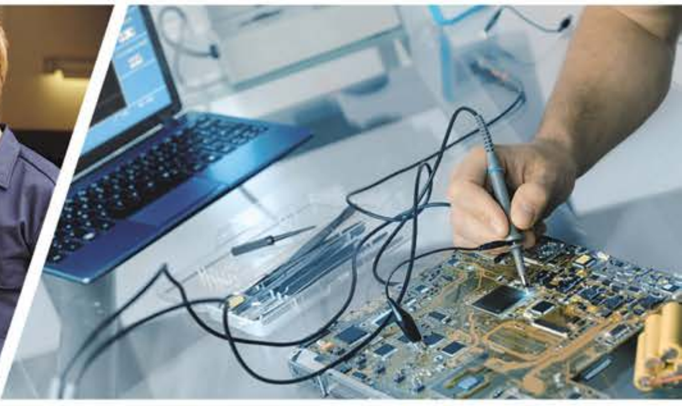
Elementary Technical Mathematics

12e



Dale Ewen

Applications Symbols Used in This Text

- Agriculture and Horticulture
- Allied Health
- Auto/Diesel Service
- Aviation
- Business and Personal Finance
- CAD/Drafting
- Culinary Arts
- Electronics
- HVAC
- Industrial and Construction Trades
- Manufacturing
- Natural Resources
- Welding

U.S. Weights and Measures

Length

Standard unit: inch (in. or ″)

12 inches = 1 foot (ft or ′)
3 feet = 1 yard (yd)
5½ yards or 16½ feet = 1 rod (rd)
5280 feet = 1 mile (mi)

Weight

Standard unit: pound (lb)

16 ounces (oz) = 1 pound
2000 pounds = 1 ton

Volume

Liquid

3 teaspoons (tsp) = 1 tablespoon (tbs)
16 tablespoons = 1 cup
2 cups = 1 pint (pt)
16 fluid ounces (fl oz) = 1 pint (pt)
2 pints = 1 quart (qt)
4 quarts = 1 gallon (gal)

Dry

2 pints (pt) = 1 quart (qt)
8 quarts = 1 peck (pk)
4 pecks = 1 bushel (bu)

Metric System Prefixes

Multiple or Submultiple* Decimal Form	Power of 10	Prefix	Prefix Symbol	Pronunciation	Meaning
1,000,000,000,000	10^{12}	tera	T	tĕr′ă	one trillion times
1,000,000,000	10^{9}	giga	G	jĭg′ă	one billion times
1,000,000	10^{6}	mega	M	mĕg′ă	one million times
1,000	10^{3}	kilo**	k	kĭl′ō or kēl′ō	one thousand times
100	10^{2}	hecto	h	hĕk′tō	one hundred times
10	10^{1}	deka	da	dĕk′ă	ten times
0.1	10^{-1}	deci	d	dĕs′ĭ	one tenth of
0.01	10^{-2}	centi**	c	sĕnt′ĭ	one hundredth of
0.001	10^{-3}	milli**	m	mĭl′ĭ	one thousandth of
0.000001	10^{-6}	micro	μ	mī′krō	one millionth of
0.000000001	10^{-9}	nano	n	năn′ō	one billionth of
0.000000000001	10^{-12}	pico	p	pē′kō	one trillionth of

*Factor by which the unit is multiplied.
**Most commonly used prefixes.

As an example, the prefixes are used below with the metric standard unit of length, metre (m).

1 *tera*metre (Tm) = 1,000,000,000,000 m	1 m = 0.000000000001 Tm
1 *giga*metre (Gm) = 1,000,000,000 m	1 m = 0.000000001 Gm
1 *mega*metre (Mm) = 1,000,000 m	1 m = 0.000001 Mm
1 *kilo*metre (km) = 1,000 m	1 m = 0.001 km
1 *hecto*metre (hm) = 100 m	1 m = 0.01 hm
1 *deka*metre (dam) = 10 m	1 m = 0.1 dam
1 *deci*metre (dm) = 0.1 m	1 m = 10 dm
1 *centi*metre (cm) = 0.01 m	1 m = 100 cm
1 *milli*metre (mm) = 0.001 m	1 m = 1,000 mm
1 *micro*metre (μm) = 0.000001 m	1 m = 1,000,000 μm
1 *nano*metre (nm) = 0.000000001 m	1 m = 1,000,000,000 nm
1 *pico*metre (pm) = 0.000000000001 m	1 m = 1,000,000,000,000 pm

12th Edition

Elementary Technical Mathematics

Dale Ewen
Parkland Community College

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Natural Resources

Welding

PREFACE

Elementary Technical Mathematics, Twelfth Edition, is intended for technical, trade, allied health, or Tech Prep programs. This book was written for students who plan to learn a technical skill, but who have minimal background in mathematics or need considerable review. To become proficient in most technical programs, students must learn basic mathematical skills. To that end, Chapters 1 through 4 cover basic arithmetic operations, fractions, decimals, percent, the metric system, and numbers as measurements. Chapters 5 through 11 present essential algebra needed in technical and trade programs. The essentials of geometry—relationships and formulas for the most common two- and three-dimensional figures—are given in detail in Chapter 12. Chapters 13 and 14 present a short but intensive study of trigonometry that includes right-triangle trigonometry as well as oblique triangles and graphing. The concepts of statistics that are most important to technical fields are discussed in Chapter 15. An introduction to binary and hexadecimal numbers is found in Chapter 16 for those who requested this material.

This text is written to match the reading level of most technical students. Visual images engage these readers and stimulate the problem-solving process. These skills are essential for success in technical courses. This text is written to be as flexible as possible for the wide range of student backgrounds and technical program needs. Sections may be easily combined for the better prepared class of students.

The following important text features have been retained from previous editions:

- A large number of applications are used from a wide variety of technical areas, including agriculture and horticulture, allied health, auto/diesel service, aviation, business and personal finance, CAD/drafting, culinary arts, electronics, HVAC, industrial and construction trades, manufacturing, natural resources, and welding.
- Chapter 1 reviews basic concepts in such a way that individuals, groups of students, or the entire class can easily study only those sections they need to review.
- A comprehensive introduction to basic algebra is presented for those students who need it as a prerequisite to more advanced algebra courses. However, the book has been written to allow the omission of selected sections or chapters without loss of continuity, to meet the needs of specific students.
- More than 6500 exercises assist student learning of skills and concepts.
- More than 750 detailed, well-illustrated examples, many with step-by-step comments, support student understanding of skills and concepts.
- Learning objectives are listed with each Chapter Opener to give a clear outline of topics covered in the chapter. This serves as a reference for students when completing homework assignments or studying for exams, and it also helps with lesson and assessment preparation for instructors.

- A chapter summary with a glossary of basic terms, a chapter review, and a chapter test appear at the end of each chapter as aids for students in preparing for quizzes and exams. Each chapter test is designed to be completed by an average student in no more than approximately 50 minutes.

REVIEW | CHAPTER 3

Give the metric prefix for each value:

1. 0.001 **2.** 1000

Give the SI abbreviation for each prefix:

3. mega **4.** micro

Write the SI abbreviation for each quantity:

5. 42 millilitres **6.** 8.3 nanoseconds

Write the SI unit for each abbreviation:

7. 18 km **8.** 350 mA **9.** 50 μs

Which is larger?

10. 1 L or 1 mL **11.** 1 kW or 1 MW

12. 1 km^2 or 1 ha **13.** 1 m^3 or 1 L

Fill in each blank:

14. 650 m = ______ km **15.** 750 mL = ______ L

16. 6.1 kg = ______ g **17.** 4.2 A = ______ μA

27. Water boils at ______ °C.

28. 180 lb = ______ kg **29.** 126 ft = ______ m

30. 360 cm = ______ in. **31.** 275 in^2 = ______ cm^2

32. 18 yd^2 = ______ ft^2 **33.** 5 m^3 = ______ ft^3

34. 15.0 acres = ______ ha

Choose the most reasonable quantity:

35. Jorge and Maria drive **a.** 1600 cm, **b.** 470 m, **c.** 12 km, or **d.** 2400 mm to college each day.

36. Chuck's mass is **a.** 80 kg, **b.** 175 kg, **c.** 14 μg, or **d.** 160 Mg.

37. An automobile's fuel tank holds **a.** 18 L, **b.** 15 kL, **c.** 240 mL, or **d.** 60 L of gasoline.

38. Jamilla, being of average height, is **a.** 5.5 m, **b.** 325 mm, **c.** 55 cm, or **d.** 165 cm tall.

39. An automobile's average fuel consumption is **a.** 320 km/L, **b.** 15 km/L, **c.** 35 km/L, or **d.** 0.75 km/L.

TEST | CHAPTER 3

1. Give the metric prefix for 1000.

2. Give the metric prefix for 0.01.

3. Which is larger, 200 mg or 1 g?

4. Write the SI unit for the abbreviation 240 μL.

5. Write the abbreviation for 30 hectograms.

6. Which is longer, 1 km or 25 cm?

Fill in each blank:

7. 4.25 km = _____ m **8.** 7.28 mm = _____ μm

9. 72 m = _____ mm **10.** 256 hm = _____ cm

11. 12 dg = _____ mg **12.** 16.2 g = _____ mg

13. 7.236 metric tons = _____ kg

14. 310 g = _____ cg **15.** 72 hg = _____ mg

16. 1.52 dL = _____ L **17.** 175 L = _____ m^3

18. 2.7 m^3 = _____ cm^3 **19.** 400 ha = _____ km^2

20. 0.2 L = _____ mL

21. What is the basic SI unit of time?

22. Write the abbreviation for 25 kilowatts.

Fill in each blank:

23. 280 W = _____ kW **24.** 13.9 mA = _____ A

25. 720 ps = _____ ns

26. What is the basic SI unit for temperature?

27. What is the freezing temperature of water on the Celsius scale?

Fill in each blank, rounding each result to three significant digits when necessary:

28. 25°C = _____ °F **29.** 28°F = _____ °C

30. 98.6°F = _____ °C **31.** 100 km = _____ mi

32. 200 cm = _____ in. **33.** 1.8 ft^3 = _____ in^3

34. 37.8 ha = _____ acres **35.** 80.2 kg = _____ lb

- The text design and second color help to make the text more easily understood, highlight important concepts, and enhance the art presentation.
- A reference of useful, frequently referenced information—such as metric system prefixes, U.S. weights and measures, metric and U.S. conversion, and formulas from geometry—is printed on the inside covers.

- The use of a basic scientific calculator has been integrated in an easy-to-use format with calculator flowcharts and displays throughout the text to reflect its nearly universal use in technical classes and on the job. The instructor should inform the students when *not* to use a calculator.

Using a Calculator to Multiply and Divide Fractions

Example 16 Multiply: $2\frac{5}{6} \times 4\frac{1}{2}$.

2 [A b/c] 5 [A b/c] 6 [×] 4 [A b/c] 1 [A b/c] 2 [=]

12 3/4

Thus, $2\frac{5}{6} \times 4\frac{1}{2} = 12\frac{3}{4}$. ◆

Example 17 Divide: $5\frac{5}{7} \div 8\frac{1}{3}$.

5 [A b/c] 5 [A b/c] 7 [÷] 8 [A b/c] 1 [A b/c] 3 [=]

24/35

Thus, $5\frac{5}{7} \div 8\frac{1}{3} = \frac{24}{35}$. ◆

- Cumulative reviews are provided at the end of every even-numbered chapter to help students review for comprehensive exams.

CUMULATIVE REVIEW | CHAPTERS 1–6

1. Find the prime factorization of 696.
2. Change 0.081 to a percent.
3. Write 3.015×10^{-4} in decimal form.
4. Write 28,500 in scientific notation.
5. 5 ha = ______ m^2 **6.** 101°F = ______ °C
7. 6250 in^2 = ______ ft^2
8. Give the number of significant digits (accuracy) of each measurement:
   a. 110 cm **b.** 6000 mi **c.** 24.005 s
9. Read the measurement shown on the vernier caliper in Illustration 1 **a.** in metric units and **b.** in U.S. units.

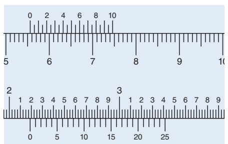

ILLUSTRATION 1

10. Read the measurement shown on the U.S. micrometer in Illustration 2.

Simplify:

12. $(2x - 5y) + (3y - 4x) - 2(3x - 5y)$
13. $(4y^3 + 3y - 5) - (2y^3 - 4y^2 - 2y + 6)$
14. $(3y^3)^3$
15. $-2x(x^2 - 3x + 4)$
16. $(6y^3 - 5y^2 - y + 2)(2y - 1)$
17. $(4x - 3y)(5x + 2y)$ **18.** $\dfrac{215\,x^2y^3}{45x^3y^5}$
19. $(16x^2y^3)(-5x^4y^5)$
20. $\dfrac{x^3 + 2x^2 - 11x - 20}{x + 5}$
21. $3x^2 - 4xy + 5y^2 - (-3x^2) + (-7xy) + 10y^2$

Solve:

22. $4x - 2 = 12$ **23.** $\frac{x}{4} - 5 = 9$
24. $4x - 3 = 7x + 15$ **25.** $\frac{5x}{8} = \frac{3}{2}$
26. $5 - (x - 3) = (2 + x) - 5$
27. $C = \frac{1}{2}(a + b + c)$ for a

- Studies show that current students will experience several career changes during their working lives. The chapter-opening pages illustrate various career paths for students to consider as their careers, technology, and the workplace evolve. The basic information provided in the chapter openers about a technical career is explored in further detail on the Cengage book companion website at www.cengage.com/mathematics/ewen.

Mathematics at Work

Electronics technicians perform a variety of jobs. Electronic engineering technicians apply electrical and electronic theory and knowledge to design, build, test, repair, and modify experimental and production electrical equipment in industrial or commercial plants for use by engineering personnel in making engineering design and evaluation decisions.

Other electronics technicians repair electronic equipment such as industrial controls, telemetering and missile control systems, radar systems, and transmitters and antennas using testing instruments. Industrial controls automatically monitor and direct production processes on the factory floor. Transmitters and antennas provide communications links for many organizations. The federal government uses radar and missile control systems for national defense as well as other applications.

Electricians install, maintain, and repair wiring, equipment, and fixtures and ensure that work is in accordance with relevant codes. They also travel to locations to repair equipment and perform preventive maintenance on a regular basis. They use schematics and manufacturers' specifications that show connections and provide instructions on how to locate problems. They also use software programs and testing equipment to diagnose malfunctions. For more information, please visit **www.cengagebrain.com** and access the Student Online Resources for this text.


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Electronics Technician
Electronics technician checking a fuse box

- Special application exercises in the areas of agriculture and horticulture, allied health, auto/diesel service, aviation, business and personal finance, CAD/drafting, culinary arts, electronics, HVAC, industrial and construction trades, manufacturing, natural resources, and welding have been submitted by faculty in these technical areas and are marked with related icons.

