \\ {\scriptstyle 1\,1} \\ 436 \\ \times\quad 23 \\ \hline 1308 \\ 20 \end{array}$$

Write the 0 that is to be attached to the result of $20 \cdot 436$ in the ones column (shown in blue). Then multiply 6 by 2. The product is 12. Write 2 in the tens column and carry 1.

$$\begin{array}{r} {\scriptstyle 1} \\ {\scriptstyle 1\,1} \\ 436 \\ \times\quad 23 \\ \hline 1308 \\ 720 \end{array}$$

Multiply 3 by 2. The product is 6. Add 6 to the carried 1 to get 7. Write the 7 in the hundreds column. There is no carry.

$$\begin{array}{r} {\scriptstyle 1} \\ {\scriptstyle 1\,1} \\ 436 \\ \times\quad 23 \\ \hline 1308 \\ 8720 \end{array}$$

Multiply 4 by 2. The product is 8. There is no carried digit to add. Write the 8 in the thousands column.

$$\begin{array}{r} {\scriptstyle 1} \\ {\scriptstyle 1\,1} \\ 436 \\ \times\quad 23 \\ \hline 1\,308 \\ 8\,720 \\ \hline 10{,}028 \end{array}$$

Draw another line beneath the two completed rows. Add column by column, working right to left. This sum gives the product of 436 and 23.

The product is 10,028.

LANGUAGE OF MATHEMATICS

In Example 4, the numbers **1,308** and **8,720** are called ***partial products.*** We added the partial products to get the answer, 10,028. The word *partial* means *only a part*, as in a *partial* eclipse of the moon.

$$\begin{array}{r} 436 \\ \times\quad 23 \\ \hline 1308 \\ 8720 \\ \hline 10{,}028 \end{array}$$

Self Check 4

Multiply: $36 \cdot 334$

Now Try ➜ **Problem 37**

When a factor in a multiplication contains one or more zeros, we must be careful to enter the correct number of zeros when writing the partial products.

EXAMPLE 5 Multiply: **a.** $406 \cdot 253$ **b.** 3,009(2,007)

Strategy We will think of 406 as $6 + 400$ and 3,009 as $9 + 3{,}000$.

WHY Thinking of the multipliers (406 and 3,009) in this way is helpful when determining the correct number of zeros to enter in the partial products.

Solution

We will use vertical form to perform each multiplication.

a. Since 406 = 6 + 400, we will multiply 253 by 6 and by 400, and add those partial products.

$$\begin{array}{r l}
253 & \\
\times \quad 406 & \\
\hline
1\ 518 & \leftarrow 6 \cdot 253 \\
101\ 200 & \leftarrow 400 \cdot 253. \text{ Think of } 400 \text{ as } 4 \cdot 100 \text{ and simply multiply} \\
\hline
102{,}718 & \quad\ \ 253 \text{ by } 4 \text{ and attach two zeros (shown in blue) to the result.}
\end{array}$$

The product is 102,718.

b. Since 3,009 = 9 + 3,000, we will multiply 2,007 by 9 and by 3,000, and add those partial products.

$$\begin{array}{r l}
2{,}007 & \\
\times \quad 3{,}009 & \\
\hline
18\ 063 & \leftarrow 9 \cdot 2{,}007 \\
6\ 021\ 000 & \leftarrow 3{,}000 \cdot 2{,}007. \text{ Think of } 3{,}000 \text{ as } 3 \cdot 1{,}000 \text{ and simply multiply} \\
\hline
6{,}039{,}063 & \quad\ \ 2{,}007 \text{ by } 3 \text{ and attach three zeros (shown in blue) to the result.}
\end{array}$$

The product is 6,039,063.

Self Check 5

Multiply:

a. 706(351)

b. 4,004(2,008)

Now Try ➡ **Problem 41**

OBJECTIVE 4 Use properties of multiplication to multiply whole numbers.

Have you ever noticed that two whole numbers can be multiplied in either order because the result is the same? For example,

$$4 \cdot 6 = 24 \qquad \text{and} \qquad 6 \cdot 4 = 24$$

This example illustrates the **commutative property of multiplication**.

Commutative Property of Multiplication

The order in which whole numbers are multiplied does not change their product.

For example:

$$7 \cdot 5 = 5 \cdot 7$$

Whenever we multiply a whole number by 0, the product is 0. For example,

$$0 \cdot 5 = 0, \qquad 0 \cdot 8 = 0, \qquad \text{and} \qquad 9 \cdot 0 = 0$$

Whenever we multiply a whole number by 1, the number remains the same. For example,

$$3 \cdot 1 = 3, \qquad 7 \cdot 1 = 7, \qquad \text{and} \qquad 1 \cdot 9 = 9$$

These examples illustrate the multiplication properties of 0 and 1.