44. Find the total piston displacement of a six-cylinder engine if each piston displaces 0.9 litres (L).

45. A four-cylinder engine has a total displacement of 2.0 L. Find the displacement of each piston.

46. An eight-cylinder engine has a total displacement of 318 in^3. Find the displacement of each piston.

47. New front brake pads are 0.375 in. thick. The average wear rate of these pads in a particular vehicle is 0.062 in. per 15,000 mi. Determine **a.** the expected wear after 45,000 mi and **b.** the expected pad thickness after 60,000 mi.

48. A certain job requires 500 person-hours to complete. How many days will it take for five people working 8 hours per day to complete the job?

49. How many gallons of herbicide are needed for 150 acres of soybeans if 1.6 gal/acre are applied?

50. Suppose 10 gal of water and 1.7 lb of pesticide are to be applied per acre. **a.** How much pesticide would you put in a 300-gal spray tank? **b.** How many acres can be covered with one tankful? (Assume the pesticide dissolves in the water and has no volume.)

51. A cattle feeder buys some feeder cattle, which average 550 lb at \$145/hundredweight (that is, \$145 per hundred pounds, or \$1.45/lb). The price he receives when he sells them as slaughter cattle is \$120/hundredweight. If he plans to make a profit of \$150 per head, what will be his cost per pound for a 500-lb weight gain?

52. An insecticide is to be applied at a rate of 2 pt/100 gal of water. How many pints are needed for a tank that holds 20 gal? 60 gal? 150 gal? 350 gal? (Assume that the insecticide dissolves in the water and has no volume.)

59. A lamp that requires 0.84 A of current is connected to a 115-V source. What is the lamp's resistance? (Resistance equals voltage divided by current.)

60. A heating element operates on a 115-V line. If it has a resistance of 18 Ω, what current does it draw? (Current equals voltage divided by resistance.)

61. A patient takes 3 tablets of clonidine hydrochloride, containing 0.1 mg each. How many milligrams are taken?

62. One dose of ampicillin for a patient with bronchitis is 2 tablets each containing 0.25 g of medication. How many grams are in one dose?

63. An order reads 0.5 mg of digitalis, and each tablet contains 0.1 mg. How many tablets should be given?

64. An order reads 1.25 mg of digoxin, and the tablets on hand are 0.25 mg. How many tablets should be given?

65. A statute mile is 5280 ft. A nautical mile used in aviation is 6080.6 ft. This gives the conversion 1 statute mile = 0.868 nautical miles. If a plane flew 350 statute miles, how many nautical miles were flown?

66. Five lathes and four milling machines are to be on one circuit. If each lathe uses 16.0 A and each milling machine uses 13.8 A, what is the amperage requirement for this circuit?

67. A steel plate 1.00 in. thick weighs 40.32 lb/ft^2. Find the weight of a 4.00 ft × 8.00 ft sheet.

68. Municipal solid waste (MSW) consists basically of trash and recycle that is produced by nonindustrial and nonagricultural sources. According to Environmental Protection Agency estimates, as of 2014, each person in the United States generated an average of 4.44 lb of MSW each day. If you are an average American, how

- Group activity projects have been moved to the Instructor Companion website.
- An instructor's edition that includes all the answers to exercises is available.

Significant changes in the twelfth edition include the following:

- New and revised applications with the help and expertise of professionals in the areas of industrial and construction trades, electronics, and CAD/drafting.
- All areas have been reviewed and updated with current information and data.
- The material on measurement has been reorganized and revised to provide better rationale for measurement accuracy and precision and for calculations with measurements. Single versus multiple measurements are compared, and the concept of random and systematic errors have been introduced.
- Major effort was made to contain cost to students by having a more space-efficient page design, reviewing art size and placement, moving Group Activities from the end of each chapter to the Instructor Companion website, and deleting dial indicator material from Section 4.9 that seemingly was not being used.
- More than 140 exercises have been updated, replaced, or improved.

Useful ancillaries available to qualified adopters of this text include the following:

WEBASSIGN From Cengage www.webassign.com/cengage

- WebAssign from Cengage *Elementary Technical Mathematics*, Twelfth Edition, is a fully customizable online solution for STEM disciplines that empowers you to help your students learn, not just do homework. Insightful tools save you time and highlight exactly where your students are struggling. Decide when and what type of help students can access while working on assignments—and incentivize independent work so help features aren't abused. Meanwhile, your students get an engaging experience, instant feedback, and better outcomes. A total win-win!

  To try a sample assignment, learn about LMS integration or connect with our digital course support; visit www.webassign.com/cengage
- **Instructor's Edition** The Instructor's Edition features an appendix containing the answers to all problems in the book. (978-1-337-63059-7)
- **Instructor Solutions Manual** (ISBN: 978-1-337-63063-4): This guide contains solutions to every exercise in the book. You can download the solutions manual from the Instructor Companion Site.
- **Instructor Companion Website:** Everything you need for your course in one place! Access the Instructor Solutions Manual, full lecture PowerPoints, Group Projects, and other support materials. This collection of book-specific lecture and class tools is available via www.cengage.com/login

Student Resources:

WEBASSIGN From Cengage www.webassign.com

- Prepare for class with confidence using WebAssign from Cengage *Elementary Technical Mathematics*, Twelfth Edition. This online learning platform fuels practice, so you truly absorb what you learn—and are better prepared come test time. Videos and tutorials walk you through concepts and deliver instant feedback and grading, so you always know where you stand in class. Focus your study time and get extra practice where you need it most. Study smarter with WebAssign!

  Ask your instructor today how you can get access to WebAssign, or learn about self-study options at www.webassign.com

- **Student Solutions Manual**
  Author: James Lapp
  (ISBN: 978-1-337-63060-3)
  The Student Solutions Manual provides worked-out solutions to all of the odd-numbered exercises in the text, as well as solutions to all chapter review and cumulative review exercises.

I am grateful for the courtesy of the L. S. Starrett Company in allowing the use of photographs of their instruments in Chapter 4. A special thank you to Sarah Alamilla, Waukesha County Technical College, and Taylor Moore, Joliet Junior College, for lending their professional expertise in reviewing and updating the applications.

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Dale Ewen

Basic Concepts

CHAPTER 1

OBJECTIVES

- Add, subtract, multiply, and divide whole numbers.
- Add, subtract, multiply, and divide whole numbers with a basic scientific calculator.
- Apply the rules for order of operations.
- Find the area and volume of geometric figures.
- Evaluate formulas.
- Find the prime factorization of whole numbers.
- Add, subtract, multiply, and divide fractions.
- Add, subtract, multiply, and divide fractions with a basic scientific calculator.
- Use conversion factors to change from one unit to another within the U.S. system of weights and measures.
- Add, subtract, multiply, and divide decimal fractions.
- Add, subtract, multiply, and divide decimal fractions with a basic scientific calculator.
- Round numbers to a particular place value.
- Apply the percent concept; change a percent to a decimal, a decimal to a percent, a fraction to a percent, and a percent to a fraction.
- Solve application problems involving the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimal fractions and percents.
- Find powers and roots of numbers using a scientific calculator.
- Solve personal finance problems involving percent.

Mathematics at Work

Modern manufacturing companies require a wide variety of technology specialists for their operations. Manufacturing technology specialists set up, operate, and maintain industrial and manufacturing equipment as well as computer-numeric-controlled (CNC) and other automated equipment that make a large variety of products according to controlled specifications. Some focus on systematic equipment maintenance and repair. Others specialize in materials transportation and distribution; that is, they are responsible for moving and distributing the products to the sales locations and/or consumers after they are manufactured. Other key team members include designers, engineers, draftspersons, and quality control specialists. Training and education for these careers are available at many community colleges and trade schools. Some require a bachelor's degree. For more information, please visit **www.cengagebrain.com** and access the Student Online Resources for this text.

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Manufacturing Technology Specialist
Technician working with numerically controlled milling machine

UNIT 1A Review of Operations with Whole Numbers

1.1 Review of Basic Operations

The **positive integers** are the numbers 1, 2, 3, 4, 5, 6, and so on. They can also be written as +1, +2, +3, and so on, but usually the *positive* (+) sign is omitted. The **whole numbers** are the numbers 0, 1, 2, 3, 4, 5, 6, and so on. That is, the whole numbers consist of the positive integers and zero.

The value of any digit in a number is determined by its place in the particular number. Each place represents a certain power of 10. By powers of 10, we mean the following:

$10^0 = 1$
$10^1 = 10$
$10^2 = 10 \times 10 = 100$ (the second power of 10)
$10^3 = 10 \times 10 \times 10 = 1000$ (the third power of 10)
$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$ (the fourth power of 10) and so on.

NOTE: A small superscript number (such as the 2 in 10^2) is called an *exponent*.

The number 2354 means 2 thousands plus 3 hundreds plus 5 tens plus 4 ones.

In the number 236,895,174, each digit has been multiplied by some power of 10, as shown below.

	(ten millions)		(hundred thousands)		(thousands)		(tens)	
	10^7		10^5		10^3		10^1	
2	3	6,	8	9	5,	1	7	4
10^8		10^6		10^4		10^2		10^0
(hundred millions)		(millions)		(ten thousands)		(hundreds)		(units)

The "+" (plus) symbol is the sign for addition, as in the expression 5 + 7. The result of adding the numbers (in this case, 12) is called the **sum**. Integers are added in columns with the digits representing like powers of 10 in the same vertical line. (*Vertical* means up and down.)

Example 1 Add: 238 + 15 + 9 + 3564.

$$\begin{array}{r} 238 \\ 15 \\ 9 \\ \underline{3564} \\ 3826 \end{array}$$

◆

Subtraction is the inverse operation of addition. Therefore, subtraction can be thought of in terms of addition. The "−" (minus) sign is the symbol for subtraction. The quantity 5 − 3 can be thought of as "what number added to 3 gives 5?" The result of subtraction is called the **difference**.

To check a subtraction, add the difference to the second number. If the sum is equal to the first number, the subtraction has been done correctly.

Example 2 Subtract: 2843 − 1928.

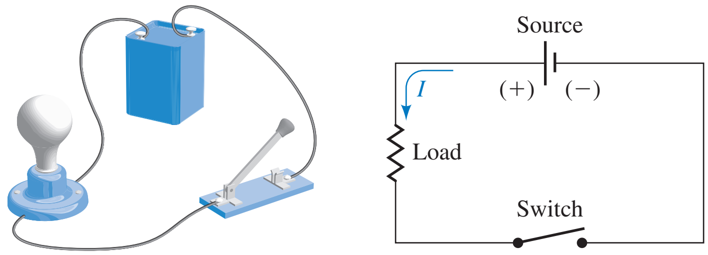
Subtract:

2843	first number
−1928	second number
915	difference

Check:

1928	second number
+915	difference
2843	This sum equals the first number, so 915 is the correct difference. ◆

Next, let's study some applications. To communicate about problems in electricity, technicians have developed a "language" of their own. It is a picture language that uses symbols and diagrams. The symbols used most often are listed in Table 2 of Appendix A. An electric circuit is a conducting loop in which electrons carrying electric energy may be transferred from a source to do useful work and returned to the source. The circuit diagram is the most common and useful way to show an electric circuit. Note how each component (part) of the picture (Figure 1.1a) is represented by its symbol in the circuit diagram (Figure 1.1b) in the same relative position.



(a) Picture diagram **(b)** Circuit diagram

Figure 1.1
Components in an electric circuit

The light bulb may be represented as a resistance. Then the circuit diagram in Figure 1.1b would appear as in Figure 1.2, where

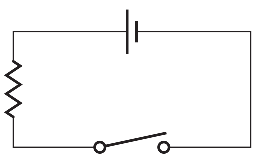
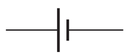


Figure 1.2

 represents the resistor

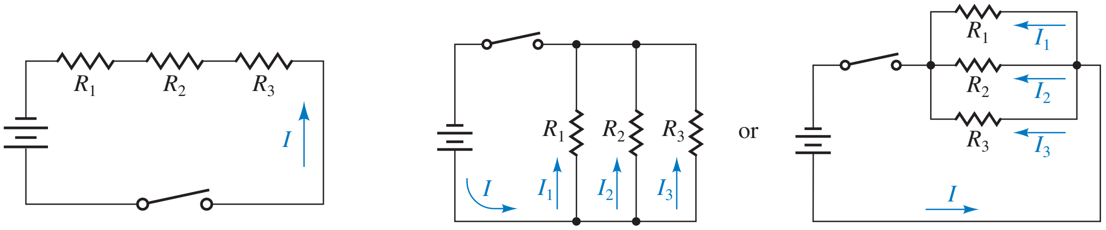
 represents the switch

 represents the source. The short line represents the negative terminal of a battery, and the long line represents the positive terminal. The current flows from positive to negative.

Energy is stored in the battery. When the switch is closed, energy is transferred to the light, and the light glows.

NOTE: In this book assume that the charge carriers are positive and draw current arrows in the direction that a positive charge would flow. This is a common practice used by most technicians and engineers. However, you may find the negative-charge–current-flow convention is also used in some books. Regardless of the convention used, the formulas and results are the same.

There are two basic types of electric circuits: series and parallel. An electric circuit with only one path for the current, I, to flow is called a *series* circuit (Figure 1.3a). An electric circuit with more than one path for the current to flow is called a *parallel* circuit (Figure 1.3b). A circuit breaker or fuse in a house is wired in series with its outlets. The outlets themselves are wired in parallel.



(a) Series circuit

(b) Parallel circuits

Figure 1.3

Two basic types of electric circuits

Example 3

In a series circuit, the total resistance equals the sum of all the resistances in the circuit. Find the total resistance in the series circuit in Figure 1.4. Resistance is measured in ohms, Ω.

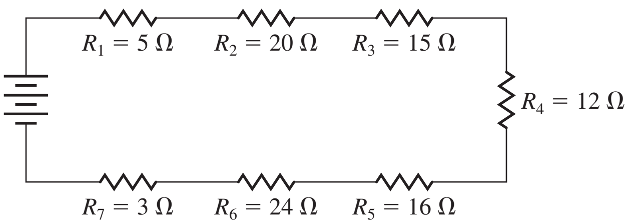


Figure 1.4

The total resistance is

$$\begin{array}{r} 5\ \Omega \\ 20\ \Omega \\ 15\ \Omega \\ 12\ \Omega \\ 16\ \Omega \\ 24\ \Omega \\ \underline{3\ \Omega} \\ 95\ \Omega \end{array}$$

◆

Example 4

Studs are upright wooden or metal pieces in the walls of a building, to which siding, insulation panels, drywall, or decorative paneling is attached. (A wall portion with seven studs is shown in Figure 1.5.) Studs are normally placed 16 in. on center and are placed double at all internal and external corners of a building. The number of studs needed in a wall can be estimated by finding the number of linear feet (ft) of the wall. How many studs are needed for the exterior walls of the building in Figure 1.6?

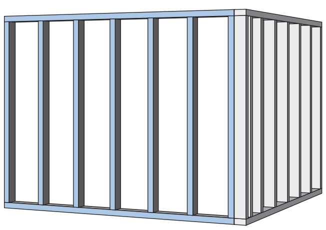


Figure 1.5

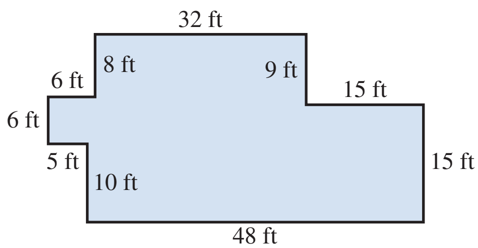


Figure 1.6

The outside perimeter of the building is the sum of the lengths of the sides of the building:

$$\begin{array}{r} 48 \text{ ft} \\ 15 \text{ ft} \\ 15 \text{ ft} \\ 9 \text{ ft} \\ 32 \text{ ft} \\ 8 \text{ ft} \\ 6 \text{ ft} \\ 6 \text{ ft} \\ 5 \text{ ft} \\ \underline{10 \text{ ft}} \\ 154 \text{ ft} \end{array}$$

Therefore, approximately 154 studs are needed in the outside wall. ◆

Repeated addition of the same number can be shortened by multiplication. The "×" (times) and the "·" (raised dot) are used to indicate multiplication. When adding the lengths of five pipes, each 7 ft long, we have 7 ft + 7 ft + 7 ft + 7 ft + 7 ft = 35 ft of pipe. In multiplication, this would be 5 × 7 ft = 35 ft. The 5 and 7 are called *factors*. The result of multiplying numbers (in this case, 35) is called the **product**. Computing areas, volumes, forces, and distances requires skills in multiplication.

Example 5 Multiply: 358 × 18.

$$\begin{array}{r} 358 \\ \underline{\times\ 18} \\ 2864 \\ \underline{358} \\ 6444 \end{array}$$ ◆

Division is the inverse operation of multiplication. The following symbols are used to show division: 15 ÷ 5, $5\overline{)15}$, 15/5, and $\frac{15}{5}$. The quantity 15 ÷ 5 can also be thought of as "what number times 5 gives 15?" The answer to this question is 3, which is 15 divided by 5. The result of dividing numbers (in this case, 3) is called the **quotient**. The number to be divided, 15, is called the *dividend*. The number you divide by, 5, is called the *divisor*.

Example 6 Divide: 84 ÷ 6.

$$\begin{array}{rl} 14 & \leftarrow \text{quotient} \\ 6\overline{)84} & \leftarrow \text{dividend} \\ \text{divisor} \quad \underline{6} & \\ 24 & \\ \underline{24} & \\ 0 & \leftarrow \text{remainder} \end{array}$$ ◆

Example 7 Divide: 115 ÷ 7.

$$\begin{array}{rl} 16 & \leftarrow \text{quotient} \\ 7\overline{)115} & \leftarrow \text{dividend} \\ \text{divisor} \quad \underline{7} & \\ 45 & \\ \underline{42} & \\ 3 & \leftarrow \text{remainder} \end{array}$$ ◆

The *remainder* (when not 0) is usually written in one of two ways: with an "r" preceding it or with the remainder written over the divisor as a fraction, as shown in Example 8. (Fractions are discussed in Unit 1B.)

Example 8

Divide: 534 ÷ 24.

$$\begin{array}{r} 22 \text{ r } 6 \quad \text{or} \quad 22\frac{6}{24} \\ 24\overline{)534} \\ \underline{48} \\ 54 \\ \underline{48} \\ 6 \end{array}$$

This quotient may be written 22 r 6 or $22\frac{6}{24}$. ◆

Example 9

Ohm's law states that in a simple electric circuit, the current I (measured in amps, A) equals the voltage E (measured in volts, V) divided by the resistance R (measured in ohms, Ω). Find the current in the circuit of Figure 1.7.

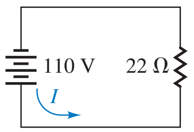


Figure 1.7

The current $I = \dfrac{E}{R} = \dfrac{110}{22} = 5$ A. ◆

Example 10

A 16-row corn planter costs $128,500. It has a 10-year life and a salvage value of $10,000. What is the annual depreciation? (Use the straight-line depreciation method.)

The straight-line depreciation method means that the difference between the cost and the salvage value is divided evenly over the life of the item. In this case, the difference between the cost and the salvage value is

$128,500	cost
−$10,000	salvage
$118,500	difference

This difference divided by 10, the life of the item, is $11,850. This is the annual depreciation. ◆

Example 11

Restaurants purchase potatoes to use for baked potatoes. The potatoes are often called bakers and can come in cases containing 90, 120, and so on, potatoes. If 3 cases of bakers with 90 potatoes per case are ordered plus 2 cases of bakers with 120 potatoes per case, how many total individual bakers are ordered?

3 cases × 90 potatoes/case	= 270 potatoes
2 cases × 120 potatoes/case	= 240 potatoes
Total	510 potatoes

◆

Using a Scientific Calculator

Use of a scientific calculator is integrated throughout this text. To demonstrate how to use a common scientific calculator, we show which keys to use and the order in which they are pushed. We have chosen to illustrate the most common types of algebraic logic calculators. Yours may differ. If so, consult your manual.

NOTE: Your calculator should be cleared before you begin any calculation.

Use a calculator to add, subtract, multiply, and divide as shown in the following examples.

Example 12 Add:

$$\begin{array}{r} 9463 \\ 125 \\ 9 \\ \underline{80} \end{array}$$

9463 [+] 125 [+] 9 [+] 80 [=]

9677

The sum is 9677. ◆

Example 13 Subtract:

$$\begin{array}{r} 3500 \\ \underline{1628} \end{array}$$

3500 [−] 1628 [=]

1872

The result is 1872. ◆

Example 14 Multiply: 125 × 68.

125 [×] 68 [=]

8500

The product is 8500. ◆

Example 15 Divide: 8700 ÷ 15.

8700 [÷] 15 [=]

580

The quotient is 580. ◆

NOTE: Your instructor will indicate which exercises should be completed using a calculator.

EXERCISES 1.1

Add:

1. 832 + 9 + 56 + 2358

2. 324 + 973 + 66 + 9430

3. $\begin{array}{r} 384 \\ 291 \\ 147 \\ \underline{632} \end{array}$

4. $\begin{array}{r} 78 \\ 107 \\ 45 \\ 217 \\ 9 \\ \underline{123} \end{array}$

5. 197 + 1072 + 10,877 + 15,532 + 768,098

6. 160,000 + 19,000 + 4,160,000 + 506,000

Subtract and check:

7. $\begin{array}{r} 7561 \\ \underline{2397} \end{array}$

8. $\begin{array}{r} 4000 \\ \underline{702} \end{array}$

9. 98,405 − 72,397

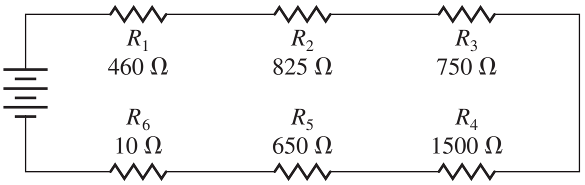
10. 417,286 − 287,156

11. $\begin{array}{r} 4000 \\ \underline{1180} \end{array}$

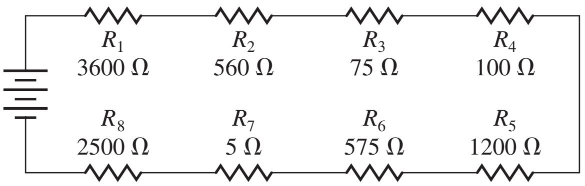
12. $\begin{array}{r} 60{,}000 \\ \underline{9{,}876} \end{array}$

Find the total resistance in each series circuit:

13.



14.



15. Approximately how many studs are needed for the exterior walls in the building shown in Illustration 1? (See Example 4.)

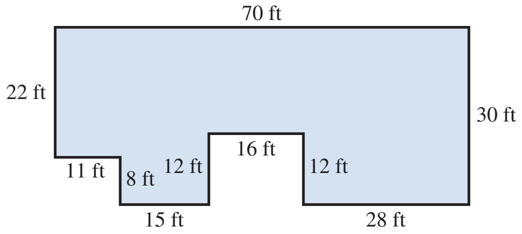


ILLUSTRATION 1

16. A pipe 24 ft long is cut into four pieces: the first 4 ft long, the second 5 ft long, and the third 7 ft long. What is the length of the remaining piece? (Assume no waste from cutting.)

17. A welder needs to weld together pipes of lengths 10 ft, 15 ft, and 14 ft. What is the total length of the new pipe?

18. A welder ordered a 125-ft^3 cylinder of argon gas, a semi-inert shielding gas for TIG welding. After a few days, only 78 ft^3 remained. How much argon was used?

19. Find the total input and output (I-O) in cubic centimetres (cm^3)* for a patient. By how much does the input of fluids exceed the output?

Input: 300 cm^3, 550 cm^3, 150 cm^3, 75 cm^3, 150 cm^3, 450 cm^3, 250 cm^3
Output: 325 cm^3, 150 cm^3, 525 cm^3, 250 cm^3, 175 cm^3

20. A student pilot must complete 40 h of total flight time as required for her private pilot certificate. She had already entered 31 h of flight time in her logbook. Monday she logged another 2 h, then Wednesday she logged another 3 h, and Friday she logged yet another 2 h. If she can fly 3 h more on Saturday, will she have enough total time as required for the certificate?

Multiply:

21. $\begin{array}{r} 567 \\ \underline{48} \end{array}$ **22.** $\begin{array}{r} 8374 \\ \underline{203} \end{array}$

23. 71,263 × 255 **24.** 1520 × 320

25. 6800 × 5200 **26.** 30,010 × 4080

Divide (use the remainder form with r):

27. $4\overline{)7236}$ **28.** $5\overline{)308{,}736}$

29. 4668 ÷ 12 **30.** 15,648 ÷ 36

31. 67,560 ÷ 80 **32.** $\dfrac{188{,}000}{120}$

33. An automobile uses gasoline at the rate of 31 miles per gallon (mi/gal or mpg) and has a 16-gallon tank. How far can it travel on one tank of gas?

34. An automobile uses gasoline at a rate of 12 kilometres per litre (km/L) and has a 65-litre tank. How far can it travel on one tank of gas?

35. A four-cylinder engine has a total displacement of 1300 cm^3. Find the displacement of each piston.

36. An automobile travels 1274 mi and uses 49 gal of gasoline. Find its mileage in miles per gallon.

37. An automobile travels 2340 km and uses 180 L of gasoline. Find its fuel consumption in kilometres per litre.

38. To replace some damaged ductwork, 20 linear feet of 8-in. × 16-in. duct is needed. The cost is $13 per 4 linear feet. What is the cost of replacement?

39. The bill for a new transmission was received. The total cost for labor was $516. If the car was serviced for 6 h, find the cost of labor per hour.

40. The cost for a set of four tires is $596. What is the cost of each tire?

41. A small Cessna aircraft has enough fuel to fly for 4 h. If the aircraft cruises at a ground speed of 125 miles per hour (mi/h or mph), how many miles can the aircraft fly in the 4 h?

42. A small plane takes off and climbs at a rate of 500 ft/min. If the plane levels off after 15 min, how high is the plane?

*Although cm^3 is the "official" metric abbreviation for cubic centimetres and will be used throughout this book, some readers may be more familiar with the abbreviation "cc," which is still used in some medical and allied health areas.

43. Inventory shows the following lengths of 3-inch steel pipe:

5 pieces 18 ft long
42 pieces 15 ft long
158 pieces 12 ft long
105 pieces 10 ft long
79 pieces 8 ft long
87 pieces 6 ft long

What is the total linear feet of pipe in inventory?

44. An order of lumber contains 36 boards 12 ft long, 28 boards 10 ft long, 36 boards 8 ft long, and 12 boards 16 ft long. How many boards are contained in the order? How many linear feet of lumber are contained in the order?

45. Two draftspersons, operating the same computer plotter, each work 8 hours per day. One produces 80 drawings per hour; the other produces 120 drawings per hour. What is the difference in their outputs after 30 work days?

46. A shipment contains a total of 5232 linear feet of steel pipe. Each piece of pipe is 12 ft long. How many pieces should be expected?

47. The wall is 10 ft high and the vertical length of the window is 54 in. The center of the window needs to be at a distance of 5/8 of the height of the wall above the floor (to meet the special Fibonacci ratio criteria). How should a window 75 in. wide be horizontally placed so that it is centered on a wall 17 ft 5 in. wide? How high is the bottom of the window above the floor?

48. A farmer expects a yield of 165 bushels per acre (bu/acre) from 260 acres of corn. If the corn is stored, how many bushels of storage are needed?

49. A farmer harvests 6864 bushels (bu) of soybeans from 156 acres. What is his yield per acre?

50. A railroad freight car can hold 2035 bu of corn. How many freight cars are needed to haul the expected 12,000,000 bu from a local grain elevator?

51. On a given day, eight steers weighed 856 lb, 754 lb, 1044 lb, 928 lb, 888 lb, 734 lb, 953 lb, and 891 lb. **a.** What is the average weight? **b.** In 36 days, 4320 lb of feed is consumed. What is the average feed consumption per day per steer?

52. What is the weight (in tons) of a stack of hay bales 6 bales wide, 110 bales long, and 15 bales high? The average weight of each bale is 80 lb. (1 ton = 2000 lb.)

53. From a 34-acre field, 92,480 lb of oats are harvested. Find the yield in bushels per acre. (1 bu of oats weighs 32 lb.)

54. A standard bale of cotton weighs approximately 500 lb. How many bales are contained in 15 tons of cotton?

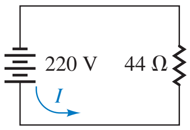
55. A tractor costs $175,000. It has a 10-year life and a salvage value of $3000. What is the annual depreciation? (Use the straight-line depreciation method. See Example 10.)

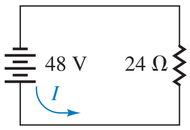
56. How much pesticide powder would you put in a 400-gal spray tank if 10 gal of spray, containing 2 lb of pesticide, are applied per acre?

57. Daylilies are to be planted along one side of a 30-ft walk in front of a house. The daylilies are planted 5 in. from each end and 10 in. apart along the walk. How many daylilies are needed?

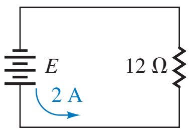
58. A potato patch has 7 rows with 75 hills of potatoes per row. If each potato hill yields 3 lb of marketable potatoes, how many pounds of marketable potatoes were produced?