Multiplication Properties of 0 and 1

The product of any whole number and 0 is 0.
The product of any whole number and 1 is that whole number.

Success Tip If one (or more) of the factors in a multiplication is 0, the product will be 0. For example,

$$16(27)(0) = 0$$

and

$$109 \cdot 53 \cdot 0 \cdot 2 = 0$$

To multiply three numbers, we first multiply two of them and then multiply that result by the third number. In the following examples, we multiply $3 \cdot 2 \cdot 4$ in two ways. The parentheses show us which multiplication to perform first. The steps of the solutions are written in horizontal form.

Method 1: Group $3 \cdot 2$

$(3 \cdot 2) \cdot 4 = 6 \cdot 4$ Multiply 3 and 2 to get 6.

$= 24$ Multiply 6 and 4 to get 24.

Method 2: Group $2 \cdot 4$

$3 \cdot (2 \cdot 4) = 3 \cdot 8$ Then multiply 2 and 4 to get 8.

$= 24$ Then multiply 3 and 8 to get 24.

Same result

LANGUAGE OF MATHEMATICS

We read $(3 \cdot 2) \cdot 4$ as "The ***quantity*** of 3 times 2," pause slightly, and then say "times 4." Read $3 \cdot (2 \cdot 4)$ as "3 times the *quantity* of 2 times 4." The word *quantity* alerts the reader to the parentheses that are used as grouping symbols.

Either way, the answer is 24. This example illustrates that changing the grouping when multiplying numbers doesn't affect the result. This property is called the **associative property of multiplication**.

Associative Property of Multiplication

The way in which whole numbers are grouped does not change their product.

For example:

$$(2 \cdot 3) \cdot 5 = 2 \cdot (3 \cdot 5)$$

Sometimes, an application of the associative property can simplify a calculation.

EXAMPLE 6 Find the product: $(17 \cdot 50) \cdot 2$

Strategy We will use the associative property to group 50 with 2.

WHY It is helpful to regroup because 50 and 2 are a pair of numbers that are easily multiplied.

Solution

We will write the solution in horizontal form.

$(17 \cdot 50) \cdot 2 = 17 \cdot (50 \cdot 2)$ Use the associative property of multiplication to regroup the factors.

$= 17 \cdot 100$ Do the multiplication within the parentheses first.

$= 1{,}700$ Since 100 has two zeros, attach two 0's after 17.

Self Check 6

Find the product: $(23 \cdot 25) \cdot 4$

Now Try ➡ **Problem 45**

OBJECTIVE 5 Estimate products of whole numbers.

Estimation is used to find an approximate answer to a problem.

EXAMPLE 7 Estimate the product: 59 · 334

Strategy We will use front-end rounding to approximate the factors 59 and 334. Then we will find the product of the approximations.

WHY Front-end rounding produces whole numbers containing many 0's. Such numbers are easier to multiply.

Solution

Both of the factors are rounded to their *largest place value* so that all but their first digit is zero.

Round to the nearest ten.

59 · 334 60 · 300

Round to the nearest hundred.

To find the product of the approximations, 60 · 300, we simply multiply 6 by 3, to get 18, and attach 3 zeros. Thus, the estimate is 18,000.

If we calculate 59 · 334, the product is exactly 19,706. Note that the estimate is close: It's only 1,706 less than 19,706.

Self Check 7

Estimate the product: 74 · 488

Now Try ➲ **Problem 51**

OBJECTIVE 6 Solve application problems by multiplying whole numbers.

Application problems that involve repeated addition are often more easily solved using multiplication.

EXAMPLE 8 **Daily pay.** In October 2016, the average U.S. manufacturing worker made $26 per hour. At that rate, how much money was earned in an 8-hour workday? (Source: Bureau of Labor Statistics)

Strategy To find the amount earned in an 8-hour workday, we will multiply the hourly rate of $26 by 8.

WHY For each of the 8 hours, the average manufacturing worker earned $26. The amount earned for the day is the sum of eight 26's: 26 + 26 + 26 + 26 + 26 + 26 + 26 + 26. This repeated addition can be calculated more simply by multiplication.

Solution

We translate the words of the problem to numbers and symbols.

The amount earned in an 8-hr workday	is equal to	the rate per hour	times	8 hours.
The amount earned in an 8-hr workday	=	26	·	8

Use vertical form to perform the multiplication:

$$\begin{array}{r} \overset{4}{2}6 \\ \times \quad 8 \\ \hline 208 \end{array}$$

In October 2016, the average U.S. manufacturing worker earned $208 in an 8-hour workday.

Self Check 8

Daily pay. In 2016, the average U.S. construction worker made $28 per hour. At that rate, how much money was earned in an 8-hour workday? (Source: Bureau of Labor Statistics)

Now Try ➲ **Problem 90**