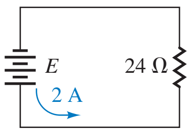
Using Ohm's law, find the current I in amps (A) in each electric circuit (see Example 9):

59.


60.


Ohm's law, in another form, states that in a simple circuit the voltage E (measured in volts, V) equals the current I (measured in amps, A) times the resistance R (measured in ohms, Ω). Find the voltage E measured in volts (V) in each electric circuit:

61.


62.


63. A hospital dietitian determines that each patient needs 4 ounces (oz) of orange juice. How many ounces of orange juice must be prepared for 220 patients?

64. During 24 hours, a patient is to receive three 60-mg doses of phenobarbital. Each tablet contains 30 mg of phenobarbital. How many milligrams of phenobarbital does the patient receive altogether in 24 hours? How many pills does the patient take in 24 hours?

65. To give 800 mg of quinine sulfate from 200-mg tablets, how many tablets would you use?

66. A nurse used two 5-g tablets of potassium permanganate in the preparation of a medication. How much potassium permanganate was used?

67. A sun room addition to a home has a wall 14 ft 6 in. long measured from inside wall to inside wall. Four windows are to be equally spaced from each other in this wall. The windows are 2 ft 6 in. wide including the inside window molding. What is the space between the wall and windows shown in Illustration 2?

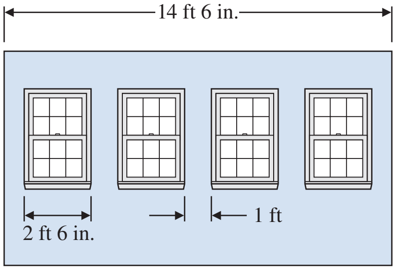


ILLUSTRATION 2

68. A solid concrete block wall is being built around a rectangular storage building with outside dimensions 12 ft 8 in. by 17 ft 4 in. using 16-in.-long by 8-in.-high by 4-in.-thick concrete block. How many blocks will be needed to build the 8-ft-high wall around the building as shown in Illustration 3? (Ignore the mortar joints and a door.) Suggestion: First, change dimensions to inches.

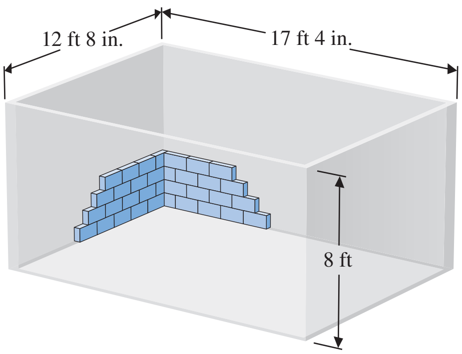


ILLUSTRATION 3

69. A sheet of plywood 8 ft long is painted with three equally spaced stripes to mark off a hazardous area as shown in Illustration 4. If each stripe is 10 in. wide, what is the space between the end of the plywood and the first stripe?

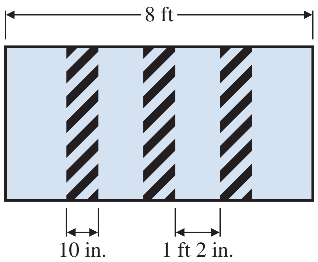


ILLUSTRATION 4

70. In a small machine shop, eight 5-gallon drums of oil are on hand. If 2 gallons are used each day and the owner wants a 30-day supply on hand, how many drums should be ordered?

71. Using a process called "cruising timber," foresters can estimate the amount of lumber in board feet in trees before they are cut down. In a stand of 1000 trees, a forester selects a representative sample of 100 trees and estimates that the sample contains 8540 board feet of lumber. If the entire stand containing 2500 trees is harvested, how many board feet would the landowner expect to harvest?

72. In tilapia aquaculture production, a feed conversion ratio of 2 lb of high-protein pelleted feed per pound of weight gain, after death losses, is not unusual. At that rate of feed conversion, if fish food costs $520 per ton (2000 lb), what would be the feed cost per pound of live fish produced?

73. A banquet facility has section areas separated with folding walls. Section A has a total of 50 seats, Section B has a total of 125 seats, Section C has a total of 110 seats, and Section D has a total of 35 seats. What is the maximum number of guests who could be seated using all sections?

74. Each beef export loin will yield 11 prime rib servings. **a.** How many beef export loins are needed to serve prime rib to 125 guests? **b.** How many total end cut servings are possible for these guests?

75. A delivery to the kitchen includes the following: 2 boxes of bakers (potatoes, 90 per box), 3 boxes of beef roast 109s (prime ribs, 4 per box), and 2 boxes of pork loins (pork, 4 per box). How many total individual items are in all the boxes?

76. The Sun Rise Restaurant has 10 tables that seat 6 people each plus 12 tables that seat 4 people each. Each server is assigned at most 6 tables. What is the least number of servers needed when the restaurant is full of customers?

77. The wait staff decides to pool (combine and divide evenly) their tips for the evening shift. The three servers have rounded tips of: $131, $152, and $128. **a.** What is the total amount to be divided? **b.** How much does each server receive?

1.2 Order of Operations

The expression 5^3 means to use 5 as a factor 3 times. We say that 5^3 is the third *power* of 5, where 5 is called the *base* and 3 is called the *exponent*. Here, 5^3 means $5 \times 5 \times 5 = 125$. The expression 2^4 means that 2 is used as a factor 4 times; that is, $2^4 = 2 \times 2 \times 2 \times 2 = 16$. Here, 2^4 is the fourth power of 2.

Just as we use periods, commas, and other punctuation marks to help make sentences more readable, we use **grouping symbols** in mathematics such as *parentheses* "( )" and *brackets* "[ ]" to help clarify the meaning of mathematical expressions. Parentheses not only give an expression a particular meaning, they also specify the order to be followed in evaluating and simplifying expressions.

What is the value of $8 - 3 \cdot 2$? Is it 10? Is it 2? Or is it some other number? It is very important that each mathematical expression have only one value. For this to happen, we all must not only perform the exact *same operations* in a given mathematical expression or problem but also perform them in exactly the *same order*. The following order of operations is followed by all:

Order of Operations

1. Always do the operations within parentheses or other grouping symbols first.
2. Then evaluate each power, if any. Examples:

$$4 \times 3^2 = 4 \times (3 \times 3) = 4 \times 9 = 36$$
$$5^2 \times 6 = (5 \times 5) \times 6 = 25 \times 6 = 150$$
$$\frac{5^3}{6^2} = \frac{5 \times 5 \times 5}{6 \times 6} = \frac{125}{36}$$

3. Next, perform multiplications and divisions in the order in which they appear as you read from left to right. For example,

$$\begin{aligned} & 60 \times 5 \div 4 \div 3 \times 2 \\ = & \ 300 \div 4 \div 3 \times 2 \\ = & \ 75 \div 3 \times 2 \\ = & \ 25 \times 2 \\ = & \ 50 \end{aligned}$$

4. Finally, perform additions and subtractions in the order in which they appear as you read from left to right.

NOTE: If two parentheses or a number and a parenthesis occur next to one another without any sign between them, multiplication is indicated.

By using the above procedure, we find that $8 - 3 \cdot 2 = 8 - 6 = 2$.

Example 1 Evaluate: $2 + 5(7 + 6)$.

$= 2 + 5(13)$ Add within parentheses.

$= 2 + 65$ Multiply.

$= 67$ Add.

NOTE: A number next to parentheses indicates multiplication. In Example 1, 5(13) means 5×13. Adjacent parentheses also indicate multiplication: (5)(13) also means 5×13. ◆

Example 2 Evaluate: $(9 + 4) \times 16 + 8$.

$= 13 \times 16 + 8$ Add within parentheses.

$= 208 + 8$ Multiply.

$= 216$ Add. ◆

Example 3 Evaluate: $(6 + 1) \times 3 + (4 + 5)$.

$= 7 \times 3 + 9$ Add within parentheses.

$= 21 + 9$ Multiply.

$= 30$ Add. ◆

Example 4 Evaluate: $4(16 - 4) + \frac{14}{7} - 8$.

$= 4(12) + \frac{14}{7} - 8$ Subtract within parentheses.

$= 48 + 2 - 8$ Multiply and divide.

$= 42$ Add and subtract. ◆

Example 5 Evaluate: $7 + (6 - 2)^2$.

$= 7 + 4^2$ Subtract within parentheses.

$= 7 + 16$ Evaluate the power.

$= 23$ Add. ◆

Example 6 Evaluate: $25 - 3 \cdot 2^3$.

$= 25 - 3 \cdot 8$ Evaluate the power.

$= 25 - 24$ Multiply.

$= 1$ Subtract. ◆

Example 7 Evaluate: $\dfrac{6^2 \cdot 4 - 2 \cdot 3^3}{5^2 + 5 \cdot 2^2}$

$= \dfrac{36 \cdot 4 - 2 \cdot 27}{25 + 5 \cdot 4}$ Evaluate each power.

$= \dfrac{144 - 54}{25 + 20}$ Multiply.

$= \dfrac{90}{45}$ Subtract in the numerator and add in the denominator.

$= 2$ Divide.

NOTE: You must evaluate the numerator and the denominator separately before you divide in the last step. ◆

If pairs of parentheses are nested (parentheses within parentheses, or within brackets), work from the innermost pair of parentheses to the outermost pair. That is, remove the innermost parentheses first, remove the next innermost parentheses second, and so on.

Example 8 Evaluate: $6 \times 2 + 3[7 + 4(8 - 6)]$.

$= 6 \times 2 + 3[7 + 4(2)]$ Subtract within parentheses.

$= 6 \times 2 + 3[7 + 8]$ Multiply.

$= 6 \times 2 + 3[15]$ Add within brackets.

$= 12 + 45$ Multiply.

$= 57$ Add. ◆

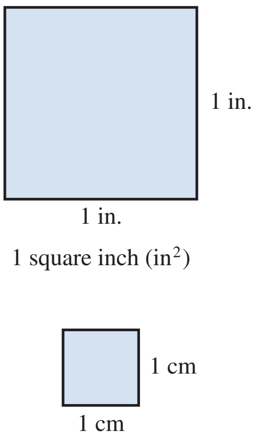
EXERCISES 1.2

Evaluate each expression:

1. $8 - 3(4 - 2)$
2. $(8 + 6)4 + 8$
3. $(8 + 6) - (7 - 3)$
4. $4 \times (2 \times 6) + (6 + 2) \div 4$
5. $2(9 + 5) - 6 \times (13 + 2) \div 9$
6. $5(8 \times 9) + (13 + 7) \div 4$
7. $27 + 13 \times (7 - 3)(12 + 6) \div 9$
8. $123 - 3(8 + 9) + 17$
9. $16 + 4(7 + 8) - 3$
10. $(18 + 17)(12 + 9) - (7 \times 16)(4 + 2)$
11. $9 - 2(17 - 15) + 18$
12. $(9 + 7)5 + 13$
13. $(39 - 18) - (23 - 18)$
14. $5(3 \times 7) + (8 + 4) \div 3$
15. $3(8 + 6) - 7(13 + 3) \div 14$
16. $6(4 \times 5) + (15 + 9) \div 6$
17. $42 + 12(9 - 3)(12 + 13) \div 30$
18. $228 - 4 \times (7 + 6) - 8(6 - 2)$
19. $38 + 9 \times (8 + 4) - 3(5 - 2)$
20. $(19 + 8)(4 + 3) \div 21 + (8 \times 15) \div (4 \times 3)$
21. $27 - 2 \times (18 - 9) - 3 + 8(43 - 15)$
22. $6 \times 8 \div 2 \times 9 \div 12 + 6$
23. $12 \times 9 \div 18 \times 64 \div 8 + 7$
24. $18 \div 6 \times 24 \div 4 \div 6$
25. $7 + 6(3 + 2) - 7 - 5(4 + 2)$
26. $5 + 3(7 \times 7) - 6 - 2(4 + 7)$
27. $3 + 17(2 \times 2) - 67$
28. $8 - 3(9 - 2) \div 21 - 7$
29. $28 - 4(2 \times 3) + 4 - (16 \times 8) \div (4 \times 4)$
30. $6 + 4(9 + 6) + 8 - 2(7 + 3) - (3 \times 12) \div 9$
31. $24/(6 - 2) + 4 \times 3 - 15/3$
32. $(36 - 6)/(5 + 10) + (16 - 1)/3$
33. $3 \times 15 \div 9 + (13 - 5)/2 \times 4 - 2$
34. $28/2 \times 7 - (6 + 10)/(6 - 2)$
35. $10 + 4^2$
36. $4 + 2 \cdot 3^2$
37. $\dfrac{3 \cdot 5 + 6 \cdot 8}{53 - 2 \cdot 5^2}$
38. $\dfrac{3 \cdot 4 + 2 \cdot 3}{42 - 20 \cdot 2 \cdot 1}$
39. $\dfrac{20 + (2 \cdot 3)^2}{7 \cdot 2^3}$
40. $\dfrac{(20 - 2 \cdot 5)^2}{3^3 - 2}$
41. $6[3 + 2(2 + 5)]$
42. $5((4 + 6) + 2(5 - 2))$
43. $5 \times 2 + 3[2(5 - 3) + 4(4 + 2) - 3]$
44. $3(10 + 2(1 + 3(2 + 6(4 - 2))))$

1.3 Area and Volume

To measure the length of an object, you must first select a suitable standard unit of length. To measure short lengths, choose a unit such as centimetres or millimetres in the metric system, or inches in the United States or, as it is still sometimes called, the English system. For long distances, choose metres or kilometres in the metric system, or yards or miles in the U.S. system.



1 square centimetre (cm²)

Figure 1.8
Square units

Area

The **area** of a plane geometric figure is the number of square units of measure it contains. To measure the surface area of an object, first select a standard unit of area suitable to the object to be measured. Standard units of area are based on the square and are called square units. For example, a square inch (in^2) is the amount of surface area within a square that measures one inch on a side. A square centimetre (cm^2) is the amount of surface area within a square that is 1 cm on a side. (See Figure 1.8.)

Example 1

What is the area of a rectangle measuring 4 cm by 3 cm?

Each square in Figure 1.9 represents 1 cm^2. By simply counting the number of squares, you find that the area of the rectangle is 12 cm^2.

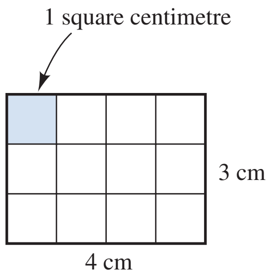


Figure 1.9

You can also find the area by multiplying the length times the width:

$$\begin{aligned} \text{Area} &= \quad l \quad\times\quad w \\ &= \underset{\text{(length)}}{4\text{ cm}} \times \underset{\text{(width)}}{3\text{ cm}} = 12\text{ cm}^2 \end{aligned}$$

Note: $\text{cm} \times \text{cm} = \text{cm}^2$ ◆

Example 2

What is the area of the metal plate represented in Figure 1.10?

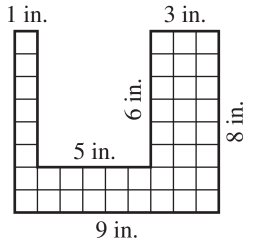


Figure 1.10

Each square represents 1 square inch. By simply counting the number of squares, we find that the area of the metal plate is 42 in^2.

Another way to find the area of the figure is to find the areas of two rectangles and then find their difference, as in Figure 1.11.

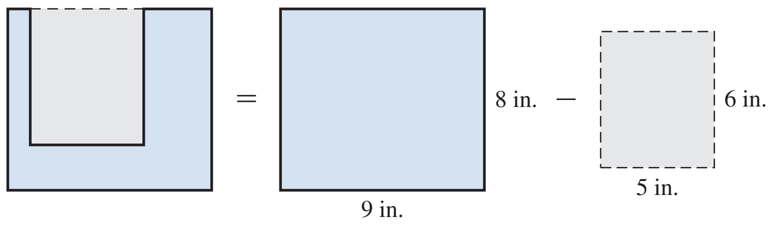


Figure 1.11

Area of outer rectangle: 9 in. × 8 in. $=$ 72 in^2
Area of inner rectangle: 5 in. × 6 in. $=$ $\underline{30\text{ in}^2}$
Area of metal plate: $=$ 42 in^2 Subtract. ◆

Volume

The **volume** of a solid geometric figure is the number of cubic units of measure it contains. In area measurement, the standard units are based on the square and called square units. For volume measurement, the standard units are based on the cube and called cubic units. For example, a cubic inch (in^3) is the amount of space contained in a cube that measures 1 in. on each edge. A cubic centimetre (cm^3) is the amount of space contained in a cube that measures 1 cm on each edge. A cubic foot (ft^3) is the amount of space contained in a cube that measures 1 ft (or 12 in.) on each edge. (See Figure 1.12.)

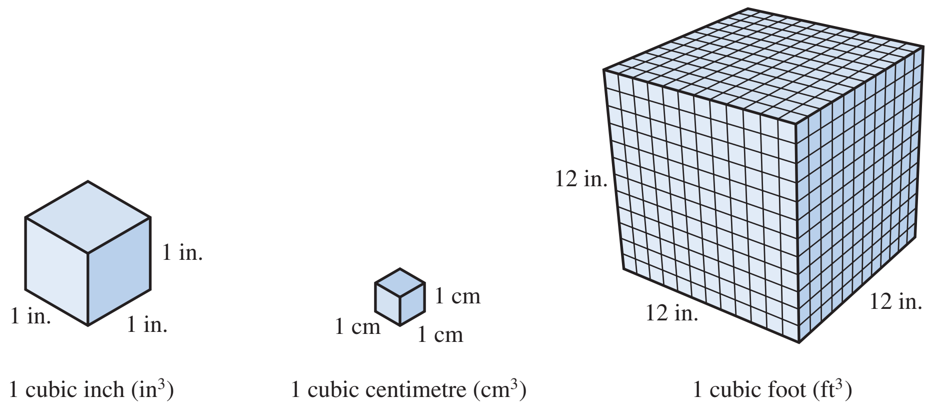


Figure 1.12
Cubic units

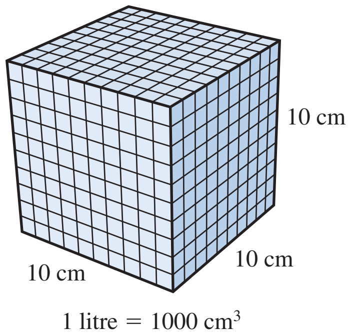


Figure 1.13
Litre

Figure 1.13 shows that the cubic decimetre (litre) is made up of 10 layers, each containing 100 cm^3, for a total of 1000 cm^3.

Example 3

Find the volume of a rectangular box 8 cm long, 4 cm wide, and 6 cm high.

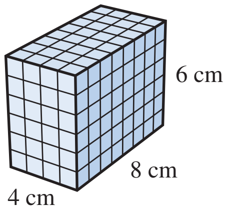


Figure 1.14

Suppose you placed one-centimetre cubes in the box, as in Figure 1.14. On the bottom layer, there would be 8×4, or 32, one-cm cubes. In all, there are six such layers, or $6 \times 32 = 192$ one-cm cubes. Therefore, the volume is 192 cm^3.

You can also find the volume of a rectangular solid by multiplying the length times the width times the height:

$$
\begin{aligned}
V &= l \times w \times h \\
&= 8\text{ cm} \times 4\text{ cm} \times 6\text{ cm} \\
&= 192\text{ cm}^3 \qquad \text{Note: cm} \times \text{cm} \times \text{cm} = \text{cm}^3
\end{aligned}
$$

◆

Example 4

How many cubic inches are in one cubic foot?

The bottom layer of Figure 1.15 contains 12×12, or 144, one-inch cubes. There are 12 such layers, or $12 \times 144 = 1728$ one-inch cubes. Therefore, 1 ft^3 = 1728 in^3. ◆

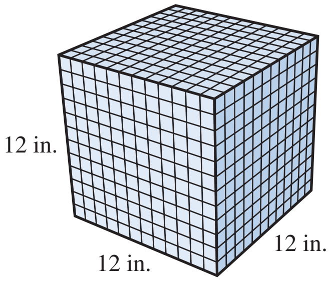
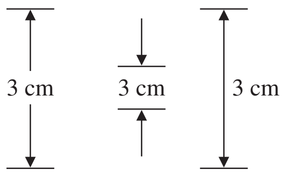


Figure 1.15
Cubic foot

EXERCISES 1.3

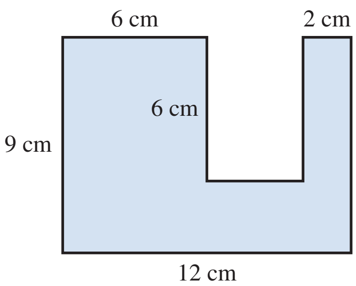
1. How many square yards (yd^2) are contained in a rectangle 12 yd long and 8 yd wide?
2. How many square metres (m^2) are contained in a rectangle 12 m long and 8 m wide?
3. At a small airport, Runway 11-29 is 4100 ft long and 75 ft wide. What is the area of the runway?
4. A small rectangular military operating zone has dimensions 12 mi by 22 mi. What is its area?
5. An automobile measures 191 in. by 73 in. Find the area it occupies.
6. Five pieces of sheet metal have been cut to form a container. The five pieces are of sizes 27 by 15, 15 by 18, 27 by 18, 27 by 18, and 15 by 18 (all in inches). What is the total area of all five pieces?

In exercises assume that corners are square and that like measurements are not repeated because the figures have consistent lengths. Note: All three of the following mean that the length of a side is 3 cm:

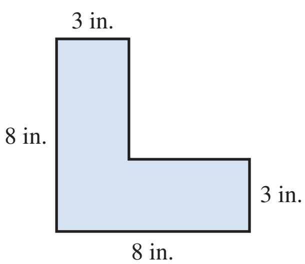


Find the area of each figure:

7.



8.



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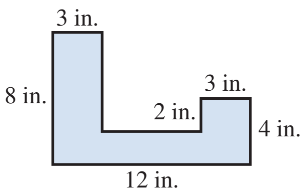

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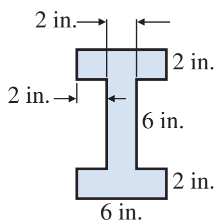
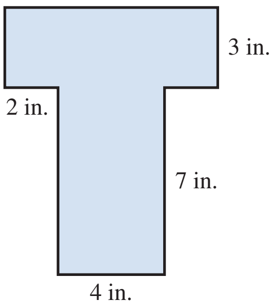
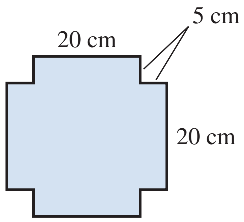


11.



12.



13. How many tiles 4 in. on a side should be used to cover a portion of a wall 48 in. long by 36 in. high? (See Illustration 1.)

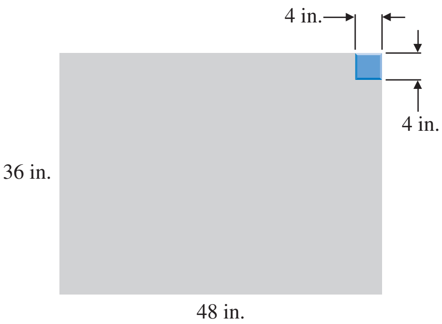


ILLUSTRATION 1

14. How many ceiling tiles 2 ft by 4 ft are needed to tile a ceiling that is 24 ft by 26 ft? (Be careful how you arrange the tiles.)

15. How many gallons of paint should be purchased to paint 20 motel rooms as shown in Illustration 2? (Do not paint the floor.) One gallon is needed to paint 400 square feet (ft^2).

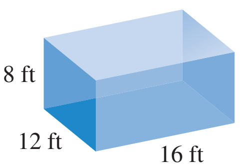


ILLUSTRATION 2

16. How many pieces of 4-ft by 8-ft drywall are needed for the 20 motel rooms in Exercise 15? All four walls and the ceiling in each room are to be drywalled. Assume that the drywall cut out for windows and doors cannot be salvaged and used again.

17. The replacement cost for construction of houses is $\$110/ft^2$. Determine how much house insurance should be carried on each of the one-story houses in Illustration 3.

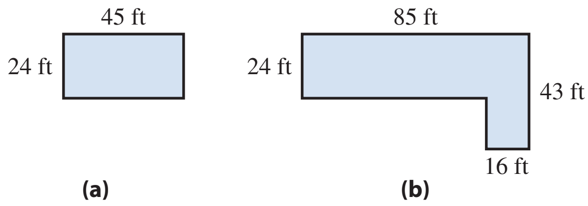


ILLUSTRATION 3

18. The replacement cost for construction of the building in Illustration 4 is $\$90/ft^2$. Determine how much insurance should be carried for full replacement.

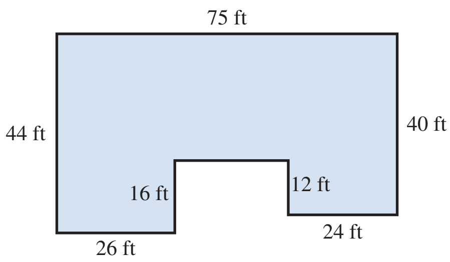
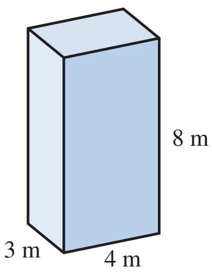


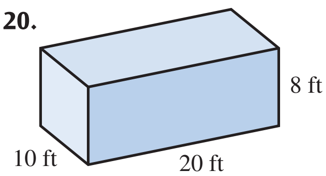
ILLUSTRATION 4

Find the volume of each rectangular solid:

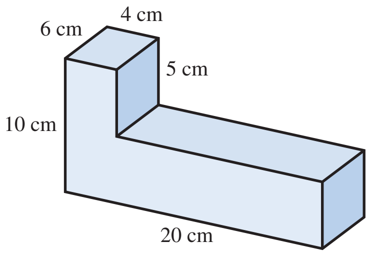
19.



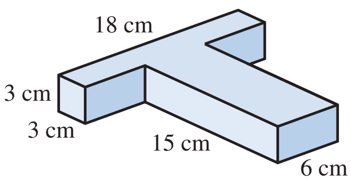
20.



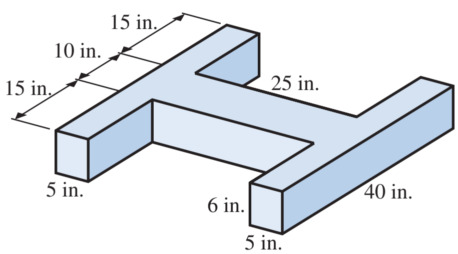
21.



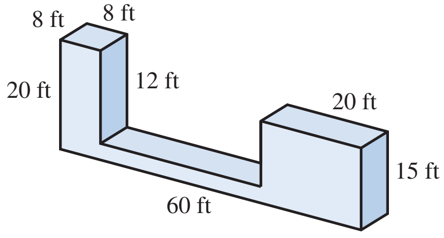
22.



23.



24.



25. Find the volume of a rectangular box 10 cm by 12 cm by 5 cm.

26. A mountain cabin has a single room 20 ft by 10 ft by 8 ft high. What is the total volume of air in the room that will be circulated through the central ventilating fan?

27. Common house duct is 8-in. by 20-in. rectangular metal duct. If the length of a piece of duct is 72 in., what is its volume?

28. A furnace filter measures 16 in. by 20 in. by 1 in. What is its volume?

29. A large rectangular tank is to be made of sheet metal as follows: 3 ft by 5 ft for the top and the base, two sides consisting of 2 ft by 3 ft, and two sides consisting of 2 ft by 5 ft. Find the volume of this container.

30. Suppose an oil pan has the rectangular dimensions 14 in. by 16 in. by 4 in. Find its volume.

31. Find the weight of a cement floor that is 15 ft by 12 ft by 2 ft if 1 ft^3 of cement weighs 193 lb.

32. A trailer 5 ft by 6 ft by 5 ft is filled with coal. Given that 1 ft^3 of coal weighs 40 lb and 1 ton = 2000 lb, how many tons of coal are in the trailer?

33. A rectangular tank is 8 ft long by 5 ft wide by 6 ft high. Water weighs approximately 62 lb/ft^3. Find the weight of water if the tank is full.

34. A rectangular tank is 9 ft by 6 ft by 4 ft. Gasoline weighs approximately 42 lb/ft^3. Find the weight of gasoline if the tank is full.

35. A building is 100 ft long, 50 ft wide, and 10 ft high. Estimate the cost of heating it at the rate of \$55 per 1000 ft^3.

36. A single-story shopping center is being designed to be 483 ft long by 90 ft deep. Two stores have been preleased. One occupies 160 linear feet and the other will occupy 210 linear feet. The owner is trying to decide how to divide the remaining space, knowing that the smallest possible space should be 4000 ft^2. How many stores can occupy the remaining area as shown in Illustration 5?

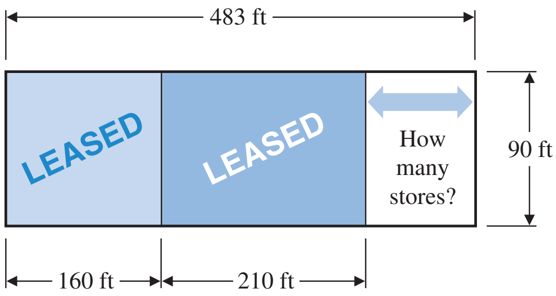


ILLUSTRATION 5

37. A trophy company needs a shipping box for a trophy 15 in. high with an 8-in.-square base. The box company is drawing the die to cut the cardboard for this box. **a.** What are the dimensions of the smallest rectangular sheet of cardboard needed to make one box that allows 1 in. for packing and 1 in. for a glue edge as shown in Illustration 6? **b.** What is the area of the rectangular cardboard to be mass produced? **c.** What is the total area of the cardboad to be removed?

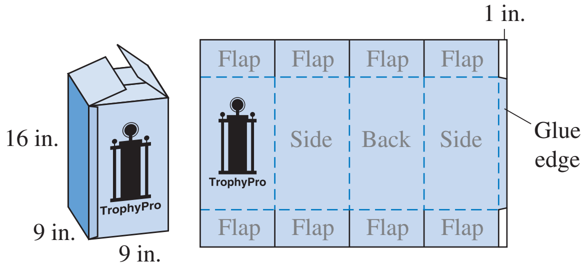


ILLUSTRATION 6

38. Styrofoam "peanuts" will be used to pack the trophy in the box in Illustration 6 to prevent the trophy from being broken during shipment. Ignoring the box wall thickness, how many cubic inches of peanuts including air space will be used for each package if the volume of the trophy is 450 in^3?

39. A standard cord of wood measures 4 ft by 4 ft by 8 ft. What is the volume in cubic feet of a cord of wood?

40. A municipal wastewater treatment plant has a settling tank that is 125 ft long and 24 ft wide with an effective depth of 12 ft. What is the surface area of the liquid in the tank and what is the volume of sewerage that the settling tank will hold?

41. Three inches of mulch need to be applied to a rectangular flower bed 8 ft by 24 ft between a house and a walk. How many cubic feet of mulch are needed?

42. How many 4 × 4 inch plant containers can be placed in a greenhouse on a table 4 ft wide and 8 ft long?

1.4 Formulas

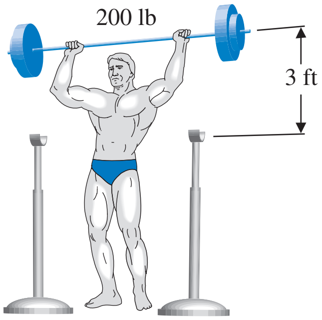


Figure 1.16
600 ft-lb of work is done moving this 200-lb weight a distance of 3 ft.

A **formula** is a statement of a rule using letters to represent the relationship of certain quantities. In physics, one of the basic rules states that *work* equals *force* times *distance*. If a person (Figure 1.16) lifts a 200-lb weight a distance of 3 ft, we say the work done is 200 lb × 3 ft = 600 foot-pounds (ft-lb). The work, W, equals the force, f, times the distance, d, or $W = f \times d$.

A person pushes against a car weighing 2700 lb but does not move it. The work done is 2700 lb × 0 ft = 0 ft-lb. An automotive technician (Figure 1.17) moves a diesel engine weighing 1100 lb from the floor to a workbench 4 ft high. The work done in moving the engine is 1100 lb × 4 ft = 4400 ft-lb.

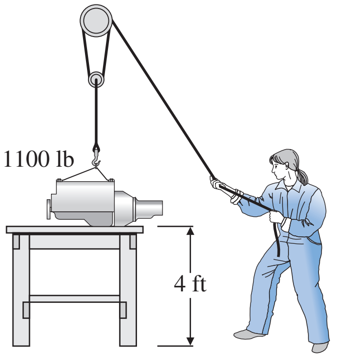


Figure 1.17
4400 ft-lb of work is done moving this 1100-lb diesel engine from the floor to this workbench 4 ft high.

To summarize, if you know the amount of force and the distance the force is applied, the work can be found by simply multiplying the force and distance. The formula $W = f \times d$ is often written $W = f \cdot d$, or simply $W = fd$. Whenever there is no symbol between a number and a letter or between two letters, it is assumed that the operation to be performed is multiplication.

Example 1

If $W = fd$, $f = 10$, and $d = 16$, find W.

$W = fd$
$W = (10)(16)$
$W = 160$ Multiply. ◆

Example 2

If $I = \frac{E}{R}$, $E = 450$, and $R = 15$, find I.

$I = \frac{E}{R}$
$I = \frac{450}{15}$
$I = 30$ Divide. ◆

Example 3

If $P = I^2R$, $I = 3$, and $R = 600$, find P.

$P = I^2R$
$P = (3)^2(600)$
$P = (9)(600)$ Evaluate the power.
$P = 5400$ Multiply. ◆

There are many other formulas used in science and technology. Some examples are given here:

a. $d = vt$ **c.** $f = ma$ **e.** $I = \dfrac{E}{R}$

b. $W = IEt$ **d.** $P = IE$ **f.** $P = \dfrac{V^2}{R}$

Formulas from Geometry

The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where b is the length of the base and h, the height, is the length of the altitude to the base (Figure 1.18). (An altitude of a triangle is a line from a vertex perpendicular to the opposite side, as shown in (a), or to the opposite side extended, as shown in (b).)

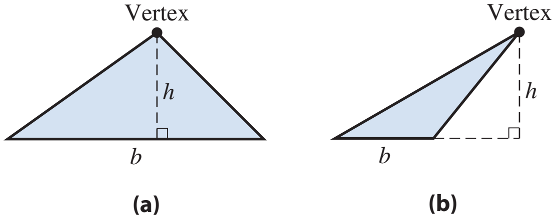


Figure 1.18

Example 4 Find the area of a triangle whose base is 18 in. and whose height is 10 in.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(18 \text{ in.})(10 \text{ in.})$$

$$= 90 \text{ in}^2 \qquad \text{Note: (in.)(in.)} = \text{in}^2$$

♦

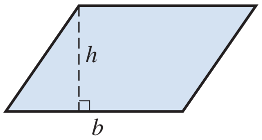


Figure 1.19

The area of a *parallelogram* (a four-sided figure whose opposite sides are parallel) is given by the formula $A = bh$, where b is the length of the base and h is the perpendicular distance between the base and its opposite side (Figure 1.19).

Example 5 Find the area of a parallelogram with base 24 cm and height 10 cm.

$$A = bh$$

$$A = (24 \text{ cm})(10 \text{ cm})$$

$$= 240 \text{ cm}^2 \qquad \text{Note: (cm)(cm)} = \text{cm}^2$$

♦

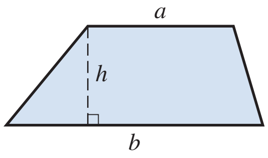


Figure 1.20

The area of a *trapezoid* (a four-sided figure with one pair of parallel sides) is given by the formula $A = \left(\frac{a+b}{2}\right)h$, where a and b are the lengths of the parallel sides (called *bases*), and h is the perpendicular distance between the bases (Figure 1.20).

Example 6 Find the area of the trapezoid in Figure 1.21.

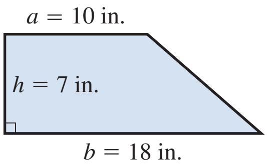


Figure 1.21

$$A = \left(\frac{a+b}{2}\right)h$$

$$A = \left(\frac{10 \text{ in.} + 18 \text{ in.}}{2}\right)(7 \text{ in.})$$

$$= \left(\frac{28 \text{ in.}}{2}\right)(7 \text{ in.}) \qquad \text{Add within parentheses.}$$

$$= (14 \text{ in.})(7 \text{ in.}) \qquad \text{Divide.}$$

$$= 98 \text{ in}^2 \qquad \text{Multiply.}$$

♦

EXERCISES 1.4

Use the formula $W = fd$, where f represents a force and d represents the distance that the force is applied. Find the work done, W:

1. $f = 30, d = 20$
2. $f = 17, d = 9$
3. $f = 1125, d = 10$
4. $f = 203, d = 27$
5. $f = 176, d = 326$
6. $f = 2400, d = 120$

From formulas **a.** *to* **f.** *on page 19, choose one that contains all the given letters. Then use the formula to find the unknown letter:*

7. If $m = 1600$ and $a = 24$, find f.
8. If $V = 120$ and $R = 24$, find P.
9. If $E = 120$ and $R = 15$, find I.
10. If $v = 372$ and $t = 18$, find d.
11. If $I = 29$ and $E = 173$, find P.
12. If $I = 11$, $E = 95$, and $t = 46$, find W.

Find the area of each triangle:

13. $b = 10$ in., $h = 8$ in.
14. $b = 36$ cm, $h = 20$ cm
15. $b = 54$ ft, $h = 30$ ft
16. $b = 188$ m, $h = 220$ m

Find the area of each rectangle:

17. $b = 8$ m, $h = 7$ m
18. $b = 24$ in., $h = 15$ in.
19. $b = 36$ ft, $h = 18$ ft
20. $b = 250$ cm, $h = 120$ cm

Find the area of each trapezoid:

21. $a = 7$ ft, $b = 9$ ft, $h = 4$ ft
22. $a = 30$ in., $b = 50$ in., $h = 24$ in.
23. $a = 96$ cm, $b = 24$ cm, $h = 30$ cm
24. $a = 450$ m, $b = 750$ m, $h = 250$ m
25. The volume of a rectangular solid is given by $V = lwh$, where l is the length, w is the width, and h is the height of the solid. Find V if $l = 25$ cm, $w = 15$ cm, and $h = 12$ cm.
26. Find the volume of a box with dimensions $l = 48$ in., $w = 24$ in., and $h = 96$ in.
27. Given $v = v_0 + gt$, $v_0 = 12$, $g = 32$, and $t = 5$, find v.
28. Given $Q = CV$, $C = 12$, and $V = 2500$, find Q.
29. Given $I = \dfrac{E}{Z}$, $E = 240$, and $Z = 15$, find I.
30. Given $P = I^2R$, $I = 4$, and $R = 2000$, find P.

1.5 Prime Factorization

Divisibility

A number is **divisible** by a second number if, when you divide the first number by the second number, you get a zero remainder. That is, the second number *divides* the first number.

Example 1 12 is divisible by 3, because 3 divides 12. ◆

Example 2 124 is not divisible by 7, because 7 does not divide 124. Check with a calculator. ◆

There are many ways of classifying the positive integers. They can be classified as even or odd, as divisible by 3 or not divisible by 3, as larger than 10 or smaller than 10, and so on. One of the most important classifications involves the concept of a **prime number**: an integer greater than 1 that has no divisors except itself and 1. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

An integer is **even** if it is divisible by 2; that is, if you divide it by 2, you get a zero remainder. An integer is **odd** if it is not divisible by 2.

In multiplying two or more positive integers, the positive integers are called the *factors* of the product. Thus, 2 and 5 are factors of 10, since $2 \times 5 = 10$. The numbers 2, 3, and 5 are factors of 30, since $2 \times 3 \times 5 = 30$. Those prime numbers whose product equals the given integer are called **prime factors**. The process of finding the prime factors of a positive integer is called **prime factorization**. The prime factorization of a given number is the product of its prime factors. That is, each of the factors is prime, and their product equals the given number. One of the most useful applications of prime factorization is in finding the least common denominator (LCD) when adding and subtracting fractions. This application is found in Section 1.7.

Example 3 Factor 28 into prime factors.

a. $28 = 4 \cdot 7$
$= 2 \cdot 2 \cdot 7$

b. $28 = 7 \cdot 4$
$= 7 \cdot 2 \cdot 2$

c. $28 = 2 \cdot 14$
$= 2 \cdot 7 \cdot 2$

In each case, you have three prime factors of 28; one factor is 7, the other two are 2's. The factors may be written in any order, but we usually list all the factors in order from smallest to largest. It would not be correct in the examples above to leave $7 \cdot 4$, $4 \cdot 7$, or $2 \cdot 14$ as factors of 28, since 4 and 14 are not prime numbers. ◆

Short division, a condensed form of long division, is a helpful way to find prime factors. Find a prime number that divides the given number. Divide, using short division. Then find a second prime number that divides the result. Divide, using short division. Keep repeating this process of stacking the quotients and divisors (as shown below) until the final quotient is also prime. The prime factors will be the product of the divisors and the final quotient of the repeated short divisions.

Example 4 Find the prime factorization of 45.

$3\underline{|45}$ Divide by 3.
$3\underline{|15}$ Divide by 3.
$\;\;5$

The prime factorization of 45 is $3 \cdot 3 \cdot 5$. ◆

Example 5 Find the prime factorization of 60.

$2\underline{|60}$ Divide by 2.
$2\underline{|30}$ Divide by 2.
$3\underline{|15}$ Divide by 3.
$\;\;5$

The prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$. ◆

Example 6 Find the prime factorization of 17.

17 has no factors except for itself and 1. Thus, 17 is a prime number. When asked for factors of a prime number, write "prime" as your answer. ◆

Divisibility Tests

To eliminate some of the guesswork involved in finding prime factors, divisibility tests can be used. Such tests determine whether or not a particular positive integer divides another integer without carrying out the division. Divisibility tests and prime factorization are used to reduce fractions to lowest terms and to find the lowest common denominator. (See Unit 1B.)

The following divisibility tests for certain positive integers are most helpful.

Divisibility by 2

If a number ends with an even digit, then the number is divisible by 2.

NOTE: Zero is even.

Example 7 Is 4258 divisible by 2?

Yes; since 8, the last digit of the number, is even, 4258 is divisible by 2. ◆

NOTE: Check each example with a calculator.

Example 8 Is 215,517 divisible by 2?

Since 7 (the last digit) is odd, 215,517 is not divisible by 2. ◆

Divisibility by 3

If the sum of the digits of a number is divisible by 3, then the number itself is divisible by 3.

Example 9 Is 531 divisible by 3?

The sum of the digits $5 + 3 + 1 = 9$. Since 9 is divisible by 3, then 531 is divisible by 3. ◆

Example 10 Is 551 divisible by 3?

The sum of the digits $5 + 5 + 1 = 11$. Since 11 is not divisible by 3, then 551 is not divisible by 3. ◆

Divisibility by 5

If a number has 0 or 5 as its last digit, then the number is divisible by 5.

Example 11 Is 2372 divisible by 5?

The last digit of 2372 is neither 0 nor 5, so 2372 is not divisible by 5. ◆

Example 12 Is 3210 divisible by 5?

The last digit of 3210 is 0, so 3210 is divisible by 5. ◆

Example 13 Find the prime factorization of 204.

2⌊204	Last digit is even.
2⌊102	Last digit is even.
3⌊ 51	Sum of digits is divisible by 3.
17	

The prime factorization of 204 is $2 \cdot 2 \cdot 3 \cdot 17$. ◆

Example 14 Find the prime factorization of 630.

2⌊630	Last digit is even.
3⌊315	Sum of digits is divisible by 3.
3⌊105	Sum of digits is divisible by 3.
5⌊ 35	Last digit is 5.
7	

The prime factorization of 630 is $2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$. ◆

NOTE: As a general rule of thumb:

1. Keep dividing by 2 until the quotient is not even.
2. Keep dividing by 3 until the quotient's sum of digits is not divisible by 3.
3. Keep dividing by 5 until the quotient does not end in 0 or 5.

That is, if you divide out all the factors of 2, 3, and 5, the remaining factors, if any, will be much smaller and easier to work with, and perhaps prime.

NOTE: Some people prefer to use the divisibility tests for 2 and 5 before using the divisibility test for 3.

EXERCISES 1.5

Which numbers are divisible **a.** *by 3 and* **b.** *by 4?*

1. 15 **2.** 28 **3.** 96
4. 172 **5.** 78 **6.** 675

Classify each number as prime or not prime:

7. 53 **8.** 57 **9.** 93
10. 121 **11.** 16 **12.** 123
13. 39 **14.** 87

Test for divisibility by 2:

15. 458 **16.** 12,746 **17.** 315,817
18. 877,778 **19.** 1367 **20.** 1205

Test for divisibility by 3 and check your results with a calculator:

21. 387 **22.** 1254 **23.** 453,128
24. 178,213 **25.** 218,745 **26.** 15,690

Test for divisibility by 5 and check your results with a calculator:

27. 70 **28.** 145 **29.** 366
30. 56,665 **31.** 63,227 **32.** 14,601

Test the divisibility of each first number by the second number:

33. 56; 2 **34.** 42; 3 **35.** 218; 3
36. 375; 5 **37.** 528; 5 **38.** 2184; 3
39. 198; 3 **40.** 2236; 3 **41.** 1,820,670; 2
42. 2,817,638; 2 **43.** 7,215,720; 5 **44.** 5,275,343; 3

Find the prime factorization of each number (use divisibility tests where helpful):

45. 20 **46.** 18 **47.** 66 **48.** 30
49. 36 **50.** 25 **51.** 27 **52.** 59
53. 51 **54.** 56 **55.** 42 **56.** 63
57. 120 **58.** 72 **59.** 171 **60.** 360
61. 105 **62.** 78 **63.** 252 **64.** 444

UNIT 1A REVIEW

1. Add: 33 + 104 + 75 + 29

2. Subtract: 2301 − 506

3. Multiply: 3709 × 731 **4.** Divide: 9300 ÷ 15

5. Josh has the following lengths of 3-inch plastic pipe:

3 pieces 12 ft long
8 pieces 8 ft long
9 pieces 10 ft long
12 pieces 6 ft long

Find the total length of pipe on hand.

6. If one bushel of corn weighs 56 lb, how many bushels are contained in 14,224 lb of corn?

Evaluate each expression:

7. $6 + 2(5 \times 4 - 2)$ **8.** $3^2 + 12 \div 3 - 2 \times 3$

9. $12 + 2[3(8 - 2) - 2(3 + 1)]$

10. In Illustration 1, find the area.

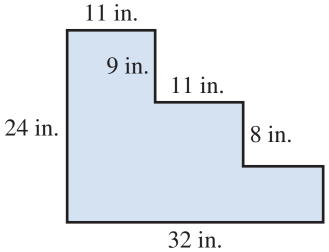


ILLUSTRATION 1

11. In Illustration 2, find the volume.

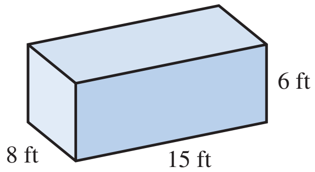


ILLUSTRATION 2

12. If $d = vt$, $v = 45$, and $t = 4$, find d.

13. If $I = \dfrac{E}{R}$, $E = 120$, and $R = 12$, find I.

14. If $A = \dfrac{1}{2}bh$, $b = 40$, and $h = 15$, find A.

Classify each number as prime or not prime:

15. 51 **16.** 47

Test for divisibility of each first number by the second number:

17. 195; 3 **18.** 821; 5

Find the prime factorization of each number:

19. 40 **20.** 135

UNIT 1B Review of Operations with Fractions

1.6 Introduction to Fractions

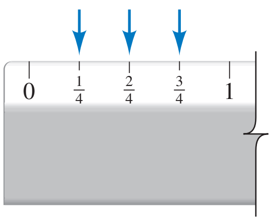


Figure 1.22

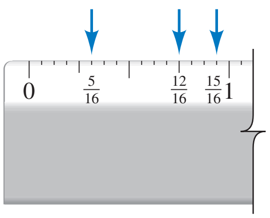


Figure 1.23

The U.S. system of measurement, which is derived from and sometimes called the English system, is a system whose units are often expressed as common fractions and mixed numbers. The metric system of measurement is a system whose units are expressed as decimal fractions and powers of 10. We must feel comfortable working with both systems of measurement and working with both fractions and decimals. The metric system is developed in Chapter 3.

A **common fraction** may be defined as the ratio or quotient of two integers (say, a and b) in the form $\frac{a}{b}$ (where $b \neq 0$). Examples are $\frac{1}{2}$, $\frac{7}{11}$, $\frac{3}{8}$, and $\frac{37}{22}$. The integer below the line is called the *denominator*. It gives the denomination (size) of equal parts into which the fraction unit is divided. The integer above the line is called the *numerator*. It numerates (counts) the number of times the denominator is used. Look at one inch on a ruler, as shown in Figures 1.22 and 1.23.

$\frac{1}{4}$ in. means 1 of 4 equal parts of an inch.

$\frac{2}{4}$ in. means 2 of 4 equal parts of an inch.

$\frac{3}{4}$ in. means 3 of 4 equal parts of an inch.

$\frac{5}{16}$ in. means 5 of 16 equal parts of an inch.

$\frac{12}{16}$ in. means 12 of 16 equal parts of an inch.

$\frac{15}{16}$ in. means 15 of 16 equal parts of an inch.

Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are *equal* or *equivalent* if $ad = bc$. That is, $\frac{a}{b} = \frac{c}{d}$ if $ad = bc$ ($b \neq 0$ and $d \neq 0$). For example, $\frac{2}{4}$ and $\frac{8}{16}$ are names for the same fraction, because $(2)(16) = (4)(8)$. There are many other names for this same fraction, such as $\frac{1}{2}$, $\frac{9}{18}$, $\frac{10}{20}$, $\frac{5}{10}$, $\frac{3}{6}$, and so on.

$\frac{2}{4} = \frac{1}{2}$, because $(2)(2) = (4)(1)$ $\qquad$ $\frac{2}{4} = \frac{5}{10}$, because $(2)(10) = (4)(5)$

Figure 1.24 may help you visualize the relative sizes of fractions. Note that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$, $\frac{7}{8}$ is greater than $\frac{3}{4}$, and $\frac{3}{16}$ is less than $\frac{1}{4}$.

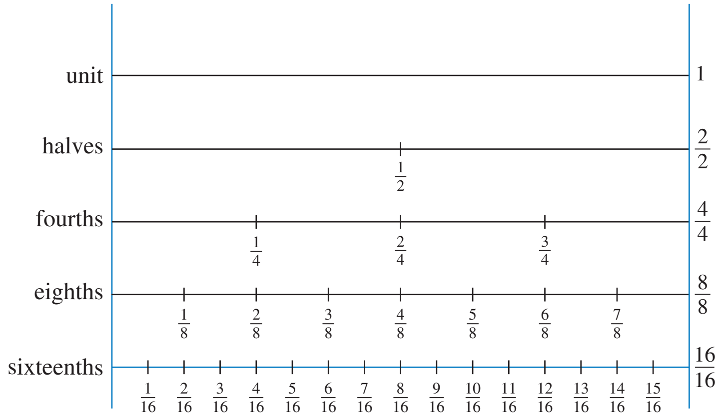


Figure 1.24

Relative sizes of fractions

We have two rules for finding equal (or *equivalent*) fractions:

Equal or Equivalent Fractions

1. The numerator and denominator of any fraction may be *multiplied* by the same number (except zero) without changing the value of the given fraction, thus producing an equivalent fraction. For example, $\frac{4}{9} = \frac{4 \cdot 5}{9 \cdot 5} = \frac{20}{45}$.
2. The numerator and denominator of any fraction may be *divided* by the same number (except zero) without changing the value of the given fraction. For example, $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$.

We use these rules for equivalent fractions (a) to reduce a fraction to lowest terms and (b) to change a fraction to higher terms when adding and subtracting fractions with different denominators.

To *simplify* a fraction means to find an equivalent fraction whose numerator and denominator are *relatively prime*—that is, a fraction whose numerator and denominator have no common divisor. This is also called **reducing a fraction to lowest terms**.

To reduce a fraction to lowest terms, write the prime factorization of the numerator and the denominator. Then divide (cancel) numerator and denominator by any pair of common factors. You may find it helpful to use the divisibility tests in Section 1.5 to write the prime factorizations.

Example 1 Simplify: $\frac{35}{50}$.

$$\frac{35}{50} = \frac{\cancel{5} \cdot 7}{2 \cdot \cancel{5} \cdot 5} = \frac{7}{10}$$

Note the use of the divisibility test for 5. A last digit of 0 or 5 indicates the number is divisible by 5. ◆

Example 2 Simplify: $\frac{63}{99}$.

$$\frac{63}{99} = \frac{\cancel{3} \cdot \cancel{3} \cdot 7}{\cancel{3} \cdot \cancel{3} \cdot 11} = \frac{7}{11}$$

Note the use of the divisibility test for 3 twice. ◆

Example 3 Simplify: $\frac{84}{300}$.

$$\frac{84}{300} = \frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{3}\cdot 7}{\cancel{2}\cdot\cancel{2}\cdot\cancel{3}\cdot 5\cdot 5} = \frac{7}{25}$$

◆

Simplifying Special Fractions

1. Any number (except zero) divided by itself is equal to 1. For example,
$$\frac{3}{3} = 1; \frac{5}{5} = 1; \frac{173}{173} = 1$$
2. Any number divided by 1 is equal to itself. For example,
$$\frac{5}{1} = 5; \frac{9}{1} = 9; \frac{25}{1} = 25$$
3. Zero divided by any number (except zero) is equal to zero. For example,
$$\frac{0}{6} = 0; \frac{0}{13} = 0; \frac{0}{8} = 0$$
4. Any number *divided by zero* is not meaningful and is called *undefined*. For example, $\frac{4}{0}$ is undefined.

A **proper fraction** is a fraction whose numerator is less than its denominator. Examples of proper fractions are $\frac{2}{3}$, $\frac{5}{14}$, and $\frac{3}{8}$. An **improper fraction** is a fraction whose numerator is greater than or equal to its denominator. Examples of improper fractions are $\frac{7}{5}$, $\frac{11}{11}$, and $\frac{9}{4}$.

A **mixed number** is an integer plus a proper fraction. Examples of mixed numbers are $1\frac{3}{4}\left(1 + \frac{3}{4}\right)$, $14\frac{1}{9}$, and $5\frac{2}{15}$.

Changing an Improper Fraction to a Mixed Number

To change an improper fraction to a mixed number, divide the numerator by the denominator. The quotient is the whole-number part. The remainder over the divisor is the proper fraction part of the mixed number.

Example 4 Change $\frac{17}{3}$ to a mixed number.

$$\frac{17}{3} = 17 \div 3 = 3\overline{)17}\ \ 5 \text{ r } 2 = 5\frac{2}{3}$$

◆

Example 5 Change $\frac{78}{7}$ to a mixed number.

$$\frac{78}{7} = 78 \div 7 = 7\overline{)78}\ \ 11 \text{ r } 1 = 11\frac{1}{7}$$

◆

If the improper fraction is not reduced to lowest terms, you may find it easier to reduce it before changing it to a mixed number. Of course, you may reduce the proper fraction after the division if you prefer.

Example 6 Change $\frac{324}{48}$ to a mixed number and simplify.

Method 1: Reduce the improper fraction to lowest terms first.

$$\frac{324}{48} = \frac{\cancel{2}\cdot\cancel{2}\cdot 3\cdot 3\cdot 3\cdot\cancel{3}}{\cancel{2}\cdot\cancel{2}\cdot 2\cdot 2\cdot\cancel{3}} = \frac{27}{4} = \underset{6 \text{ r } 3}{4\overline{)27}} \qquad = 6\frac{3}{4}$$

Method 2: Change the improper fraction to a mixed number first.

$$\frac{324}{48} = \begin{array}{r} 6 \text{ r } 36 \\ 48\overline{)324} \\ \underline{288} \\ 36 \end{array} = 6\frac{36}{48} = 6\frac{\cancel{2}\cdot\cancel{2}\cdot 3\cdot\cancel{3}}{\cancel{2}\cdot\cancel{2}\cdot 2\cdot 2\cdot\cancel{3}} = 6\frac{3}{4}$$ ◆

One way to change a mixed number to an improper fraction is to multiply the integer by the denominator of the fraction and then add the numerator of the fraction. Then place this sum over the original denominator.

Example 7 Change $2\frac{1}{3}$ to an improper fraction.

$$2\frac{1}{3} = \frac{(2 \times 3) + 1}{3} = \frac{7}{3}$$ ◆

Example 8 Change $5\frac{3}{8}$ to an improper fraction.

$$5\frac{3}{8} = \frac{(5 \times 8) + 3}{8} = \frac{43}{8}$$ ◆

Example 9 Change $10\frac{5}{9}$ to an improper fraction.

$$10\frac{5}{9} = \frac{(10 \times 9) + 5}{9} = \frac{95}{9}$$ ◆

A number containing an integer and an improper fraction may be simplified as follows.

Example 10 Change $3\frac{8}{5}$ **a.** to an improper fraction and then **b.** to a mixed number in simplest form.

a. $3\frac{8}{5} = \frac{(3 \times 5) + 8}{5} = \frac{23}{5}$

b. $\frac{23}{5} = 23 \div 5 = \underset{4 \text{ r } 3}{5\overline{)23}} \qquad = 4\frac{3}{5}$

◆

A calculator with a fraction key may be used to simplify fractions as follows. The fraction key often looks similar to $\boxed{A\,{}^{b}/_{c}}$.

Example 11 Reduce $\frac{108}{144}$ to lowest terms.

108 [A b/c] 144 [=]

3/4

Thus, $\frac{108}{144} = \frac{3}{4}$ in lowest terms. ◆

A calculator with a fraction key may be used to change an improper fraction to a mixed number, as follows.

Example 12 Change $\frac{25}{6}$ to a mixed number.

25 [A b/c] 6 [=]

4 1/6

Thus, $\frac{25}{6} = 4\frac{1}{6}$ ◆

A calculator with a fraction key may be used to change a mixed number to an improper fraction as follows. The improper fraction key often looks similar to [d/e].

NOTE: Most scientific calculators are programmed so that several keys will perform more than one function. These calculators have what is called a *second function key*. To access this function, press the second function key first.

Example 13 Change $6\frac{3}{5}$ to an improper fraction.

6 [A b/c] 3 [A b/c] 5 [d/e] [=]

33/5

Thus, $6\frac{3}{5} = \frac{33}{5}$. ◆

EXERCISES 1.6

Simplify:

1. $\frac{12}{28}$	**2.** $\frac{9}{12}$	**3.** $\frac{36}{42}$	**4.** $\frac{12}{18}$	**9.** $\frac{48}{60}$	**10.** $\frac{72}{96}$	**11.** $\frac{9}{9}$	**12.** $\frac{15}{1}$
5. $\frac{9}{48}$	**6.** $\frac{8}{10}$	**7.** $\frac{13}{39}$	**8.** $\frac{24}{36}$	**13.** $\frac{0}{8}$	**14.** $\frac{6}{6}$	**15.** $\frac{9}{0}$	**16.** $\frac{6}{8}$

17. $\frac{14}{16}$ **18.** $\frac{7}{28}$ **19.** $\frac{27}{36}$ **20.** $\frac{15}{18}$

21. $\frac{12}{16}$ **22.** $\frac{9}{18}$ **23.** $\frac{20}{25}$ **24.** $\frac{12}{36}$

25. $\frac{12}{40}$ **26.** $\frac{54}{72}$ **27.** $\frac{112}{128}$ **28.** $\frac{330}{360}$

29. $\frac{112}{144}$ **30.** $\frac{525}{1155}$

Change each fraction to a mixed number in simplest form:

31. $\frac{78}{5}$ **32.** $\frac{11}{4}$ **33.** $\frac{28}{3}$ **34.** $\frac{21}{3}$

35. $\frac{45}{36}$ **36.** $\frac{67}{16}$ **37.** $\frac{57}{6}$ **38.** $\frac{84}{9}$

39. $5\frac{15}{12}$ **40.** $2\frac{70}{16}$

Change each mixed number to an improper fraction:

41. $3\frac{5}{6}$ **42.** $6\frac{3}{4}$ **43.** $2\frac{1}{8}$ **44.** $5\frac{2}{3}$

45. $1\frac{7}{16}$ **46.** $4\frac{1}{2}$ **47.** $6\frac{7}{8}$ **48.** $8\frac{1}{5}$

49. $10\frac{3}{5}$ **50.** $12\frac{5}{6}$

51. Pies in a restaurant are cut into 6 pieces each. Twenty-eight pieces would be equivalent to $\frac{28}{6}$ pies. Change this improper fraction to a mixed number reduced to lowest terms.

52. The chef asks the prep cook to convert each of the following as indicated: **a.** $1\frac{1}{3}$ cups of butter, to an improper fraction. **b.** $\frac{15}{4}$ cups of milk, to a mixed number. **c.** $\frac{3}{2}$ pints, to a mixed number.

1.7 Addition and Subtraction of Fractions

Technicians must be able to compute fractions accurately, because mistakes on the job can be quite costly. Also, many shop drawing dimensions contain fractions.

Adding Fractions

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad (c \neq 0)$$

That is, to add two or more fractions with the same denominator, first add their numerators. Then place this sum over the common denominator and simplify.

Example 1

Add: $\frac{1}{8} + \frac{3}{8}$.

$$\frac{1}{8} + \frac{3}{8} = \frac{1+3}{8} = \frac{4}{8} = \frac{1}{2}$$ Add the numerators and simplify. ◆

Example 2

Add: $\frac{2}{16} + \frac{5}{16}$.

$$\frac{2}{16} + \frac{5}{16} = \frac{2+5}{16} = \frac{7}{16}$$ Add the numerators. ◆

Example 3

Add: $\frac{2}{31} + \frac{7}{31} + \frac{15}{31}$.

$$\frac{2}{31} + \frac{7}{31} + \frac{15}{31} = \frac{2+7+15}{31} = \frac{24}{31}$$ Add the numerators. ◆

To add fractions with different denominators, we first need to find a common denominator. When reducing fractions to lowest terms, we *divide* both numerator and denominator by the same nonzero number, which does not change the value of the fraction. Similarly, we can *multiply* both numerator and denominator by the same nonzero number without changing the value of the fraction. It is customary to find the **least common denominator (LCD)** for fractions with unlike denominators. The LCD is the smallest positive integer that has all the denominators as divisors. Then, multiply both numerator and denominator of each fraction by a number that makes the denominator of the given fraction the same as the LCD.

To find the least common denominator (LCD) of a set of fractions:

1. Factor each denominator into its prime factors.
2. Write each prime factor the number of times it appears *most* in any *one* denominator in Step 1. The LCD is the product of these prime factors.

Example 4 Find the LCD of the following fractions: $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{18}$.

STEP 1 Factor each denominator into prime factors. (Prime factorization may be reviewed in Section 1.5.)

$$6 = 2 \cdot 3$$
$$8 = 2 \cdot 2 \cdot 2$$
$$18 = 2 \cdot 3 \cdot 3$$

STEP 2 Write each prime factor the number of times it appears *most* in any *one* denominator in Step 1. The LCD is the product of these prime factors.

Here, 2 appears once as a factor of 6, three times as a factor of 8, and once as a factor of 18. So 2 appears at most *three* times in any one denominator. Therefore, you have $2 \cdot 2 \cdot 2$ as factors of the LCD. The factor 3 appears at most twice in any one denominator, so you have $3 \cdot 3$ as factors of the LCD. Now 2 and 3 are the only factors of the three given denominators. The LCD for $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{18}$ must be $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72$. Note that 72 does have divisors 6, 8, and 18. This procedure is shown in Table 1.1.

Table 1.1

Prime factor	Denominator	Number of times the prime factor appears: in given denominator	Number of times the prime factor appears: *most* in any one denominator
2	$6 = 2 \cdot 3$	once	
	$8 = 2 \cdot 2 \cdot 2$	three times	three times
	$18 = 2 \cdot 3 \cdot 3$	once	
3	$6 = 2 \cdot 3$	once	
	$8 = 2 \cdot 2 \cdot 2$	none	
	$18 = 2 \cdot 3 \cdot 3$	twice	twice

From the table, we see that the LCD contains the factor 2 three times and the factor 3 two times. Thus, LCD $= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72$. ♦

Example 5 Find the LCD of $\frac{3}{4}$, $\frac{3}{8}$, and $\frac{3}{16}$.

$$4 = 2 \cdot 2$$
$$8 = 2 \cdot 2 \cdot 2$$
$$16 = 2 \cdot 2 \cdot 2 \cdot 2$$

The factor 2 appears at most *four* times in any one denominator, so the LCD is $2 \cdot 2 \cdot 2 \cdot 2 = 16$. Note that 16 does have divisors 4, 8, and 16. ◆

Example 6 Find the LCD of $\frac{2}{5}$, $\frac{4}{15}$, and $\frac{3}{20}$.

$$5 = 5$$
$$15 = 3 \cdot 5$$
$$20 = 2 \cdot 2 \cdot 5$$

The LCD is $2 \cdot 2 \cdot 3 \cdot 5 = 60$. ◆

Of course, if you can find the LCD by inspection, you need not go through the method shown in the examples.

Example 7 Find the LCD of $\frac{3}{8}$ and $\frac{5}{16}$.

By inspection, the LCD is 16, because 16 is the smallest number that has divisors 8 and 16. ◆

After finding the LCD of the fractions you wish to add, change each of the original fractions to a fraction of equal value, with the LCD as its denominator.

Example 8 Add: $\frac{3}{8} + \frac{5}{16}$.

First, find the LCD of $\frac{3}{8}$ and $\frac{5}{16}$. The LCD is 16. Now change $\frac{3}{8}$ to a fraction of equal value with a denominator of 16.

Write: $\frac{3}{8} = \frac{?}{16}$. Think: $8 \times ? = 16$.

Since $8 \times 2 = 16$, we multiply both the numerator and the denominator by 2. The numerator is 6, and the denominator is 16.

$$\frac{3}{8} \times \frac{2}{2} = \frac{6}{16}$$

Now, using the rule for adding fractions, we have

$$\frac{3}{8} + \frac{5}{16} = \frac{6}{16} + \frac{5}{16} = \frac{6+5}{16} = \frac{11}{16}$$ ◆

Now try adding some fractions for which the LCD is more difficult to find.

Example 9 Add: $\frac{3}{4} + \frac{1}{6} + \frac{5}{16} + \frac{7}{12}$.

First, find the LCD.

$$\begin{aligned} 4 &= 2 \cdot 2 \\ 6 &= 2 \cdot 3 \\ 16 &= 2 \cdot 2 \cdot 2 \cdot 2 \\ 12 &= 2 \cdot 2 \cdot 3 \end{aligned}$$

Note that 2 is used as a factor at most four times in any one denominator and 3 as a factor at most once. Thus, the LCD $= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 48$.

Second, change each fraction to an equivalent fraction with 48 as its denominator.

$$\frac{3}{4} = \frac{?}{48} \qquad \frac{3 \times 12}{4 \times 12} = \frac{36}{48}$$

$$\frac{1}{6} = \frac{?}{48} \qquad \frac{1 \times 8}{6 \times 8} = \frac{8}{48}$$

$$\frac{5}{16} = \frac{?}{48} \qquad \frac{5 \times 3}{16 \times 3} = \frac{15}{48}$$

$$\frac{7}{12} = \frac{?}{48} \qquad \frac{7 \times 4}{12 \times 4} = \frac{28}{48}$$

$$\begin{aligned} \frac{3}{4} + \frac{1}{6} + \frac{5}{16} + \frac{7}{12} &= \frac{36}{48} + \frac{8}{48} + \frac{15}{48} + \frac{28}{48} \\ &= \frac{36 + 8 + 15 + 28}{48} \\ &= \frac{87}{48} \end{aligned}$$

Simplifying, we have

$$\frac{87}{48} = 1\frac{39}{48} = 1\frac{\cancel{3} \cdot 13}{\cancel{3} \cdot 16} = 1\frac{13}{16}$$ ◆

Subtracting Fractions

$$\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c} \qquad (c \neq 0)$$

That is, to subtract two (or more) fractions with the same denominator, first subtract their numerators. Then place the difference over the common denominator and simplify.

Example 10 Subtract: $\frac{3}{5} - \frac{2}{5}$.

$$\frac{3}{5} - \frac{2}{5} = \frac{3 - 2}{5} = \frac{1}{5}$$ Subtract the numerators. ◆

Example 11 Subtract: $\frac{5}{8} - \frac{3}{8}$.

$$\frac{5}{8} - \frac{3}{8} = \frac{5 - 3}{8} = \frac{2}{8} = \frac{\cancel{2} \cdot 1}{\cancel{2} \cdot 4} = \frac{1}{4}$$ Subtract the numerators and simplify. ◆

To subtract two fractions that have different denominators, first find the LCD. Then express each fraction as an equivalent fraction using the LCD, and subtract the numerators.

Example 12 Subtract: $\frac{5}{6} - \frac{4}{15}$.

$$\frac{5}{6} - \frac{4}{15} = \frac{25}{30} - \frac{8}{30}$$ First, change the fractions to the LCD, 30.

$$= \frac{25 - 8}{30} = \frac{17}{30}$$ Subtract the numerators. ◆

Adding Mixed Numbers

To add mixed numbers, find the LCD of the fractions. Add the fractions, then add the whole numbers. Finally, add these two results and simplify.

Example 13 Add: $2\frac{1}{2}$ and $3\frac{3}{5}$.

$$2\frac{1}{2} = 2\frac{5}{10}$$ First, change the proper fractions to the LCD, 10.

$$\underline{3\frac{3}{5}} = \underline{3\frac{6}{10}}$$

$$5\frac{11}{10} = 5 + \frac{11}{10} = 5 + 1\frac{1}{10} = 6\frac{1}{10}$$ ◆

Subtracting Mixed Numbers

To subtract mixed numbers, find the LCD of the fractions. Subtract the fractions, then subtract the whole numbers and simplify.

Example 14 Subtract: $8\frac{2}{3}$ from $13\frac{3}{4}$.

$$13\frac{3}{4} = 13\frac{9}{12}$$ First, change the proper fractions to the LCD, 12.

$$\underline{8\frac{2}{3}} = \underline{8\frac{8}{12}}$$

$$5\frac{1}{12}$$ ◆

If the larger of the two mixed numbers does not also have the larger proper fraction, borrow 1 from the whole number. Then add it to the proper fraction before subtracting.

Example 15 Subtract: $2\frac{3}{5}$ from $4\frac{1}{2}$.

$$4\frac{1}{2} = 4\frac{5}{10} = 3\frac{15}{10}$$ First, change the proper fractions to the LCD, 10. Then, borrow 1 from 4 and add $\frac{10}{10}$ to $\frac{5}{10}$.

$$\underline{2\frac{3}{5}} = \underline{2\frac{6}{10}} = \underline{2\frac{6}{10}}$$

$$1\frac{9}{10}$$ ◆

Example 16 Subtract: $2\frac{3}{7}$ from $8\frac{1}{4}$.

$$\begin{aligned} 8\frac{1}{4} &= 8\frac{7}{28} = 7\frac{35}{28} \\ 2\frac{3}{7} &= 2\frac{12}{28} = 2\frac{12}{28} \\ & \qquad\qquad\; 5\frac{23}{28} \end{aligned}$$

First, change the proper fractions to the LCD, 28. Then, borrow 1 from 8 and add $\frac{28}{28}$ to $\frac{7}{28}$. ◆

Example 17 Subtract: $12 - 4\frac{3}{8}$.

$$\begin{aligned} 12 &= 11\frac{8}{8} \\ 4\frac{3}{8} &= \;\;4\frac{3}{8} \\ & \qquad 7\frac{5}{8} \end{aligned}$$

First, borrow 1 from 12. Then, rewrite it in terms of the LCD, 8, as shown. ◆

Applications Involving Addition and Subtraction of Fractions

An electrical circuit with more than one path for the current to flow is called a *parallel circuit*. See Figure 1.25. The current in a parallel circuit is divided among the branches in the circuit. How it is divided depends on the resistance in each branch. Since the current is divided among the branches, the total current (I_T) of the circuit is the same as the current from the source. This equals the sum of the currents through the individual branches of the circuit. That is, $I_T = I_1 + I_2 + I_3 + \cdots$.

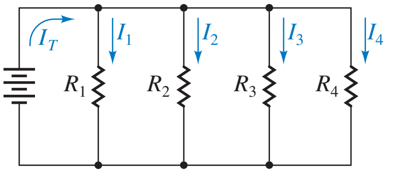


Figure 1.25

Example 18 Find the total current in the parallel circuit in Figure 1.26.

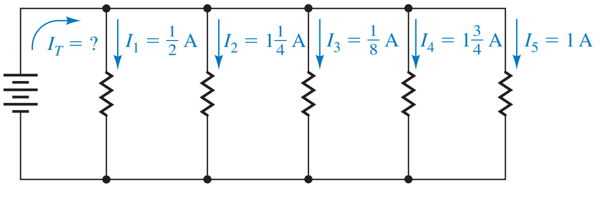


Figure 1.26